GRAPHICS

AND

STRUCTURAL DESIGN

BY

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SECOND EDITION

FIRST THOUSAND

NEW YORK

JOHN WILEY & SONS, Inc.

London: CHAPMAN & HALL, Limited

1915

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE

This text is intended for the author's classes in General Engineering Design in Sibley College, Cornell University. The treatment of the subject has been kept as general as possible and is that used for his students during the past four or five years. The problems chosen for discussion are those on the border line between Civil and Mechanical Engineering. To the general designer knowledge of this subject is indispensable while acquaintance with the methods used in determining the stresses in, and the subsequent designing of, structures is of the greatest benefit to others in all designing for strength whether of machines or structures.

Although the book is called "Graphics and Structural Design" the determination of the stresses has not been confined to graphical means; the other usual methods have been included.

The design of a plate girder may seem out of keeping with the purpose of the book. The author formerly used a crane runway girder instead of the railway bridge, but decided that the railway girders made the more comprehensive and better problem. The design of the latter permitted the use of the moment table and acquainted the student with the usual method of treating locomotive and train loads.

Incidentally, one who could design a railway plate girder was well prepared to design a runway girder, although the reverse was not so generally true.

The author's practice is to use a set of problems paralleling the drawing-room work, to assign a number of these problems each week to the students upon which they recite. Although this reduces slightly the time spent over the drawing boards the work

accomplished by the students is not apparently diminished and the work seems to be generally better understood.

Although intended primarily for a textbook, it is hoped that this may prove a satisfactory reference book for designers whose work is not too highly specialized.

The author desires to acknowledge his indebtedness to many manufacturers, all of whom have been most generous and among whom are American Bridge Co., Pennsylvania Steel Co., Cambria Steel Co., Bethlehem Steel Co., McClintock-Marshall Construction Co., Jeffrey Mfg. Co., Best & Co., and Bucyrus Co.

Among the periodicals he is especially indebted to the Engineering News.

The section on Specifications has been drawn largely from the specifications of the American Railway Engineering and Maintenance of Way Association, the specifications of Mr. C. C. Schneider in the Transactions of the American Society of Civil Engineers, and the specifications of the American Bridge Co.

H. D. HESS.

ITHACA, N. Y.
May, 1913.

PREFACE TO SECOND EDITION

A number of changes have been made, including a more extended treatment of steel mill-buildings and of reinforced concrete, modifications and explanations of the specifications, and other changes due to the advances made in the subject.

H. D. H.

ITHACA, N. Y.
August, 1915.

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CHAPTER I

MATERIALS

THE principal materials used in the structures considered in this book are iron castings, hard, medium and soft steels, steel castings, timber (oak, hemlock, long-leaf Southern and yellow pine, spruce and white pine) and concrete, cement, slag and stone.

Cast Iron is made by remelting pig iron or pig iron and cast-iron scrap in a cupola. The quality of the product is a varying one, as the entire charge in the cupola is never melted at any one time. The quality, therefore, depends upon the time at which it was run off during the heat.

For machining, cast iron should be soft, and should have an ultimate strength in tension of from 16,000 to 20,000 lbs. per sq. in. Although cast iron has no well-defined elastic limit, it may be assumed for practical purposes at 8000 lbs. per sq. in. Cast iron is exceedingly strong in direct compression; when no bending is introduced its ultimate strength in compression should reach 90,000 to 100,000 lbs. per sq. in. The resilience of cast iron being very low indicates little ability to resist shock.

Steel is made by refining pig iron in the Bessemer converter, also by refining pig iron or pig iron and steel scrap in the open-hearth furnace. Small quantities of special steels are also being made in electric furnaces. The refining processes remove the impurities and the carbon, the steel being afterwards recarbonized. The ordinary merchant and structural steels are soft and medium

steels, the carbon contents ranging from 0.08 to 0.30 of 1 per cent. The ultimate strength of steel varies with the percentage of carbon from 48,000 lbs. per sq. in. for very soft or rivet steel to 70,000 lbs. per sq. in. for medium steel. Steel, having great resilience, is peculiarly adapted to resisting shock.

Steel Castings. — This steel is also made in open-hearth furnaces but may be made in small converters. The steel is poured into molds as is done for iron castings. To obtain fluidity it is necessary that steel for castings have a much higher temperature than is required for cast iron. The mold frames must, therefore, be much more substantial and the molds must be thoroughly dried before the metal is poured into them.

Considerable contraction, with the attendant internal stresses, results from the exceedingly high temperature and necessitates the annealing of steel castings. The ultimate strength depends upon the carbon and should be 50,000 or more pounds per square inch. Steel castings have high resilience and are, therefore, superior to iron castings. However, both the material and the machining of steel castings cost more than do those of iron castings.

TABLE OF PHYSICAL PROPERTIES OF METALS

Material.	Modulus of elasticity.		Ultimate strength.		Elastic strength.	
	Tension compression.	Torsion.	Tension.	Shear.	Tension.	Shear.
Cast iron (cupola)	10,700,000 15,000,000	4,000,000 6,000,000	16,000 20,000	16,000 20,000	8,000	8,000
Cast iron (air furnace)	....	....	30,000 40,000	....	....	....
Wrought iron	28,000,000	11,000,000	47,000 57,000	35,000 43,000	....	....
Steel 0.15 carbon	30,000,000	11,800,000	60,000	45,000	40,000	....
Steel 0.25 carbon	30,000,000	11,800,000	70,000	52,000	45,000	....
Crucible steel (high carbon)	31,000,000	12,100,000	100,000	......	75,000	....
Steel castings	30,600,000	11,800,000	50,000 100,000	30,000 60,000	....	....

NOTE. — The ultimate compressive strength of cast iron is 90,000 to 100,000 lbs. per sq. in. Its elastic strength in compression may be taken at 25,000 lbs. per sq. in. The ultimate compressive strengths of the other materials will approximate their ultimate tensile strengths.

Working Fiber Stresses. — In the structures hereinafter described the usual working fiber stresses will be given in each problem. Ordinarily, in structures liable to little or no shock or vibration, where the stresses are fully determined, the maximum fiber stress on mild steel will range from 16,000 to 20,000 lbs. per sq. in. Structures, such as crane frames, liable to some shock will have the maximum stress on mild steel reduced to 11,000 or 12,000 lbs. per sq. in. In the case of highway bridges and similar structures an addition of 25 per cent is added to the live-load stresses to allow for impact, and a unit stress of 15,000 lbs. per sq. in. is allowed on soft steel, while a unit stress of 17,000 lbs. per sq. in. is allowed on medium steel. In the case of railway bridges 15,000 and 17,000 lbs. per sq. in. are used, but the impact allowance is made by a formula similar to that used in the design of the railway plate girder in Chapter XI.

Properties of Timber

Wooden beams are designed similarly to metal ones. Being liable to fail by horizontal shearing they should be examined for this. One-fifth the ultimate shearing resistance given in the tables may be taken as the working shearing strength (lengthwise).

The formula for bending is

$$M = f\frac{I}{e}.$$

M = bending moment in inch pounds.
I = moment of inertia in inches^4.
e = distance from the neutral axis to the extreme fibers in inches.
f = working fiber stress, in flexure, pounds per square inch.

For rectangular timber beams this becomes

$$M = \frac{fbd^2}{6}.$$

b = width of the beam in inches.
d = depth of the beam in inches.

The maximum longitudinal unit shearing stress is $f_s = \frac{3R}{2bd}$,

where f_s = fiber stress in shear, pounds per square inch.
R = end reaction in pounds.

Timber Columns

The following formula is suggested by the United States Government reports on timber.

$$f_c = F_c \times \frac{700 + 15c}{700 + 15c + c^2}.$$

f_c = ultimate compressive strength of the column in pounds per square inch.

F_c = ultimate crushing strength of short timber column in pounds per square inch.

$$c = \frac{l}{d},$$

where

l = length of the column in inches.

d = least diameter in inches.

The factor of safety should vary from 5 for 18 per cent moisture when used in the open to $3\frac{1}{3}$ for 10 per cent or less of moisture when used in heated buildings.

Name of wood.	Pounds per square inch F_c.
White oak, Southern long-leaf pine......	5000
Short-leaf yellow pine (Georgia)........	4500
Hemlock, chestnut and spruce..........	4000
White pine and cedar....................	3500

The properties of concrete will be treated quite fully under Reinforced Concrete; see Chapter XIV.

Bricks. — Bricks will vary in size and properties according to the locality in which they are made. Common bricks may generally be assumed about $8\frac{1}{4}$ ins. $\times$ 4 ins. $\times$ $2\frac{1}{4}$ ins., while face bricks will run $8\frac{3}{8}$ ins. $\times$ $4\frac{1}{8}$ ins. $\times$ $2\frac{1}{4}$ ins. Common bricks will

TABLE OF PROPERTIES OF TIMBERS

Name of wood.	Ultimate resistance to					Elastic limit.	Modulus of elasticity.	Modulus of		Ordinary working stresses.			Weight, pounds per cubic foot.
	Tension.	Compression.		Shear.									
		Length-wise.	Cross-wise.	Length-wise.	Cross-wise.			Ultimate bending.	Elastic bending.	Ten-sion.	Comp.	Bend-ing.	
Ash	17,000	7,200	1,900	1,100	6,280	7,900	1,640,000	10,800	7,900	2,000	1,000	1,200	39
Cedar	......	5,200	700	400	1,370	5,800	910,000	6,300	5,800	1,200	600	800	23
Chestnut	11,500	5,300	......	......	1,530	......	1,140,000	8,100	......	1,400	600	900	41
Douglas spruce	13,000	5,700	800	500	......	6,400	1,680,000	7,900	6,400	1,400	700	1,000	32
Hemlock	8,700	5,700	......	400	2,750	......	......	7,100	......	......	......	750	25
Oak (white)	13,600	8,500	2,200	1,000	4,400	9,600	2,090,000	13,100	9,600	1,700	1,000	1,500	50
Pine (white)	10,000	5,400	700	400	2,500	6,400	1,390,000	7,900	6,400	1,200	700	900	24
Pine (Southern yellow, long-leaf)	13,000	8,000	1,260	835	5,600	10,000	2,070,000	12,600	9,500	1,600	1,000	1,500	38
Pine (Cuban)	13,000	8,700	1,200	770	......	11,100	2,370,000	13,600	10,640	......	......	......	..
Pine (loblolly)	13,000	7,400	1,150	800	......	9,200	2,050,000	11,300	9,400	1,600	900	1,200	33
Spruce (Northern)	11,000	6,000	......	400	3,250	......	1,400,000	8,000	......	1,200	700	900	26
Spruce pine	12,000	7,300	1,200	800	......	8,400	1,640,000	10,000	8,400	1,200	700	900	30

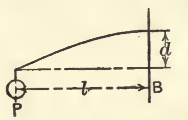
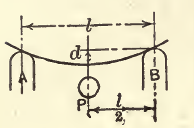
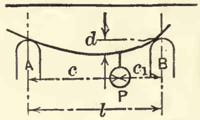
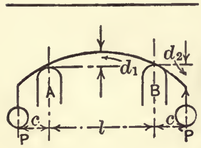
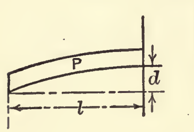
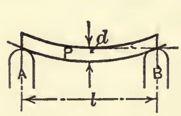
BASIS. — United States Department of Agriculture, Forestry Division, moisture 12 per cent or less.
All quantities are in pounds per square inch.

weigh 125 lbs. per cu. ft., while soft bricks will average 100 lbs. per cu. ft.

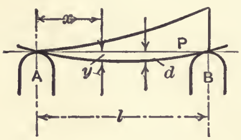
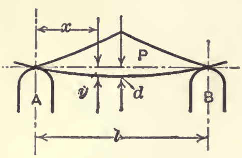
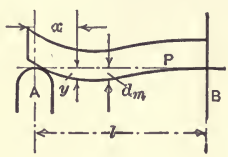
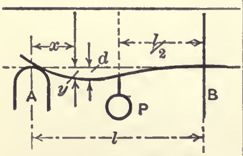
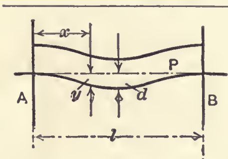
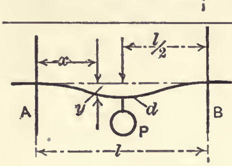
Good bricks should be sound, hard, of regular shape and size, and should have a uniform structure. When struck a sharp blow they should emit a ringing sound. They should not absorb more than $\frac{1}{10}$ their weight of moisture. Their crushing strength should reach 7000 to 10,000 lbs. per sq. in.

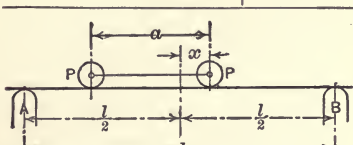
Fire Bricks. — These are bricks made of clay or of a mixture of clay and sand having ability to resist high temperatures. They are used mainly for building furnaces and lining flues and some chimneys.

BENDING MOMENTS, DEFLECTIONS, ETC., FOR BEAMS OF UNIFORM SECTION

Form of support and load.	End reactions A, B. Bending moment M.	Relation between load "P" and moment of resistance.	Maximum deflection.
	$B=P,$ $M=Px,$ $M_{\max}=Pl.$	$P=\frac{fW}{l},$ $W=\frac{Pl}{f}.$	$d=\frac{P}{EI}\cdot\frac{l^3}{3},$ $=\frac{1}{3}\frac{f\cdot l^2}{E.e}.$
	$A=B=\frac{P}{2},$ $M=\frac{Px}{2},$ $M_{\max}=\frac{Pl}{4}.$	$P=4\frac{fW}{l},$ $W=\frac{Pl}{4f}.$	$d=\frac{P}{EI}\cdot\frac{l^3}{48},$ $d=\frac{1}{12}\frac{f\cdot l^2}{E\cdot e}.$
	$A=\frac{Pc^1}{l}$, $B=\frac{Pc}{l},$ $M_{\max}=\frac{Pcc^1}{l}.$	$P=\frac{fWl}{cc^1},$ $W=\frac{Pcc^1}{l.f}.$	$d=\frac{Pcc_1}{27\,EI\,l}$ $\times(c+2c_1)$ $\times\sqrt{3c(c+2c_1)}.$
	$A=B=P,$ Between A & B, $M=P\cdot c=$ const.	$P=\frac{fW}{c},$ $W=\frac{Pc}{f}.$	$d_1=\frac{P}{EI}\frac{l^3}{8}\frac{c}{l},$ $d_1=\frac{1}{8}\frac{f}{E}\frac{l^2}{e},$ $d_2=\frac{P}{EI}\left(\frac{c^3}{3}+\frac{c^2l}{2}\right).$
	$B=P,$ $M=\frac{Px^2}{2l},$ $M_{\max}=\frac{Pl}{2}.$	$P=2\frac{fW}{l},$ $W=\frac{Pl}{2f}.$	$d=\frac{P}{EI}\cdot\frac{l^3}{8},$ $d=\frac{1}{4}\frac{fl^2}{Ee}.$
	$A=B=\frac{P}{2},$ $M=\frac{Px}{2}\left(1-\frac{x}{l}\right),$ $M_{\max}=\frac{Pl}{8}.$	$P=\frac{8fW}{l},$ $W=\frac{Pl}{8f}.$	$d=\frac{P}{EI}\cdot\frac{5\,l^3}{384},$ $d=\frac{5}{48}\frac{fl^2}{Ee}.$

BENDING MOMENTS, DEFLECTIONS, ETC., FOR BEAMS OF UNIFORM SECTION

Form of support.	End reactions A, B. Bending moment.	Relation between load "P" and moment of resistance.	Maximum deflection.
	$A = \frac{1}{3}P,\ B = \frac{2}{3}P,$ $M = \frac{P}{3}x\left(1 - \frac{x^2}{l^2}\right)$ $M_{max} = 0.128\,Pl.$	$P = 7.794\,\frac{fW}{l},$ $W = \frac{Pl}{7.794\,f}.$	$d = 0.01304\,\frac{Pl^3}{EI}.$
	$A = B = \frac{P}{2},$ $M = Px\left(\frac{1}{2} - \frac{2}{3}\frac{x^2}{l^2}\right),$ $M_{max} = \frac{Pl}{6}.$	$P = 6\,\frac{fW}{l},$ $W = \frac{Pl}{6f}.$	$d_{max} = \frac{Pl^3}{60\,EI}$ $= \frac{1}{10}\frac{fl^2}{Ee}$
	$A = \frac{3}{8}P,\ B = \frac{5}{8}P,$ $M = \frac{Px}{2}\left(\frac{3}{4} - \frac{x}{l}\right),$ $M_{max} = \frac{Pl}{8}.$	$P = \frac{8fW}{l},$ $W = \frac{Pl}{8f}.$	$d_{max} = \frac{Pl^3}{185\,EI}$
	$A = \frac{5}{16}P,\ B = \frac{11}{16}P,$ $M_{max} = \frac{3\,Pl}{16}.$	$P = \frac{16}{3}\frac{fW}{l},$ $W = \frac{3}{16}\frac{Pl}{f}.$	$d_{max} = \sqrt{\frac{1}{5}}\,\frac{Pl^3}{48\,EI}.$
	$A = B = \frac{P}{2},$ $M = \frac{Pl}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right),$ $M_{max} = \frac{Pl}{12}.$	$P = \frac{12\,fW}{l},$ $W = \frac{Pl}{12\,f}.$	$d = \frac{Pl^3}{384\,EI}$ $= \frac{1}{32}\frac{fl^2}{Ee}.$
	$A = B = \frac{P}{2},$ $M_{max} = \frac{Pl}{8},$	$P = \frac{8\,fW}{l},$ $W = \frac{Pl}{8f}.$	$d = \frac{Pl^3}{192\,EI}$ $= \frac{fl^2}{24\,Ee}.$



$$A = P\left(1 - \frac{2x}{l} + \frac{a}{l}\right).$$

$$B = P\left(1 + \frac{2x}{l} - \frac{a}{l}\right).$$

The bending moment is a maximum under load when $x = \frac{a}{4}\cdot$

$$M_{\max} = \frac{Pl}{2}\left(1 - \frac{a}{2\,l}\right)^2.$$

The letters used have the following significance and for convenience they should be expressed in the units stated.

A and B = end reactions, pounds.

P = load, pounds.

f = extreme fiber stress, pounds per square inch.

l = span of beam in inches.

e = distance neutral axis to extreme fibers having fiber stress f, inches.

E = modulus of elasticity, pounds per square inch.

c, c_1 and x = portions of span, inches.

W = resistance, $= \frac{I}{e}\cdot$

I = moment of inertia, inches^4.

d = deflection, inches.

Strength of Flat Plates

Nomenclature:

t = thickness of plate in inches.

f_m = maximum fiber stress in plate, pounds per square inch.

E = modulus of elasticity in flexure, pounds per square inch.

c factor according to Grashof or Bach.

P = concentrated load in pounds, usually assumed as acting on a circle whose radius is r_0 ins.

q = uniform load acting on plate, pounds per square inch.

r = radius of circular plate, inches.

b = longer side of rectangular plate, inches.

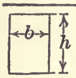
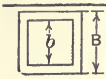
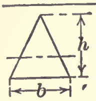
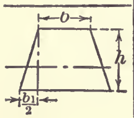
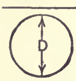
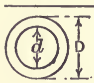
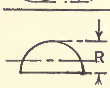
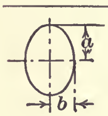
a = shorter side of rectangular plate, inches.

$\frac{b}{a} = \epsilon.$

1. *Circular plates*, carrying uniform load q.

$$f_m = c\frac{r^2}{t^2}q.$$

MOMENTS OF INERTIA, RESISTANCES, CENTERS OF GRAVITY AND LEAST RADII OF GYRATION OF GEOMETRICAL SECTIONS

Shape of section.	Moment of inertia.	Resistance.	Distance base to center of gravity.	Least radius of gyration.
	$\frac{bh^3}{12}$.	$\frac{bh^2}{6}$.	$\frac{h}{2}$.	$\frac{\text{Lesser side}}{3.46}$.
	$\frac{B^4-b^4}{12}$.	$\frac{1}{6}\frac{B^4-b^4}{B}$.	$\frac{B}{2}$.	$\sqrt{\frac{B^2+b^2}{12}}$.
	$\frac{bh^3}{36}$.	$\frac{bh^2}{24}$.	$\frac{h}{3}$.	$\frac{h}{4.24}$
	$\frac{6b^2+6bb_1+b_1^2}{36(2b+b_1)}\cdot h^3$.		$\frac{1}{3}\cdot\frac{3b+b_1}{2b+b_1}\cdot h$.	
	$\frac{D^4}{20.38}$.	$\frac{D^3}{10.19}$.	$\frac{D}{2}$.	$\frac{D}{4}$
	$0.049(D^4-d^4)$.	$0.098\frac{D^4-d^4}{D}$.	$\frac{D}{2}$.	$\frac{1}{4}\sqrt{(D^2+d^2)}$
	$0.11R^4$.	$W_1=0.191R^3$, $W_2=0.259R^3$.	$0.424R$.	$0.07R^2$.
	$0.7854ba^3$.	$0.7854ba^2$.		

Cast-iron plate supported at outer circumference, $c = 0.8$ to 1.2.
Steel plate supported at outer circumference, $c = 0.75$.
Steel plate bolted or riveted at outer circumference, $c = 0.50$.

2. *Circular plates*, carrying a central load P, applied on a circle of radius r_0. Plate supported at outer circumference.

$$f_m = \frac{c}{1.05}\left(1 - \frac{2\,r_0}{3\,r}\right)\frac{P}{t^2}.$$

For cast iron $c = 1.5$.

3. *Circular plates*, central load similar to 2 but plate bolted or riveted at outer circumference.

$$f_m = \frac{P}{t^2}\log\frac{r}{r_0}.$$

Rectangular and Square Plates

Rectangular and square plates supported at the outer edges and carrying a uniform load q.

Rectangular,

$$f_m = c\,\frac{b^2}{2\,t^2} \times \frac{q}{1 + \epsilon^2}.$$

Square,

$$f_m = \frac{c}{4}\frac{a^2}{t^2}\,q.$$

For cast iron $c = 0.75$ to 1.13.
For steel $c = 0.56$ to 0.75 (max. 1.13).

Structural Material

The principal rolled sections used are I beams, channels, angles, plates, flats and rounds. Manufacturers' handbooks afford the best sources of information regarding these sections and in actual design they should be freely consulted. The following data are abridged from these books.

SHEARED PLATES

Width in inches.	Thickness in inches.																
	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$
	Length in inches.																
15	240	320	400	500	500	550	500	475	475	475	425	400	375	360	300	280	280
16	240	320	400	500	500	550	500	475	475	475	425	400	375	360	300	280	280
17	240	320	400	500	500	550	500	475	475	475	425	400	375	360	300	280	280
18	240	360	400	500	500	500	500	550	550	550	500	500	450	400	400	400	350
19	240	360	400	500	500	500	500	550	550	550	500	500	450	400	400	400	350
20	240	360	400	500	500	500	500	550	550	550	500	500	450	400	400	400	350
21	216	360	400	500	525	525	525	550	550	550	500	500	450	400	400	400	350
22	216	360	400	500	525	525	525	550	550	550	500	500	450	400	400	400	350
23	204	360	400	500	525	525	525	550	550	550	500	500	450	400	400	400	350
24	204	360	400	500	525	550	550	550	550	550	500	475	425	400	350	350	325
25	204	360	400	500	525	550	550	550	550	550	500	475	425	400	350	350	325
26	180	360	400	500	525	550	550	550	550	550	500	475	425	400	350	350	325
27	168	340	400	500	500	550	550	500	500	500	450	450	400	380	330	300	300
28	168	340	400	500	500	550	550	500	500	500	450	450	400	380	330	300	300
29	156	340	400	500	500	550	550	500	500	500	450	450	400	380	330	300	300
30– 35	...	320	400	500	500	550	500	475	475	475	425	400	375	360	300	280	280
36– 41	...	360	400	500	500	500	500	550	550	550	500	500	450	400	400	400	350
42– 47	...	360	400	500	525	525	525	550	550	550	500	500	450	400	400	400	350
48– 53	...	360	400	500	525	550	550	550	550	550	500	475	425	400	350	350	325
54– 59	...	340	400	500	500	550	550	500	500	500	450	450	400	380	330	300	300
60– 65	...	320	400	500	500	550	500	475	475	475	425	400	375	360	300	280	280
66– 71	...	300	350	430	450	475	425	425	425	410	375	340	330	320	280	260	260
72– 77	...	260	300	400	425	450	400	400	400	390	350	320	320	300	260	240	240
78– 83	...	240	275	380	400	420	375	375	375	370	325	300	300	300	240	220	220
84– 89	...	200	250	350	375	385	350	350	350	350	300	280	275	275	230	210	210
90– 95	...	180	230	330	340	350	350	325	325	325	275	260	260	260	220	200	200
96–101	...	120	175	240	250	275	275	275	275	275	240	240	220	220	200	180	180
102–107	...	...	150	200	230	230	250	250	250	250	230	230	210	210	190	170	170
108–113	...	...	...	180	180	200	220	225	225	225	220	220	200	200	180	160	160
114–119	...	...	...	...	...	180	200	210	210	210	200	200	180	180	170	150	150
120–125	...	...	...	...	...	120	150	150	180	180	175	175	160	160	160	144	144

EDGED PLATES

Width in inches.	Thickness in inches.													
	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
	Length in feet.													
4	50	50	50	50	50	50	40	40	30	30	30	28	28	28
5	30	42	42	42	42	40	30	30	30	30	30	30	30	30
6	30	42	42	42	42	40	35	30	30	30	30	30	30	30
7	25	42	42	42	42	40	35	30	30	30	30	30	30	30
8	25	42	42	42	42	42	38	36	32	30	29	28	26	25
9	25	42	42	42	42	42	38	34	32	30	29	28	26	25
10	25	42	42	42	42	42	38	33	32	30	29	28	26	25
11	25	42	42	42	42	42	38	33	31	29	28	27	25	24
12	25	42	42	42	42	42	37	32	30	28	27	26	24	23
13	..	42	42	42	42	42	37	32	30	27	25	24	22	20
14	..	42	42	42	42	40	35	30	28	26	25	23	22	20
$14\frac{1}{2}$	..	42	42	42	42	36	33	30	28	25	..	..	..	..

In the tables of angles the areas are given, and the weights in pounds per foot of section can be obtained by multiplying the areas by 3.4.

The moments of inertia and location of the centers of gravity of the angles are given in the tables following.

The radii of gyration are readily found from

$$r = \sqrt{\frac{I}{A}};$$

I = inertia;

A = area in square inches.

All data are based upon dimensions in inches and weights in pounds.

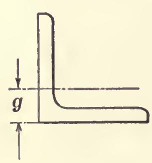
DIMENSIONS AND AREAS OF ANGLES

Uneven legs.	Even legs.	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
....	8 ×8	....	....	....	....	....	....	7.75	8.68	9.61	10.5	11.4	12.3	13.2	14.1	15.0
....	6 ×6	....	....	....	....	4.36	5.06	5.75	6.43	7.11	7.78	8.44	9.09	9.74	10.4	11.0
*7 ×3½	....	....	....	....	....	....	4.40	5.00	5.59	6.17	6.75	7.31	7.87	8.42	8.97	9.50
6 ×4	*5 ×5	....	....	....	....	3.61	4.18	4.75	5.31	5.86	6.41	6.94	7.47	7.99	8.50	9.00
6 ×3½	....	....	....	....	....	3.42	3.97	4.50	5.03	5.55	6.06	6.56	7.06	7.55	8.03	8.50
5 ×4	....	....	....	....	....	3.23	3.75	4.25	4.75	5.23	5.72	6.19	6.65	7.11	.....	.....
5 ×3½	....	....	....	....	2.56	3.05	3.53	4.00	4.47	4.92	5.37	5.81	6.25	6.67	.....	.....
5 ×3	4 ×4	....	....	....	2.40	2.86	3.31	3.75	4.18	4.61	5.03	5.44	5.84	.....	.....	.....
*4 ×3½	....	....	....	....	2.25	2.67	3.09	3.50	3.90	4.30	4.68	5.06	5.43	.....	.....	.....
4 ×3	3½×3½	....	....	....	2.09	2.48	2.87	3.25	3.62	3.98	4.34	4.69	5.03	.....	.....	.....
3½×3	....	....	....	....	1.93	2.30	2.65	3.00	3.34	3.67	4.00	4.31	4.62	.....	.....	.....
3½×2½	3 ×3	....	....	1.44	1.78	2.11	2.43	2.75	3.06	3.36	3.65	.....	.....	.....	.....	.....
3 ×2½	*2¾×2¾	....	....	1.31	1.62	1.92	2.22	2.50	2.78	....	.....	.....	.....	.....	.....	.....
3 ×2	2½×2½	....	....	1.19	1.47	1.73	2.00	2.25	....	....	.....	.....	.....	.....	.....	.....
2½×2	*2¼×2¼	....	0.81	1.06	1.31	1.55	1.78	2.00	....	....	.....	.....	.....	.....	.....	.....
....	2 ×2	....	0.72	0.94	1.15	1.36	1.56	....	....	....	.....	.....	.....	.....	.....	.....
....	1¾×1¾	....	0.62	0.81	1.00	1.17	1.30	....	....	....	.....	.....	.....	.....	.....	.....
....	1½×1½	0.36	0.53	0.69	0.84	0.99	....	....	....	....	.....	.....	.....	.....	.....	.....

* Special sections.

MOMENTS OF INERTIA AND CENTERS OF GRAVITY OF ANGLES

EVEN LEGS

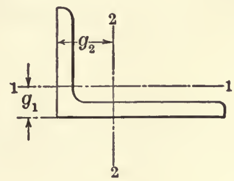


Thickness.	$\frac{1}{4}$		$\frac{3}{8}$		$\frac{1}{2}$		$\frac{5}{8}$		$\frac{3}{4}$		$\frac{7}{8}$		1	
Legs.	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*
8 ×8	....	....	....	....	48.6	2.2	59.4	2.2	69.7	2.3	79.6	2.3	89.0	2.4
6 ×6	....	....	15.4	1.6	19.9	1.7	24.2	1.7	28.2	1.8	31.9	1.8	35.5	1.9
*5 ×5	....	....	8.7	1.4	11.3	1.4	13.6	1.5	....	....	....	....	....	....
4 ×4	....	....	4.4	1.1	5.6	1.2	6.7	1.2	7.7	1.3	8.6	1.3	....	....
3½×3½	....	....	2.9	1.0	3.6	1.1	4.3	1.1	5.0	1.2	5.5	1.2	....	....
3 ×3	1.2	0.84	1.8	0.89	2.2	0.93	2.6	0.98	....	....	....	....	....	....
*2¾×2¾	0.95	0.78	1.3	0.82	1.7	0.87	....	....	....	....	....	....	....	....
2½×2½	0.70	0.72	0.98	0.76	1.2	0.81	....	....	....	....	....	....	....	....
2¼×2¼	0.50	0.65	0.70	0.70	....	....	....	....	....	....	....	....	....	....
2 ×2	0.35	0.59	0.48	0.64	0.59	0.68	....	....	....	....	....	....	....	....

* Special sections.

MOMENTS OF INERTIA AND CENTERS OF GRAVITY OF ANGLES

UNEVEN LEGS

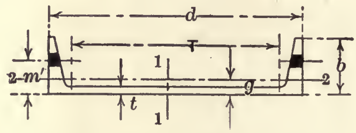


Thickness.	Axis.	1/4		3/8		1/2		5/8		3/4		7/8		1	
Legs.		*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*	*I*	*g*
*7 ×3½	1	....	....	....	....	4.4	0.78	5.3	0.82	6.1	0.87	6.8	0.91	7.5	0.96
	2	....	....	....	....	25.4	2.5	30.9	2.6	36.0	2.6	40.8	2.7	45.4	2.7
6 ×4	1	....	....	4.9	0.94	6.3	0.99	7.5	1.0	8.7	1.1	9.8	1.1	10.7	1.2
	2	....	....	13.5	1.94	17.4	1.99	21.1	2.03	24.5	2.1	27.7	2.1	30.8	2.2
6 ×3½	1	....	....	3.3	0.79	4.3	0.83	5.1	0.88	5.8	0.93	6.6	0.97	7.2	1.0
	2	....	....	12.9	2.0	16.6	2.1	20.1	2.1	23.3	2.2	26.4	2.2	29.2	2.3
*5 ×4	1	....	....	4.7	1.0	6.0	1.1	7.1	1.1	....	....	....	....	....	....
	2	....	....	8.1	1.5	10.5	1.6	12.6	1.6	....	....	....	....	....	....
5 ×3½	1	....	....	3.2	0.86	4.1	0.91	4.8	0.95	5.6	1.0	6.2	1.0	....	....
	2	....	....	7.8	1.6	10.0	1.7	12.0	1.7	13.9	1.7	15.7	1.8	....	....
5 ×3	1	....	....	2.0	0.70	2.6	0.75	3.1	0.80	3.5	0.84	3.9	0.88	....	....
	2	....	....	7.4	1.7	9.5	1.8	11.4	1.8	13.2	1.8	14.8	1.9	....	....
*4 ×3½	1	....	....	3.0	0.96	3.8	1.00	4.5	1.00	....	....	....	....	....	....
	2	....	....	4.2	1.2	5.3	1.2	6.4	1.3	....	....	....	....	....	....
4 ×3	1	....	....	1.9	0.78	2.4	0.83	2.9	0.87	3.3	0.92	3.7	0.96	....	....
	2	....	....	4.0	1.3	5.1	1.3	6.0	1.4	6.9	1.4	7.8	1.5	....	....
3½×3	1	....	....	1.9	0.83	2.3	0.88	2.8	0.92	3.2	0.96	3.5	1.0	....	....
	2	....	....	2.7	1.1	3.5	1.1	4.1	1.2	4.9	1.2	5.2	1.3	....	....
3½×2½	1	0.78	0.61	1.1	0.66	1.4	0.70	1.6	0.75	1.8	0.79	....	....	....	....
	2	1.8	1.1	2.6	1.2	3.2	1.2	3.9	1.3	4.4	1.3	....	....	....	....
3 ×2½	1	0.74	0.66	1.0	0.71	1.3	0.75	1.5	0.79	....	....	....	....	....	....
	2	1.2	0.91	1.7	0.96	2.1	1.0	2.5	1.0	....	....	....	....	....	....
3 ×2	1	0.39	0.49	0.54	0.54	0.67	0.58	....	....	....	....	....	....	....	....
	2	1.1	0.99	1.5	1.0	1.9	1.1	....	....	....	....	....	....	....	....
2½×2	1	0.37	0.54	0.51	0.58	0.64	0.63	....	....	....	....	....	....	....	....
	2	0.65	0.79	0.91	0.83	1.1	0.88	....	....	....	....	....	....	....	....

* Special sections.

The properties of angles of intermediate thickness can be interpolated from the tables with sufficient accuracy for most purposes.

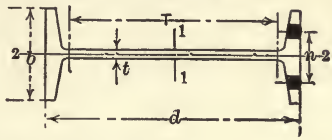
PROPERTIES OF STANDARD CHANNELS



	Weight per foot.	Flange, *b*.	Web, *t*.	Gauge, *m*.	Tangent, *T*.	Max. bolt or rivet.	Moment of inertia, axis 1–1.	Moment of inertia, axis 2–2.	Distance base to c. of g.	Area.	Radius of gyration. Axis 1–1.	Radius of gyration. Axis 2–2.
15	55.00	3.82	0.82	2.50	12.25	$\frac{3}{4}$	430	12.2	0.82	16.18	5.16	0.87
	50.00	3.72	0.72	2.50	12.25		403	11.2	0.80	14.71	5.23	0.87
	45.00	3.62	0.62	2.00	12.25		375	10.3	0.79	13.24	5.32	0.88
	40.00	3.52	0.52	2.00	12.25		348	9.4	0.78	11.76	5.44	0.89
	35.00	3.43	0.43	2.00	12.25		320	8.5	0.79	10.29	5.57	0.91
	33.00	3.40	0.40	2.00	12.25		312	8.2	0.79	9.90	5.62	0.91
12	40.00	3.42	0.76	2.00	10.00	$\frac{3}{4}$	197	6.6	0.72	11.76	4.09	0.75
	35.00	3.30	0.64	2.00	10.00		179	5.9	0.69	10.29	4.17	0.76
	30.00	3.17	0.51	1.75	10.00		162	5.2	0.68	8.82	4.28	0.77
	25.00	3.05	0.39	1.75	10.00		144	4.5	0.68	7.35	4.43	0.78
	20.50	2.94	0.28	1.75	10.00		128	3.9	0.70	6.03	4.61	0.81
10	35.00	3.18	0.82	1.75	8.25	$\frac{3}{4}$	116	4.7	0.69	10.29	3.35	0.67
	30.00	3.04	0.68	1.75	8.25		103	4.0	0.65	8.82	3.42	0.67
	25.00	2.89	0.63	1.75	8.25		91	3.4	0.62	7.35	3.52	0.68
	20.00	2.74	0.38	1.50	8.25		79	2.9	0.61	5.88	3.66	0.70
	15.00	2.60	0.24	1.50	8.25		67	2.3	0.64	4.46	3.87	0.72
9	25.00	2.81	0.61	1.50	7.25	$\frac{3}{4}$	71	3.0	0.62	7.35	3.10	0.64
	20.00	2.65	0.45	1.50	7.25		61	2.5	0.58	5.88	3.21	0.65
	15.00	2.49	0.29	1.38	7.25		51	2.0	0.59	4.41	3.40	0.66
	13.25	2.43	0.23	1.38	7.25		47	1.8	0.61	3.89	3.49	0.67
8	21.25	2.62	0.58	1.50	6.25	$\frac{3}{4}$	48	2.3	0.59	6.25	2.76	0.60
	18.75	2.53	0.49	1.50	6.25		44	2.0	0.57	5.51	2.82	0.60
	16.25	2.44	0.40	1.50	6.25		40	1.8	0.56	4.78	2.89	0.61
	13.75	2.35	0.31	1.38	6.25		36	1.6	0.56	4.04	2.98	0.62
	11.25	2.26	0.22	1.38	6.25		32	1.3	0.58	3.35	3.10	0.63
7	19.75	2.51	0.63	1.50	5.50	$\frac{5}{8}$	33	1.8	0.58	5.81	2.39	0.56
	17.25	2.41	0.53	1.50	5.50		30	1.6	0.55	5.07	2.44	0.56
	14.75	2.30	0.42	1.50	5.50		27	1.4	0.53	4.34	2.50	0.57
	12.25	2.20	0.32	1.25	5.50		24	1.2	0.53	3.60	2.59	0.57
	9.75	2.09	0.21	1.25	5.50		21	0.98	0.55	2.85	2.72	0.59
6	15.50	2.28	0.56	1.25	4.50	$\frac{5}{8}$	19.5	1.3	0.55	4.56	2.07	0.53
	13.00	2.16	0.44	1.25	4.50		17.3	1.1	0.52	3.82	2.13	0.53
	10.50	2.04	0.32	1.25	4.50		15.1	0.88	0.50	3.09	2.21	0.53
	8.00	1.92	0.20	1.13	4.50		13.0	0.70	0.52	2.38	2.34	0.54
5	11.50	2.04	0.48	1.13	3.75	$\frac{1}{2}$	10.4	0.82	0.51	3.38	1.75	0.49
	9.00	1.89	0.33	1.13	3.75		8.9	0.64	0.48	2.65	1.83	0.49
	6.50	1.75	0.19	1.13	3.75		7.4	0.48	0.49	1.95	1.95	0.50
4	7.25	1.73	0.33	1.00	2.75	$\frac{1}{2}$	4.6	0.44	0.46	2.13	1.46	0.46
	6.25	1.65	0.25	1.00	2.75		4.2	0.38	0.46	1.84	1.51	0.45
	5.25	1.58	0.18	1.00	2.75		3.8	0.32	0.46	1.55	1.56	0.45
3	6.00	1.60	0.36	0.88	1.75	$\frac{1}{2}$	2.1	0.31	0.46	1.76	1.08	0.42
	5.00	1.50	0.26	0.88	1.75		1.8	0.25	0.44	1.47	1.12	0.41
	4.00	1.41	0.17	0.88	1.75		1.6	0.20	0.44	1.19	1.17	0.41

Note. — This table is taken from the handbook of the Cambria Steel Co.

PROPERTIES OF STANDARD I BEAMS



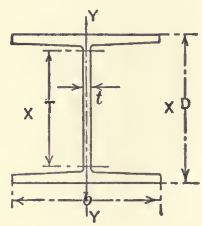
Depth of beam.	Weight per foot.	Area of section.	Thickness of web.	Width of flange.	Moment of inertia, axis 1-1.	Section modulus, axis 1-1.	Radius of gyration, axis 1-1.	Moment of inertia, axis 2-2.	Radius of gyration, axis 2-2.	Max. rivet diam., inches.		
d		A	t	b	I	S	r	I'	r'		n	T
Ins.	Lbs.	Sq. ins.	In.	Ins.	Ins.^{4}	Ins.^{3}	Ins.	Ins.^{4}	In.	Ins.	Ins.	Ins.
3	5.50	1.63	0.17	2.33	2.5	1.7	1.23	0.46	0.53	3/8	1 7/16	1 13/16
3	6.50	1.91	0.26	2.42	2.7	1.8	1.19	0.53	0.52			
3	7.50	2.21	0.36	2.52	2.9	1.9	1.15	0.60	0.52			
4	7.50	2.21	0.19	2.66	6.0	3.0	1.64	0.77	0.59	1/2	1 1/2	2 11/16
4	8.50	2.50	0.26	2.73	6.4	3.2	1.59	0.85	0.58			
4	9.50	2.79	0.34	2.81	6.7	3.4	1.54	0.93	0.58			
4	10.50	3.09	0.41	2.88	7.1	3.6	1.52	1.01	0.57			
5	9.75	2.87	0.21	3.00	12.1	4.8	2.05	1.23	0.65	1/2	1 3/4	3 5/8
5	12.25	3.60	0.36	3.15	13.6	5.4	1.94	1.45	0.63			
5	14.75	4.34	0.50	3.29	15.1	6.1	1.87	1.70	0.63			
6	12.25	3.61	0.23	3.33	21.8	7.3	2.46	1.85	0.72	5/8	2	4 7/16
6	14.75	4.34	0.35	3.45	24.0	8.0	2.35	2.09	0.69			
6	17.25	5.07	0.47	3.57	26.2	8.7	2.27	2.36	0.68			
7	15.00	4.42	0.25	3.66	36.2	10.4	2.86	2.67	0.78	5/8	2 1/4	5 3/8
7	17.50	5.15	0.35	3.76	39.2	11.2	2.76	2.94	0.76			
7	20.00	5.88	0.46	3.87	42.2	12.1	2.68	3.24	0.74			
8	18.00	5.33	0.27	4.00	56.9	14.2	3.27	3.78	0.84	3/4	2 1/4	6 3/16
8	20.25	5.96	0.35	4.08	60.2	15.0	3.18	4.04	0.82			
8	22.75	6.69	0.44	4.17	64.1	16.0	3.10	4.36	0.81			
8	25.25	7.43	0.53	4.26	68.0	17.0	3.03	4.71	0.80			
9	21.00	6.31	0.29	4.33	84.9	18.9	3.67	5.16	0.90	3/4	2 1/2	7 1/16
9	25.00	7.35	0.41	4.45	91.9	20.4	3.54	5.65	0.88			
9	30.00	8.82	0.57	4.61	101.9	22.6	3.40	6.42	0.85			
9	35.00	10.29	0.73	4.77	111.8	24.8	3.30	7.31	0.84			
10	25.00	7.37	0.31	4.66	122.1	24.4	4.07	6.89	0.97	3/4	2 5/8	7 15/16
10	30.00	8.82	0.45	4.80	134.2	26.8	3.90	7.65	0.93			
10	35.00	10.29	0.60	4.95	146.4	29.3	3.77	8.52	0.91			
10	40.00	11.76	0.75	5.10	158.7	31.7	3.67	9.50	0.90			
12	31.50	9.26	0.35	5.00	215.8	36.0	4.83	9.50	1.01	3/4	2 3/4	9 3/4
12	35.00	10.29	0.44	5.09	228.3	38.0	4.71	10.07	0.99			
12	40.00	11.76	0.56	5.21	245.9	41.0	4.57	10.95	0.96			
15	42.00	12.48	0.41	5.50	441.8	58.9	5.95	14.62	1.08	3/4	3	12 1/2
15	45.00	13.24	0.46	5.55	455.8	60.8	5.87	15.09	1.07			
15	50.00	14.71	0.56	5.65	483.4	64.5	5.73	16.04	1.04			
15	55.00	16.18	0.66	5.75	511.0	68.1	5.62	17.06	1.03			
15	60.00	17.65	0.75	5.84	538.6	71.8	5.52	18.17	1.01			
18	55.0	15.93	0.46	6.00	795.6	88.4	7.07	21.19	1.15	0.875	3 1/4	15 3/16
18	60.0	17.65	0.56	6.10	841.8	93.5	6.91	22.38	1.13	1		
18	65.0	19.12	0.64	6.18	881.5	97.9	6.79	23.47	1.11	1		
18	70.0	20.59	0.72	6.26	921.2	102.4	6.69	24.62	1.09	1		
20	65.0	19.08	0.50	6.25	1169.5	117.0	7.83	27.86	1.21	1	3 1/2	16 15/16
20	70.0	20.59	0.58	6.33	1219.8	122.0	7.70	29.04	1.19			
20	75.0	22.06	0.65	6.40	1268.8	126.9	7.58	30.25	1.17			
24	80.0	23.32	0.50	7.00	2087.2	173.9	9.46	42.86	1.36	1	4	20 11/16
24	85.0	25.00	0.57	7.07	2167.8	180.7	9.31	44.35	1.33			
24	90.0	26.47	0.63	7.13	2238.4	186.5	9.20	45.70	1.31			
24	95.0	27.94	0.69	7.19	2309.0	192.4	9.09	47.10	1.30			
24	100.0	29.41	0.75	7.25	2379.6	198.3	8.99	48.55	1.28			

Note. — This table is taken from the handbook of the Cambria Steel Co.

Grey Mill Sections

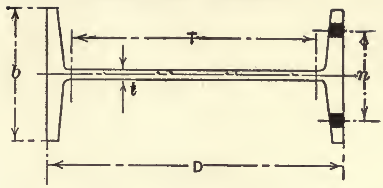
The Bethlehem Steel Company on their Grey Mill are able to roll sections with much wider flanges than are possible with the usual methods of manufacture. These sections make better columns and can be used upon longer spans without lateral bracing. The following tables give some of these sections with their properties. In H columns, under each section-number, only the first three weights and the maximum weight are given although a large number of intermediate weights are rolled. The beam and girder-beam sections are given almost completely.

H COLUMNS



Section No.	Weight per foot.	Area of section.	D	b	t	T	Axis XX. Moment of inertia.	Axis XX. Section modulus.	Axis XX. Radius of gyration, inches.	Axis YY. Moment of inertia.	Axis YY. Section modulus.	Axis YY. Radius of gyration, inches.
	Lbs.	Sq. ins.					I	S	r	I'	S'	r'
H 14	83.5	24.5	$13\frac{3}{4}$	13.9	0.43	11.1	884.9	128.7	6.01	294.5	42.3	3.47
	91.0	26.8	$13\frac{7}{8}$	14.0	0.47	11.1	976.8	140.8	6.04	325.4	46.6	3.49
	99.0	29.1	14	14.0	0.51	11.1	1070.6	153.0	6.07	356.9	51.0	3.50
	287.5	84.5	$16\frac{7}{8}$	14.9	1.41	11.1	3836.1	454.7	6.74	1226.7	164.7	3.81
H 12	64.5	19.0	$11\frac{3}{4}$	11.9	0.39	9.2	499.0	84.9	5.13	168.6	28.3	2.98
	71.5	21.0	$11\frac{7}{8}$	12.0	0.43	9.2	556.6	93.7	5.15	188.2	31.5	3.00
	78.0	22.9	12	12.0	0.47	9.2	615.6	102.6	5.18	208.1	34.7	3.01
	161.0	47.3	$13\frac{1}{2}$	12.5	0.94	9.2	1444.3	214.0	5.53	477.0	76.5	3.18
H 10	49.0	14.4	$9\frac{7}{8}$	10.0	0.36	7.7	263.5	53.4	4.28	89.1	17.9	2.49
	54.0	15.9	10	10.0	0.39	7.7	296.8	59.4	4.32	100.4	20.1	2.51
	59.5	17.6	$10\frac{1}{8}$	10.0	0.43	7.7	331.9	65.6	4.35	112.2	22.3	2.53
	123.5	36.3	$11\frac{1}{2}$	10.5	0.86	7.7	790.4	137.5	4.67	259.3	49.5	2.67
H 8	31.5	9.2	$7\frac{7}{8}$	8.0	0.31	6.1	105.7	26.9	3.40	35.8	8.9	1.98
	34.5	10.2	8	8.0	0.31	6.1	121.5	30.4	3.46	41.1	10.3	2.01
	39.0	11.5	$8\frac{1}{8}$	8.0	0.35	6.1	139.5	34.3	3.48	47.2	11.7	2.03
	92.5	26.6	$9\frac{1}{2}$	8.5	0.78	6.1	385.3	81.1	3.80	125.1	29.6	2.17

GIRDER BEAMS



Depth of beam, inches.	Weight per foot, pounds.	Area of section, square inches.	Thickness of web, inches.	Width of flange, inches.	Neutral axis perpendicular to web at center.			Neutral axis coincident with center line of web.		Maximum rivet diam., inches.		
					Moment of inertia.	Radius of gyration.	Section modulus.	Moment of inertia.	Radius of gyration.			
D			t	b	I	r	$\frac{I}{c}$	I'	r'		n	T
30	200.0	58.71	0.750	15.00	9150.6	12.48	610.0	630.2	3.28	1	11.00	25.2
30	180.0	53.00	0.690	13.00	8194.5	12.43	546.3	433.3	2.86	1	9.00	25.2
28	180.0	52.86	0.690	14.35	7264.7	11.72	518.9	533.3	3.18	1	10.25	23.4
28	165.0	48.47	0.660	12.50	6562.7	11.64	468.8	371.9	2.77	1	8.50	23.4
26	160.0	46.91	0.630	13.60	5620.8	10.95	432.4	435.7	3.05	1	9.50	21.6
26	150.0	43.94	0.630	12.00	5153.9	10.83	396.5	314.6	2.68	1	8.00	21.6
24	140.0	41.16	0.600	13.00	4201.4	10.10	350.1	346.9	2.90	1	9.00	20.0
24	120.0	35.38	0.530	12.00	3607.3	10.10	300.6	249.4	2.66	1	8.00	20.3
20	140.0	41.19	0.640	12.50	2934.7	8.44	293.5	348.9	2.91	1	8.50	15.7
20	112.0	32.81	0.550	12.00	2342.1	8.45	234.2	239.3	2.70	1	8.00	16.4
18	92.0	27.12	0.480	11.50	1591.4	7.66	176.8	182.6	2.59	1	7.50	14.8
15	140.0	41.27	0.800	11.75	1592.7	6.21	212.4	331.0	2.83	1	7.75	10.1
15	104.0	30.50	0.600	11.25	1220.1	6.32	162.7	213.0	2.64	1	7.25	11.1
15	73.0	21.49	0.430	10.50	883.4	6.41	117.8	123.2	2.39	1	6.50	12.1
12	70.0	20.58	0.460	10.00	538.8	5.12	89.8	114.7	2.36	1	6.00	9.0
12	55.0	16.18	0.370	9.75	432.0	5.17	72.0	81.1	2.24	1	6.00	9.5
10	44.0	12.95	0.310	9.00	244.2	4.34	48.8	57.3	2.10	$\frac{7}{8}$	5.50	7.8
9	38.0	11.22	0.300	8.50	170.9	3.90	38.0	44.1	1.98	$\frac{7}{8}$	5.25	9.9
8	32.5	9.54	0.290	8.00	114.4	3.46	28.6	32.9	1.86	$\frac{7}{8}$	5.00	6.0

BEAMS

D	Weight	Area	t	b	I	r	$\frac{I}{c}$	I'	r'	Rivet	n	T
30	120.0	35.3	0.54	10.5	5239.0	12.2	349.0	165.0	2.2	1	6.50	26.4
28	105.0	30.9	0.50	10.0	4014.0	11.4	287.0	131.0	2.1	1	6.00	24.7
26	90.0	26.5	0.46	9.5	2977.0	10.6	229.0	101.0	1.9	1	5.50	23.0
24	84.0	24.8	0.46	9.3	2382.0	9.8	198.0	91.0	1.9	$\frac{7}{8}$	5.25	21.0
24	73.0	21.5	0.39	9.0	2091.0	9.9	174.0	74.0	1.9	$\frac{7}{8}$	5.25	21.3
20	82.0	24.2	0.57	8.9	1560.0	8.0	156.0	80.0	1.8	$\frac{7}{8}$	5.00	17.1
20	72.0	21.4	0.43	8.7	1466.0	8.3	146.0	76.0	1.9	$\frac{7}{8}$	5.00	17.1
20	69.0	20.3	0.52	8.1	1269.0	7.9	127.0	51.0	1.6	$\frac{7}{8}$	4.50	17.5
20	64.0	18.9	0.45	8.1	1222.0	8.0	122.0	50.0	1.6	$\frac{7}{8}$	4.50	17.5
20	59.0	17.4	0.38	8.0	1172.0	8.2	117.0	48.0	1.7	$\frac{7}{8}$	4.50	17.5
18	59.0	17.4	0.50	7.7	883.0	7.1	98.0	39.0	1.5	$\frac{7}{8}$	4.25	15.7
18	54.0	15.9	0.41	7.6	842.0	7.3	94.0	38.0	1.5	$\frac{7}{8}$	4.25	15.7
18	52.0	15.2	0.38	7.6	825.0	7.4	92.0	37.0	1.6	$\frac{7}{8}$	4.25	15.7
18	48.5	14.2	0.32	7.5	798.0	7.5	89.0	36.0	1.6	$\frac{7}{8}$	4.25	15.7
15	71.0	20.9	0.52	7.5	796.0	6.2	106.0	61.0	1.7	$\frac{7}{8}$	4.25	11.7
15	64.0	18.8	0.60	7.2	665.0	6.0	89.0	42.0	1.5	$\frac{7}{8}$	4.00	12.3
15	54.0	15.9	0.41	7.0	610.0	6.2	81.0	38.0	1.6	$\frac{7}{8}$	4.00	12.3
15	46.0	13.5	0.44	6.8	485.0	6.0	65.0	25.0	1.4	$\frac{7}{8}$	3.75	12.9
15	41.0	12.0	0.34	6.7	457.0	6.2	61.0	24.0	1.4	$\frac{7}{8}$	3.75	12.9
15	38.0	11.3	0.29	6.7	443.0	6.3	59.0	23.0	1.4	$\frac{7}{8}$	3.75	12.9
12	36.0	10.6	0.31	6.3	269.0	5.0	45.0	21.0	1.4	$\frac{3}{4}$	3.50	9.9
12	32.0	9.4	0.33	6.2	228.0	4.9	38.0	16.0	1.3	$\frac{3}{4}$	3.50	10.2
12	28.5	8.4	0.25	6.1	216.0	5.1	36.0	15.0	1.3	$\frac{3}{4}$	3.50	10.2
10	28.5	8.3	0.39	6.0	134.0	4.0	27.0	12.0	1.2	$\frac{3}{4}$	3.25	8.4
10	23.5	6.9	0.25	5.9	123.0	4.2	25.0	11.0	1.3	$\frac{3}{4}$	3.25	8.4
9	24.0	7.0	0.36	5.6	92.0	3.6	20.0	9.0	1.1	$\frac{3}{4}$	3.00	7.5
9	20.0	6.0	0.25	5.4	85.0	3.8	19.0	8.0	1.2	$\frac{3}{4}$	3.00	7.5
8	19.5	5.8	0.32	5.3	61.0	3.2	15.0	7.0	1.1	$\frac{3}{4}$	2.75	6.6
8	17.5	5.2	0.25	5.2	57.0	3.3	14.0	6.0	1.1	$\frac{3}{4}$	2.75	6.6

CHAPTER II

GRAPHICS

Statics. — Statics treats of forces at rest and therefore in equilibrium, hence the resultant force in any direction and the moments of the forces about any point must be zero. For coplanar forces the condition of equilibrium is expressed in the following equations:

$$\Sigma \text{ horizontal forces} = 0,$$
$$\Sigma \text{ vertical forces} = 0,$$
$$\Sigma \text{ moment of forces about any point} = 0.$$

Graphic Statics. — Graphic statics relates to the solution of statical problems by geometrical constructions.

Force. — Force is an action upon a body tending to change its state of rest or motion. A force is completely known when its magnitude, direction, line of action and point of application are known.

Magnitude. — Forces may be measured by any unit of weight. It is most convenient in this work to use the pound. The magnitude of a force can be represented graphically by the length of a line, the length being drawn to a previously chosen scale, for instance, if the scale is 1 inch to 1000 pounds then a line $\frac{1}{2}$-inch long represents a force of 500 pounds.

Direction. — An arrow placed on the line is used to indicate the direction of the force.

Line of Action. — The line of action is along the line representing the force and it is along this line that the force tends to produce motion.

Point of Application. — The point of application is the place assumed as a point where the force acts upon the body.

Coplanar and Noncoplanar Forces. — Coplanar forces have their lines of action in a common plane, while the lines of action of noncoplanar forces do not lie in the same plane. Where not otherwise stated coplanar forces are to be understood.

Concurrent and Nonconcurrent Forces. — Concurrent forces have their lines of action intersect in a point, while the lines of action of nonconcurrent forces fail to meet in a common point.

Equilibrium. — A number of forces are in equilibrium when they produce no tendency to motion in the body upon which they act.

Couple. — Two equal and parallel but opposite forces constitute a couple. The arm of the couple is the perpendicular distance between their two lines of action.

Moment of a Couple. — The moment of a couple is the product of either force by the arm of the couple.

Resultant. — The resultant of a system of forces is the single force or simplest system that would produce the same effect as the other forces and could, therefore, replace them.

Equilibrant. — The equilibrant is equal in magnitude to the resultant but opposite to it in direction. It is, therefore, the single force or the simplest system that will exactly neutralize the given system of forces.

Let the force F_1 acting on the point d be represented in direction and magnitude by the line ad, and similarly the force F_2, also acting on d, be represented by dc, then completing the parallelogram and drawing the diagonal db gives R, the magnitude and line of action of the resultant. Its direction, to produce the same effect as the two forces F_1 and F_2, must be as indicated by the arrow.

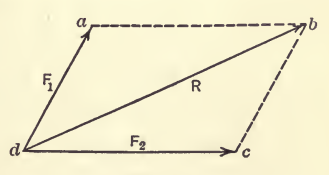


FIG. 1.

Force Triangle. — The triangle dbc would have given the resultant force as well as the parallelogram. Suppose the two known forces are laid off so that the arrows run in the same

direction, that is either clockwise or counterclockwise; then the line closing the triangle with its arrow following around in the same direction as the other two arrows, here counterclockwise, will be equal to the resultant but opposite in direction; hence, E is the force which would hold the two forces F_1 and F_2 in equilibrium and is their *equilibrant.*

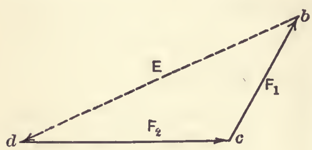


FIG. 2.

Force Polygon.—If more than three forces meet at a point and are in equilibrium a force polygon may be constructed by finding the resultant of two of the forces, combining this resultant with a third force to find a second resultant, and so on until all the forces have been considered and a final resultant determined.

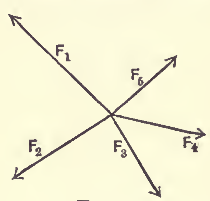


FIG. 3.

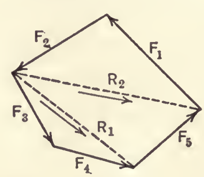


FIG. 4.

R_1 is the resultant of F_3 and F_4; R_2 is the resultant of R_1 and F_5, and F_2 is the equilibrant of the forces F_1, F_5, F_4 and F_3.

It is evident that in the construction the drawing of the resultants R_1 and R_2 might have been dispensed with, it only being necessary to lay the forces off so that the arrows would follow in the same direction. It follows then that any number of concurrent coplanar forces will be in equilibrium if their force polygon closes and that any side of the force polygon represents the equilibrant of all the other forces.

Equilibrium of Coplanar, Nonconcurrent Forces. — In the case of coplanar nonconcurrent forces the closing of the force polygon is not alone sufficient proof of equilibrium. This will be seen from the following simple example. The three forces F_1, F_2 and F_3 are equal and act in the same plane making an angle of

120 degrees with each other. It is evident that, being an equilateral triangle, the force polygon closes. For the system to be in equilibrium, however, the equilibrant of F_2 and F_3 should coincide with E. It is evident that the equilibrant of the system would be a clockwise moment $F_1 \times a$. Hence, when a system of nonconcurrent forces is in equilibrium the force

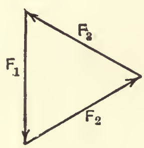


Fig. 5.

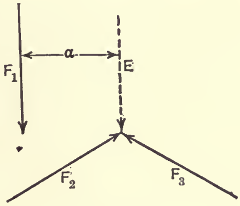


Fig. 6.

polygon must close and the sum of the moments of the forces must be zero. It follows that three nonconcurrent forces cannot be in equilibrium unless the forces are parallel and that the resultant of the group of nonconcurrent forces may be either a single force or a couple.

Components. — In a system of forces having a resultant each force is a component of the resultant. Hence, a force may have any number of components.

Composition of Forces. — Composition of forces consists in finding for a system of forces an equivalent system having a smaller number of forces. The simplest case of composition of forces is finding a single force replacing a system of forces.

Resolution of Forces. — Resolution of forces consists in finding for a system of forces an equivalent system having a greater number of forces. An illustration of this is where a given force is resolved into two or more forces or components.

Equilibrium Polygon

A system of forces is shown in Fig. 7. Fig. 8 is the force polygon. F_1 and F_2 intersect and the line of action of their resultant must pass through their point of intersection and be

parallel to a, the resultant of F_1 and F_2 in the force polygon. Similarly, the resultant b of a and F_3 must pass through the point of intersection of a and F_3 in Fig. 7, and be parallel to b in the force polygon, Fig. 8. In this way we finally reach R, the resultant of F_4, with b, the resultant of all preceding forces F_1, F_2 and

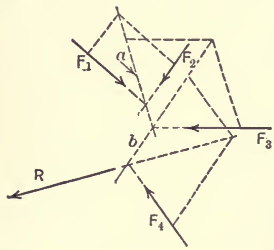


FIG. 7.

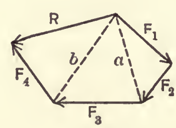


FIG. 8.

F_3. In Fig. 7, R is located by the fact that it must pass through the intersection of F_4 and b. It must also be equal and parallel to R in Fig. 8.

Figure 10 is an equilibrium or funicular polygon.

In the above system the force polygon, Fig. 9, closes but F_5 does not coincide with the resultant R of the other forces

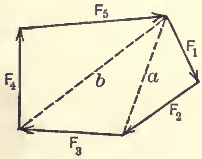


FIG. 9.

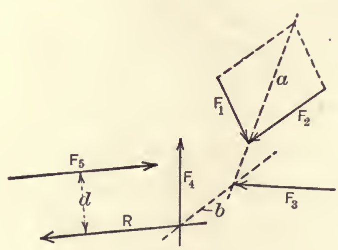


FIG. 10.

F_1, F_2, F_3 and F_4, in the equilibrium polygon, hence, the system lacks equilibrium due to the clockwise moment $F_5 \times d$. The system is in equilibrium for translation. The resultant of the system is $F_5 \times d$. The equilibrant of the system is $- F_5 \times d$.

The method just given applies when the several forces can be conveniently made to intersect. It should be noticed that an infinite number of equilibrium polygons are possible for a given system of forces.

When the forces do not intersect conveniently the equilibrium polygon is drawn as follows:

Take any point O, called a pole, outside of the forces F_1, F_2 and F_3 and connect the extremities of these forces with the lines called rays, a, b, c and d. The force F_1 will be held in equilibrium by the two rays a and b, acting as indicated by the arrows inside the triangle formed by the force F_1 and the rays a and b. In the same way b and c hold F_2, and c and d hold F_3 in equilibrium, also R, the resultant of F_1, F_2 and F_3, is held in

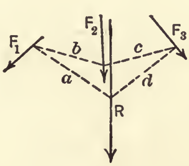


FIG. 11.

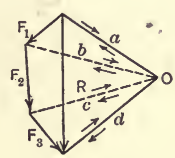


FIG. 12.

equilibrium by the rays a and d. Now if two forces represented by the rays a and b hold F_1 in equilibrium, then in the equilibrium polygon these three forces must intersect in a point. Take any point on F_1 in Fig. 11, and through it draw the lines called strings parallel respectively to b and a in Fig. 12. F_2 is held in equilibrium by the rays b and c; therefore, in Fig. 11, produce b until it intersects the force F_2 and through this point of intersection draw the string c parallel to the ray c, Fig. 12. Through the point of intersection of string c and force F_3 in Fig. 11 draw the string d parallel to ray d in Fig. 12 and produce it until it cuts string a. Now, since the resultant R, Fig. 12, is held in equilibrium by the rays a and d, these three forces R, a and d must intersect in a point in Fig. 11. The intersection of a and d locates R in Fig. 11, its magnitude and direction being given by Fig. 12.

Figure 12 is the force polygon. Fig. 11 is the equilibrium polygon. The difference between these two polygons should be carefully noted. The force polygon gives the direction and magnitude of the forces while the equilibrium polygon gives the lines of action and the direction but not the magnitude.

Graphic Moments

Let F_1, F_2, F_3 and F_4 be four forces constituting a system whose bending moment is desired about a point p. Draw the force

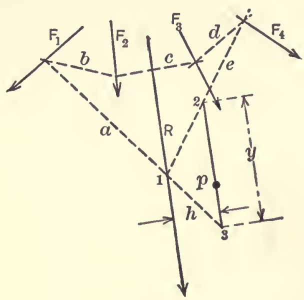


Fig. 13.

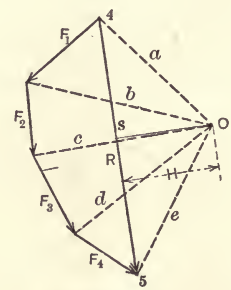


Fig. 14.

polygon (Fig. 14), take a pole O, draw the several rays and draw the pole distance Os perpendicular to the resultant R; this distance is H. In Fig. 13 draw the equilibrium polygon making the strings parallel to the rays in Fig. 14. Through the intersection of strings a and e draw the resultant R. Through the point p draw y parallel to R and limited by the string e and the string a produced. The bending moment of R about the point p is $R \times h$. The triangles 1 2 3 and 4 O 5 are similar, hence

$$R : H :: y : h \quad \text{or} \quad R \times h = H \times y.$$

That is, the bending moment of any system of coplanar forces about any point in the plane is the product of the pole distance H, and the line y, drawn through the point p, parallel to the resultant of the forces R and limited by the two strings e and a which intersect on the resultant.

Moment of Parallel Forces. — The equilibrium polygon may be used to determine the moment of any or all of a system of parallel forces.

The bending moment on the beam at the section y–y due to the forces R_1 and F_1 at the left of that section is equal to 1–2,

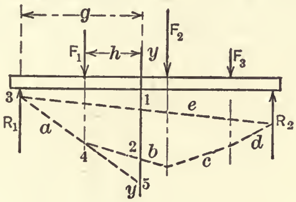


Fig. 15.

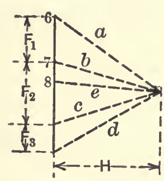


Fig. 16.

the intercept in the equilibrium polygon multiplied by the pole distance H. Expressed algebraically the bending at y–y equals

$$M = (R_1 \times g) - (F_1 \times h). \tag{1}$$

Now triangles 135 and 680 are similar, hence 1–5 : g :: 6–8 : H. (2)

Also triangles 245 and 670 are similar, so that 2–5 : h :: 6–7 : H. (3)

In equation (2), 6–8 $= R_1$, therefore $R_1 \times g =$ 1–5 $\times H$.
In equation (3), 6–7 $= F_1$, therefore $F_1 \times h =$ 2–5 $\times H$, and

$$M = (1\text{–}5 \times H) - (2\text{–}5 \times H) = 1\text{–}2 \times H.$$

The bending moment at any point on a beam due to a system of parallel forces is equal to the ordinate of the equilibrium polygon, cut off by a line drawn through the given point and parallel to the resultant of all the forces multiplied by the pole distance. By bending moment on any beam section is meant the algebraic sum of the moments to the left of that section. The intercept in the equilibrium polygon is a distance and should be measured to the same scale as that to which the beam is laid off. The pole distance is a force and should be measured by the scale used in the force polygon.

USES OF FORCE AND EQUILIBRIUM POLYGONS

Reactions of a Beam. — A supported beam of span L carries a load F a distance a from the left support.

Lay off F in Fig. 18, take the pole O and draw the rays a and b. Through any point 2 on line of action of load F in Fig. 17 draw the two strings parallel to the rays a and b. Through the points 1 and 3 where the rays a and b respectively cut the lines of the

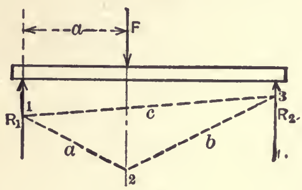


FIG. 17.

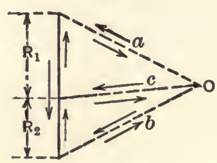


FIG. 18.

reactions R_1 and R_2 draw the string c. Now in Fig. 18 draw the ray c through the pole O and parallel to the string c in Fig. 17. This ray c cuts the force F into two parts representing the two reactions, R_1 being held in equilibrium by the rays a and c and R_2 by the rays c and b.

Problem. — In Fig. 19 find the reactions when

$$F_1 = 10 \text{ tons } (20{,}000 \text{ lbs.}).$$
$$F_2 = 2 \text{ tons } (4000 \text{ lbs.}).$$

Answer. — $R_1 = 9300$ lbs. $\quad R_2 = 14{,}700$ lbs.

Problem. — Draw force and equilibrium polygons for the beam in Fig. 20, finding reactions R_1 and R_2 and the maximum bending moment. Check the answers by algebraic calculations.

Problem. — Draw force and equilibrium polygons for the beam in Fig. 21, find the reactions R_1 and R_2 and the maximum bending moment. Check the answers by algebraic calculations.

In treating uniform loads graphically it is necessary to divide the load into small sections and the load corresponding to each section is considered at its center. See Figs. 22 and 23.

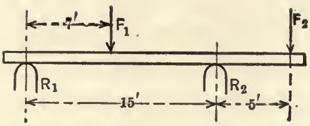


FIG. 19.

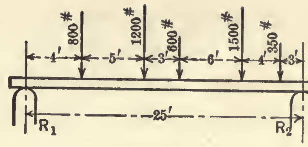


FIG. 20.

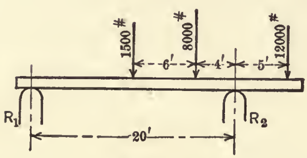


FIG. 21.

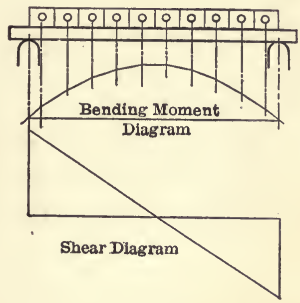


FIG. 22.

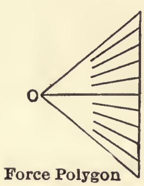


FIG. 23.

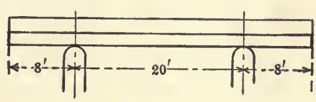
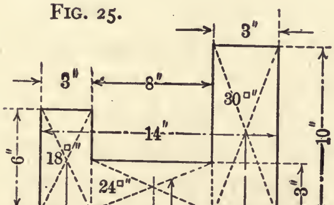


FIG. 24.

FIG. 25.



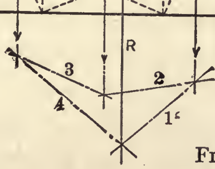


FIG. 27.

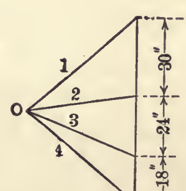


FIG. 26.

Problem. — The beam shown in Fig. 24 carries a uniform load of 1000 lbs. per foot of length; draw the shear and bending-moment diagrams, using the force and equilibrium polygons.

Determination of the Center of Gravity. — Divide Fig. 25 into convenient regular sections, in this case three rectangles. Assume forces acting through the centers of gravity of these rectangles representing their areas. Then, by means of force and equilibrium polygons (Figs. 26 and 27), locate the resultant or equilibrant of these three forces. The location of the vertical equilibrant is shown.

Similar treatment of these areas taken as acting horizontally will give the horizontal equilibrant and the center of gravity of the figure will lie at the intersection of these two equilibrants.

Deflection of Beams

For beams carrying irregular loadings the graphical determination of deflections is convenient. Books on mechanics of materials deduce the general equation of the elastic curve of beams as $\frac{d^2y}{dx^2} = \frac{M}{EI}$. M is the bending moment in inch pounds.

If the equilibrium polygon representing the bending moment upon the beam is known the curve of its deflection may be drawn in the following way:

The given bending-moment diagram is divided into sections and the areas of these sections are represented by the lines A, B, C, etc., in Figs. 28 (a) and (d). In the funicular polygon, the pole O is taken a distance to represent EI, E being the modulus of elasticity of the material of the beam, expressed in pounds per square inch, and I the moment of inertia of the section, in inches^4, and referred to that axis of the beam about which the deflection occurs. Fig. 28 (c) is an equilibrium polygon drawn in the usual way for the two Figs. 28 (a) and 28 (d). The intercepts y in the diagram, when properly scaled, give the deflection at that point on the beam. To determine the scale if 1 inch measured horizontally in Fig. 28 (a) represents s inches of span, 1 inch in

Fig. 28 (b) equals P pounds, H the pole distance in Fig. 28 (b) measured in inches, H^1 the pole distance of Fig. 28 (d) also measured in inches, further in Fig. 28 (d) 1 inch equals k square inches of the bending moment area of Fig. 28 (a). Then 1 inch of intercept in Fig. 28 (c) represents $\frac{k \cdot P \cdot s^3 \cdot H \cdot H^1}{E \cdot I}$ inches.

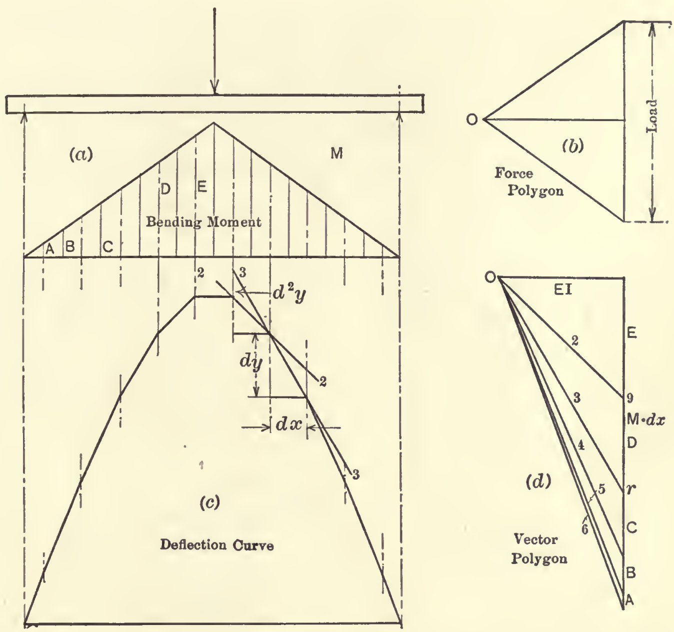


FIG. 28.

The correctness of this method may be shown by taking a section of the moment curve and dividing it into strips having a width dx. The area of one of these sections is $M \cdot dx$. Comparing the similar triangles Or 9 and the infinitesimal triangle, we have $\frac{M \cdot dx}{E \cdot I} = \frac{d^2y}{dx}$ or $\frac{M}{E \cdot I} = \frac{d^2y}{dx^2}$. This is the general equation of the elastic curve, as well as the equation of the line in Fig. 28 (c).

Continuous Beams. — Another use of the curves of deflection is the solution of restrained or continuous beams with irregular loads and spans.

The following is a modification of the method credited to Dr. Geo. Wilson, "Proceedings of the Royal Society," Vol. 62, Nov., 1897.

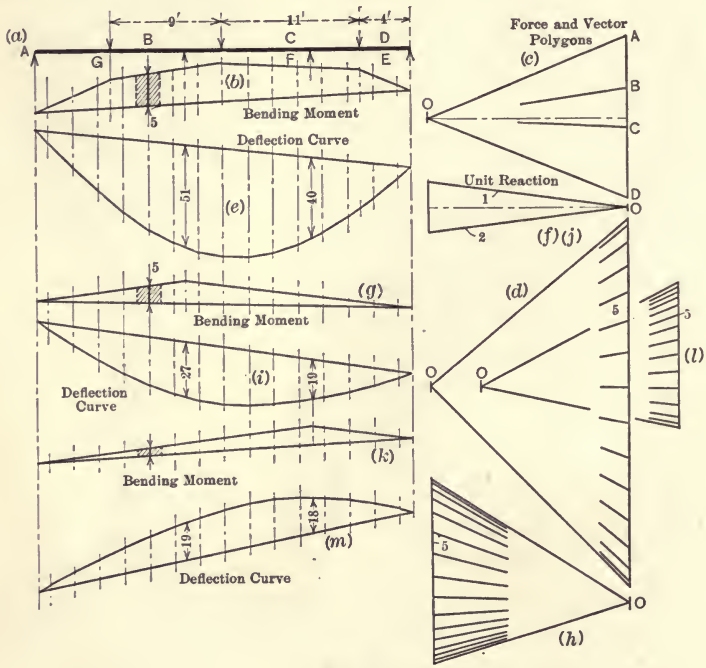


FIG. 29.

The method consists of plotting the bending moment and deflection curves due to the external loads assumed as carried by the outer supports. Similar curves are then drawn for the same beam excepting that now the actual loads are removed and assumed reactions are applied at the intermediate supports, the beam being held as before at the outside reactions. If the beam is level the deflections caused by the first loading must be equal and be opposite to those produced by the second loading. Equat-

ing these values of the deflections at the several supports gives sufficient data for the determination of the reactions and when these are known, a revised moment diagram for the actual beam is readily made. The solution is simplified by using the same pole distance for all the force or vector polygons, and using the same scale for the forces in all diagrams. An example will show the method in detail.

A continuous beam, Fig. 29 (a), carries three loads on four supports. The loads are, $AB = 1000$ lbs.; $BC = 800$ lbs.; and $CD = 1400$ lbs. The lengths of the spans beginning with that at A are 12 ft., 10 ft. and 8 ft. The loads are placed as shown.

The curve of deflection Fig. 29 (e) is first found for a beam supported at the ends and carrying the three loads, AB, BC and CD. The force polygon Fig. 29 (c) is drawn, from which the bending moment diagram readily follows in Fig. 29 (b). This polygon Fig. 29 (b) is then divided into a number of small sections of uniform width, similar to the one cross-hatched. Since they have the same widths their areas may be represented by their middle ordinates, shown heavy in the cross-hatched section and marked 5. These middle ordinates may now be laid off in the vertical line of the vector polygon Fig. 29 (d). From this vector polygon the deflection curve Fig. 29 (e) is obtained in a manner similar to that used for the bending-moment diagram Fig. 29 (b).

Under the support GF the deflection in Fig. 29 (e) scales 51, while under FE it scales 40.

Now removing the loads AB, BC and CD, assume a unit load GF applied below the beam and acting along the line of the reaction. As before construct a force polygon Fig. 29 (f), the equilibrium polygon of bending-moment diagram Fig. 29 (g) and finally the deflection curve Fig. 29 (i). In this polygon the deflections for the unit load at the lines of the reaction scale 27 and 19. In the same way place a unit load at EF and draw the polygons l, k and m. The deflections over the reactions are 19 and 18. The reactions will be some multiples of the unit loads applied. These unit loads may be any convenient one as 1,

100, 500 or 1000 lbs., depending upon the beam and its loading. If we call the reaction *GF*, p times the unit load and *FE*, q times the unit load, then the deflections due to *GF* will be $27 \times p$ and $19 \times p$, respectively, while those due to *FE* will be $19 \times q$ and $18 \times q$. Since the deflections at the points of support due to the reactions equal the deflections due to the loads we have in this problem, under *GF*

$$(27 \times p) + (19 \times q) = 51$$

and under *FE*

$$(19 \times p) + (18 \times q) = 40.$$

Solving, these equations give $p = 1.26$ and $q = 0.89$. The unit load at the reactions having been taken at 1000 lbs., the reactions become $GF = 1.26 \times 1000 = 1260$ lbs., while $FE = 0.89 \times 1000 = 890$ lbs.

The reaction *AG* may be found by taking moments about the right support *ED*.

$$AG = \frac{(1000\times 24)+(800\times 15)+(1400\times 4)-(890\times 8)-(1260\times 18)}{30}$$

$$= 390 \text{ lbs.}$$

Since the sum of the reactions equals the sum of the loads the reaction *ED* equals $(1000 + 800 + 1400) - (390 + 1260 + 890) = 660$ lbs.

It should be noted that the pole distances were all equal and the same scale of forces was used in all force polygons. When all the forces and reactions are known the bending moments may be found either graphically by constructing the force and equilibrium polygons as usual, or the moment at any section may be found by computing the moment of the forces acting on one side of the section.

CHAPTER III

STRESSES IN STRUCTURES

THE application of force and equilibrium polygons to the determination of stresses in structures is fairly simple, especially if care is taken in properly marking the structure acted on by the external forces and the lines of the diagram as drawn.

In the simple truss, Fig. 31, having located the forces at the several points, place a letter in each triangle of the truss and a letter between each pair of external forces. A member in the

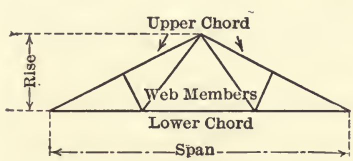


FIG. 30.

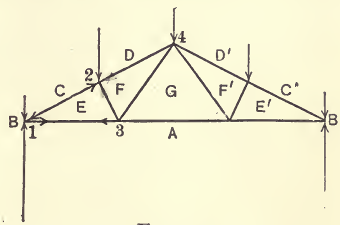


FIG. 31.

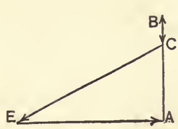


FIG. 32.

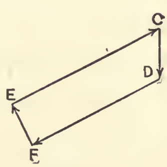


FIG. 33.

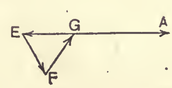


FIG. 34.

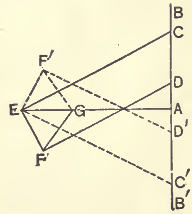


FIG. 35.

structure will be designated by the letters on each side of it, while the stress in the member will be designated in the stress diagram by the same letters at its extremities. The truss being symmetrical and symmetrically loaded the reactions will be equal and, since the sum of the vertical forces must be zero, each reaction must equal one-half the total load, hence the reaction is

$$AB = \frac{BC + CD + DD' + D'C' + C'B'}{2}.$$

To assist the explanation, numbers have been placed at some of the points but this is not usually required. Considering the forces acting at apex 1, *AB* and *BC* are known and the resultant of these is held in equilibrium by forces whose lines of action are *CE* and *EA*. The values of these two forces can, therefore, be found by drawing the force polygon of the four forces acting at point 1. This has been done in Fig. 32. It is important to place the directions on these forces and then indicate these directions as acting to or from their point of application. This has been done at apex 1. As soon as an arrow is placed at 1 on *CE* the arrow can be placed on *CE* at apex 2, since it must be opposite in direction to the first arrow. Similarly, the arrow on *EA* can be placed at apex 3. Now considering the forces acting at apex 2, *EC* and *CD* are known and, therefore, the other two, *DF* and *FE*, being known in direction, can be found in magnitude as in Fig. 33.

Again taking the forces at apex 3, we now know *EF* and *EA* from which *FG* and *GA* can be found. This has been done in Fig. 34. In this manner, force polygons could be drawn for all the apices.

Generally, instead of drawing these force polygons separately as just done, they are superimposed upon each other making a diagram, Fig. 35. Where the truss is symmetrical about a vertical axis through the center of its span and the corresponding loads on the two sides are equal, it is only necessary to make the portion of the diagram shown in full lines. The other half is shown dotted and by rotation about *AE* could be superimposed upon the full-line diagram. In this case, drawing this dotted diagram is unnecessary.

Tension or Compression in the Member

Where the forces acting upon the structure are indicated as shown, a piece having the arrows ⟵⟶ shows that piece to be in compression, while ⟶⟵ indicates tension.

NOTE. — In analyzing stresses in this way the members at the apices are assumed as pin connected and the external forces of the pieces are indicated by the arrows as acting on the pin. That is, the forces acting on the pin and not the forces of the pin on the pieces are indicated by the arrows.

Frequently trusses carry loads attached to the lower chords. The solution graphically follows the procedure just described.

If the lower apex loads *BC*, *CD*, etc., are equal the reactions *AB* and *AB′* will be equal and each will be one-half the sum of the loads on the lower chord. Taking the forces at point 1, we can lay off *AB*, knowing both its direction and magnitude, then laying off *BF* and *FA* parallel to the respective truss members

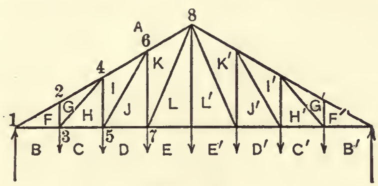


FIG. 36.

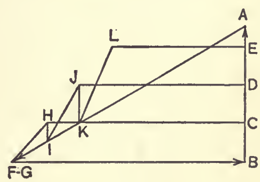


FIG. 37.

the stresses *BF* and *FA* become known in magnitude. Now going to point 2, since *AF* and *AG* will be parallel lines passing through a common point *A* they will coincide and their intercept on *FG* will be a point or its value is zero. Hence the stress in *FG* due to loads carried on the lower chord is zero.

The points *F* and *G* will be coincident on Fig. 37. Now going to point 3, *GF*, *FB* and *BC* are known, hence *CH* and *HG* are readily drawn and their magnitudes determined.

The method of determining the direction of the forces is the same as previously explained and, as before, the members in the upper chord are shown to be in compression, those in the lower chord in tension.

The vertical members, excepting *LL′*, will be in compression while the diagonal members are in tension.

Moving Loads Carried under Trusses. — Not infrequently trolley or hoist runway tracks are carried by the lower chords of trusses. In this event the track is preferably fastened to the

lower chord at the apices of the triangles, and the load may come upon any apex. It will generally be easy under these circumstances to make a diagram for the load under each apex in one-half the lower chord, and by comparing the diagrams, which are preferably made to the same scale, the maximum stress in any member due to any of the several positions of the load is readily found. A diagram made for one position of the load will illustrate the method.

Fig. 38.

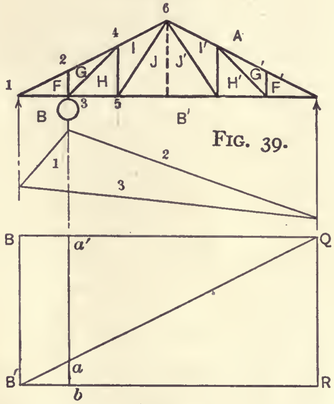


Fig. 39.

Fig. 40.

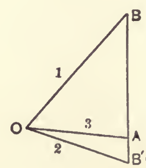


Fig. 41.

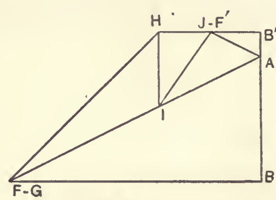


Fig. 42.

The load is assumed at apex 3. The reactions AB and AB' must first be found. Draw the force polygon, Fig. 41, and the equilibrium polygon, Fig. 39. When the ray 3, in Fig. 41, is drawn parallel to the closing side of the equilibrium polygon, Fig. 39, the load BB' is divided into the two reactions.

When it is required to find the reactions for all positions of the load, the work can be abridged by laying off the load BB' at the left reaction and taking any point Q in the other reaction and drawing BQ and $B'Q$, then the intercept a–a' in the triangle under the load will be the left reaction for that position of the load. If BQ and $B'R$ are drawn parallel to the lower chord a–a' will be the left reaction and a–b the right reaction.

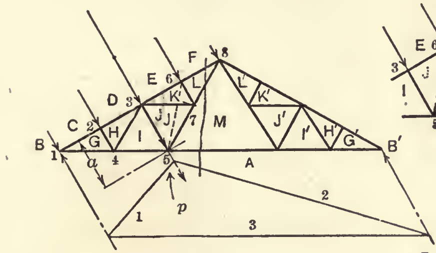
Note. — The student should prove the truth of this last diagram, using both algebraic and graphical demonstrations.

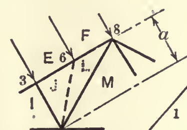
Wind Load on Trusses

The wind load will be assumed normal to the left side of the roof and both ends of the truss will be fixed.

The reactions *AB* and *AB′* will be parallel to the wind loads *BC*, *CD*, *DE* and *EF*. The sum of the reactions will equal the sum of the loads. The magnitude of the reactions can be found by the force and equilibrium polygons, Figs. 44 and 45. In Fig. 44, *BB′* is laid off parallel to the wind loads and equal to their sum. The resultant normal wind pressure on the roof

Fig. 43.





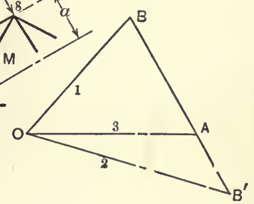


Fig. 44.

Fig. 45.

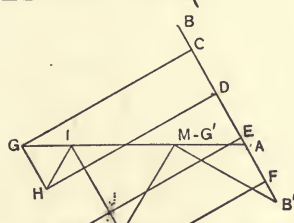


Fig. 46.

equals the sum of the wind loads *BC*, *CD*, *DE*, *EF* and *FB′* and will act coincident with the force *DE*. Take any point *p* in the line of this resultant and through it draw the strings parallel to their respective rays 1 and 2 in Fig. 44. Then in Fig. 45 draw the closing line of the equilibrium polygon and in Fig. 44 draw ray 3 parallel to string 3 in Fig. 45. *BB′* will then be divided into two parts corresponding to the two reactions. Having found the two reactions the stress diagram follows naturally as in

Fig. 46. No difficulty is experienced until apex 3 is reached. Here there are three forces known completely and three known in direction only. It is, therefore, impossible to determine the magnitude of the unknown forces unless more conditions are assumed. One solution of the problem is as follows. It can be shown that the forces acting in *FL*, *LM* and *MA* will be the same whether the truss is left as shown by the full lines *JK* and *KL*, or if these are removed and replaced by the member shown dotted, *JL*.

Taking moments about point 5, cutting the truss as shown by the line and equating internal and external moments we have

$$\Sigma \text{ external moments} = (\text{force } FL) \times a.^*$$

As the forces *LM* and *MA* pass through point 5, their moments about 5 are zero. Hence, the force in *FL* depends only upon the external moments and the lever arm a, and is consequently independent of the truss members to the left of the cut. In a similar way stresses in *LM* and *MA* can be shown to be independent of truss members to the left of the cut. Having found the temporary forces acting at 3, when *JK* is removed complete the diagram for the forces at 5 and 6, then remove the dotted member *JL* and replace the full members *LK* and *KJ*. Now, taking the forces acting at 6, *EF* and *FL* are known completely; therefore, the magnitudes of *EK* and *KL* can be found. From here taking the forces at point 7 and then returning to point 3 all the forces acting in the left half of the truss become known. The diagram can be closed by finally taking the forces acting at the right reaction.

The stresses due to the wind in the members *H'G'* to *L'M* in the right half of the truss will be seen to be zero. It is evident that for the wind acting on the right side of the truss, the members in the right half will be stressed the same as similar members in the left half when the wind acts as shown in Fig. 46; hence, redrawing the diagram is unnecessary.

In trusses of long span it may be necessary to have one end free to allow for expansion; this may be done by allowing the

* Note. — Consult Method of Moments. Chapter IV.

truss to rest upon a plate secured to the wall. The plate fastened to the lower chord of the truss has slotted holes which permit the truss to slide upon the wall plate by overcoming the friction between the plates. The friction may be greatly reduced by using rollers under the truss. The following example illustrates the method of determining the reactions and stresses due to wind loading when the truss is free at one end. The plates are placed at the right-hand end of the truss. The coefficient of friction is assumed as one-third; hence the tangent $\alpha = \frac{1}{3}$ or the inclination

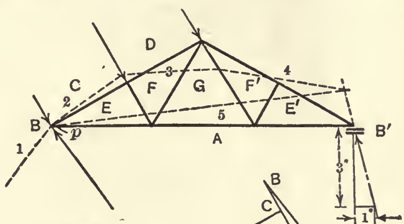


FIG. 47.

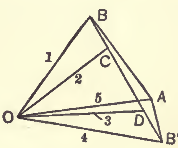


FIG. 48.

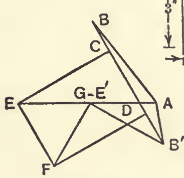


FIG. 49.

is one in three. The line of action of the right reaction is, therefore, known. The first part of the problem requires the determination of the magnitude of the right reaction and of both the magnitude and the direction of the left reaction. It is known, however, that the left reaction must pass through point p of the truss. First draw the force polygon, Fig. 48. Through B' draw a line parallel to the right reaction. Take any pole O and draw the rays 1, 2, 3 and 4. In Fig. 47, draw the corresponding strings.

Since the left reaction AB, the force BC and rays 1, 2 and 5 are in equilibrium they must intersect in a common point in the equilibrium polygon; this must be point p, the only point common to BC and the left reaction AB. Hence, through p draw a string parallel to ray 1. From Fig. 48 the force BC is held in equilibrium by the strings 1 and 2; hence, in Fig. 47 they must

have a common point. Through p draw also the string 2 parallel to ray 2. Force CD and rays 2 and 3 are in equilibrium and must pass through a common point in the equilibrium polygon, Fig. 47. From Fig. 48 rays 4 and 5 hold reaction AB' in equilibrium and rays 1 and 5 hold reaction AB in equilibrium; hence, string 5 must pass through the point of intersection of string 4 with reaction AB' and it must also pass through the intersection of string 1 with reaction AB which is in point p. It is noticed that string 1 becomes merely a point in Fig. 47. This work would have been simplified by using a resultant wind pressure equal to BB', acting in the same line as CD in Fig. 47.

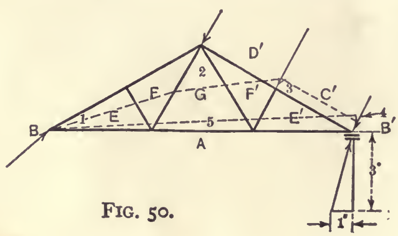


FIG. 50.

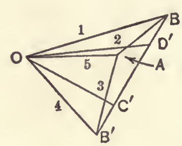


FIG. 51.

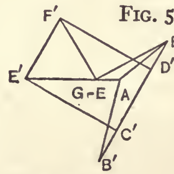


FIG. 52.

The stress diagram is shown in Fig. 49, its method of construction differing in no way from those previously described. It should be noted that the diagram is for the entire truss, the loading being unsymmetrical, and that the diagram checks if the force $B'E'$ when drawn through the points B' and E' in Fig. 49 is parallel to the line $B'E'$ of the truss, Fig. 47.

The diagram should be constructed for the wind on the free side and such construction is shown in Figs. 50 to 52.

When rollers are used at the free end the reaction at that end is assumed vertical.

It is shown, page 54, how the stresses may be obtained algebraically by the method of moments and how the moments

due to external loads may be found graphically. A combination of these two methods may be used thus:

Figure 56 is the force polygon for the external loads, with pole O and the rays drawn. From this the equilibrium polygon, Fig. 55, has been drawn. Now the bending moment at any section is the product of the intercept under that section in the equilibrium polygon multiplied by the pole distance H. Hence to determine the stress in any member DF cut the truss at $Z–Z$ and replace the cut members by the forces that would have to act in them to produce equilibrium. Now, if possible, take

FIG. 53.

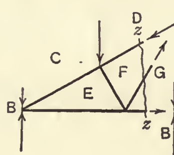
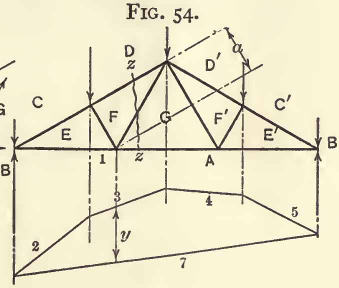


FIG. 54.



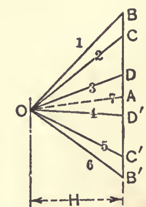


FIG. 55. FIG. 56.

moments about a point common to all but one of these forces, in this case point 1. Let a be the lever arm of the force DF about point 1, then making the sum of the internal and external moments zero we have

$$(H \times y) + (DF \times a) = 0,$$

from which

$$DF = -\frac{(H \times y)}{a}.$$

The Character of the Stress, whether tension or compression, can be determined as follows:

Calling clockwise rotation positive and counterclockwise negative find the character of the moment of the external forces $H \times y$. This is found to be positive. Since the sum of the internal and external moments about a point is zero the moment

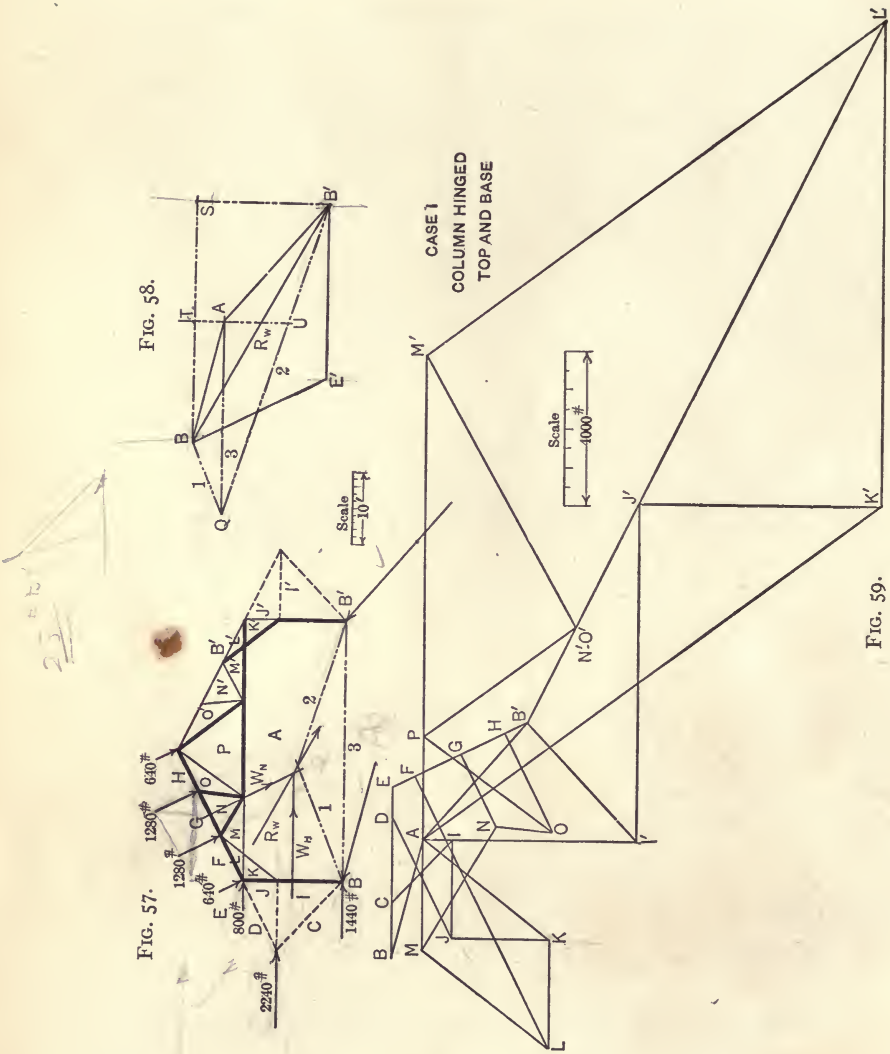


Fig. 57. Fig. 58. Fig. 59.

of the internal forces must be of opposite character to the moment of the external forces hence $DF \times a$ is negative and DF must act in the opposite direction to that assumed. This makes DF in compression.

Stresses in a Bent

The determination of the dead-load and snow-load stresses in the truss of a bent presents no greater difficulties than that of a truss carried upon brick walls, as there are no stresses in the knee braces due to these loads. Where the wind load is carried by the columns and truss forming the bent the magnitude of the forces acting in the truss members and of the stresses acting in the column section vary greatly with the distance from the top of the column to the foot of the knee brace and with the manner of securing the column to the foundation.

The wind upon the truss may be considered as either horizontal or normal but will here be assumed as normal. The columns will first be considered as hinged top and bottom and then as fixed at the base and hinged at the top. In this latter case it can be shown that the point of contraflexure lies between the base of the column and the foot of the knee brace and between one-half and five-eighths of this distance from the base of the column. It is generally assumed as one-half for convenience, and this is sufficiently accurate. To expedite the determination of the stresses the forces acting upon the column have been transferred to the truss through the extra truss members added temporarily to the columns. It should be noted that vertical sections can be taken through all members of the truss without cutting these added pieces, hence the stresses in these members will be independent of the stresses in the added pieces and the diagram drawn with their assistance will give the correct stresses in the permanent members of the truss. The wind load has been assumed as 12 lbs. per sq. ft., normal to the roof surface. The following dimensions have also been used: span 36 ft. 0 ins., bay widths 16 ft., base of column to lower chord of truss 14 ft., lower chord of truss to foot of knee brace 5 ft. 0 ins.

In Case I the columns are hinged both top and bottom, while in Case II they are fixed at the base and hinged at the top. The

lettering has been made identical in both cases and, the scales being the same, the influence of the method of securing the columns upon the stresses in corresponding members can be seen at a glance by comparing the stresses in the stress diagrams, Figs. 59 and 62. The following explanation refers to Case I, but applies also to Case II. It is first necessary to estimate the apex wind loads on the sides and roof. On the side of the building

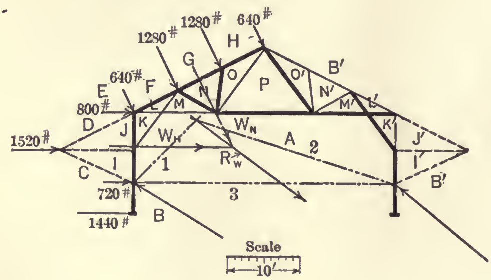


FIG. 60.

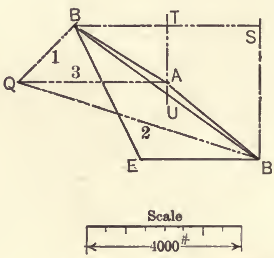


FIG. 61.

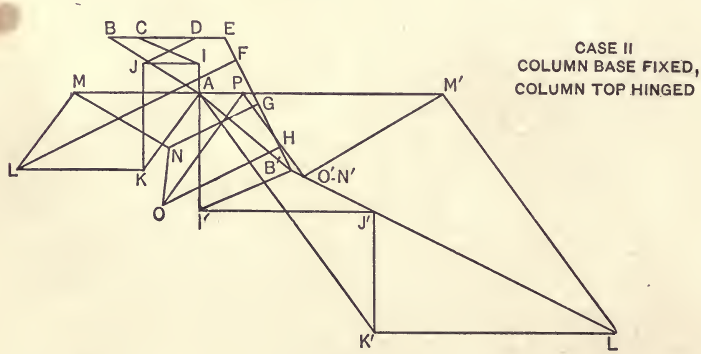


FIG. 62.

the wind is assumed as acting at the base of the column, the foot of the knee brace and the top of the column. The normal wind pressure on the roof has a resultant W_N equal to the sum of apex loads EF, FG, GH and HB', and acting centrally with this side of the roof as shown.

* Similarly, the horizontal wind pressure W_H equals the total horizontal wind pressure between two bents and acts centrally upon the side of the building. Now these two wind forces acting on the building have a resultant acting through their point of intersection given in direction and magnitude in the force polygon, Fig. 58. Taking any point in this resultant R_W in Fig. 57, an equilibrium polygon can be drawn, since it is known that the resultants AB and AB' must pass through the column bases. In Case II these resultants pass through points in the columns midway between the base of the column and the foot of the knee brace. Having drawn the strings 1, 2, and 3 in Fig. 57, draw the rays 1 and 2 in Fig. 58, and their point of intersection Q will be the desired pole. The usual assumption is that the columns share the horizontal components of the wind forces equally. In Fig. 58 the horizontal component of the wind forces is BS, which has TU drawn perpendicular to it from its center. If, through the pole Q, ray 3 is drawn parallel to string 3 in the equilibrium polygon in Fig. 57 it will cut TU at the point of intersection of the resultants AB and AB' acting at the column bases. In Case II these resultants act on the columns midway between the bases and the knee braces. Having found the resultant wind forces acting on the columns the drawing of the stress diagram presents no unusual difficulties. To transfer the forces from the column bases to the truss members CI, IJ, JD, $B'I'$, $I'J'$ and $J'B'$ have been added.

The stresses given by the diagram for the column AI, AI', $K'J'$ and KJ will not be the correct stresses for the actual structure, but the stresses of all other truss members including the knee braces are the desired ones. In the drawings of the truss the members in compression have been drawn heavy.

Maximum Bending due to Moving Loads. — It has been shown, page 26, how the equilibrium polygon can be used to

* Note. — In Case II the wind pressure on the side effecting the stresses in the truss is assumed as that from the point of contraflexure to the top of the column and the resultant acts midway between these two points.

determine the bending moments upon a beam. By a simple extension of the principles a diagram can be made showing the maximum bending that will occur along the beam as a system of moving loads passes over the span.

The loads *BC*, *CD*, *DE* and *EF* in Fig. 63 are a constant distance apart and roll across the girder *ab*. The several positions of the system of loads relative to the girder can either be shown by redrawing several additional positions of the loads or by redrawing the girder, moving the girder under the loads. The

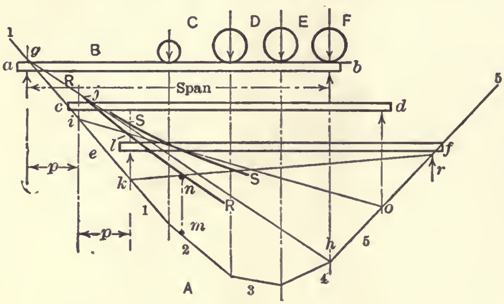


Fig. 63.

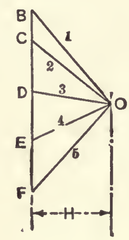


Fig. 64.

latter method, being the easier, has been used and two additional positions of the girder *c–d* and *e–f* have been drawn. Fig. 64 is the force polygon for the four wheel-loads, a pole *O* was chosen and the rays 1 to 5 inclusive were drawn.

Draw the equilibrium polygon for the several loads, the strings being drawn parallel to their respective rays. Strings 1 and 2 will intersect on the force *BC*, strings 2 and 3 will intersect on force *CD*, etc. Now draw the closing line of the first equilibrium polygon by drawing *gh* through the points of intersection of the extended reactions *AB* and *AF* of the girder. Then for the first position of the loading upon the girder the bending moment at any point on the girder will be the product of the intercept measured in feet to the scale of the span by the pole distance *H* measured by the scale of forces. In the same way move the girder to the second position *c–d* and locate the closing line of the second equilibrium polygon in the line *io*. Proceed in this way until the loads

have been rolled across the girder. It now remains to compare the bending moments at definite points on the girder for the several positions of the loads. Suppose it is desired to know the maximum bending moments at p intervals across the girder, one would begin by comparing the intercepts in the several equilibrium polygons a horizontal distance p from the left end. Several of these ordinates ji, kl and mn have been drawn. The comparison may be assisted by drawing a curve through the points of intersection of these ordinates with the closing lines of their respective equilibrium polygons.

This is shown by the curves *RR* and *SS*.

The intercepts between the curves *RR* and *SS* and the broken line of the equilibrium polygon will give the bending moments exactly only at the vertexes of the equilibrium polygons; therefore, when the maximum appears to be between vertexes, the ordinate should be checked by drawing an equilibrium polygon with the loads in a position to bring a vertex at the desired point.

Diagram of Maximum Live-load Shears. — This will be explained by a loading somewhat resembling the usual locomotive and train load.

In making the diagram of maximum shears, Fig. 66, lay off the forces *BC*, *CD*, etc., and take a pole distance *OB* equal to the girder span. Draw the rays 1 to 10 and beginning at *O* draw the strings 1 to 10 parallel to their respective rays and in accordance with the usual method, *i.e.*, *CD* in the force polygon is held in equilibrium by rays 2 and 3; hence, in the equilibrium polygon, strings 2 and 3 must intersect force *CD* in a common point. The side of the equilibrium polygon under the uniform load is a parabola and can be drawn tangent to the broken line. To understand the theory of the diagram draw the closing line *OA*, string 11, of the equilibrium polygon; this is also ray 11 of the force polygon and divides the line of the forces *B–H* into the reactions. *B–A* is the left reaction when the load *BC* is over the left support with the other loads as shown. Suppose the loads moved to the right a distance a or the same relative position of the loads

and girder is more readily obtained by moving the span a distance a to the left as designated by position 2. The closing line in this case is string 12 and the reaction is MN. As the loads travel to the right the shears under load BC can be obtained by measuring from point B the distance the load BC is to the right of the left reaction and at this point measuring the ordinate from OB to the broken line 2, 3 – – – to 10 of the equilibrium polygon. This of course must be measured by the same scale as that to which the forces are laid off. When the usual locomotive wheel loads are

FIG. 65.

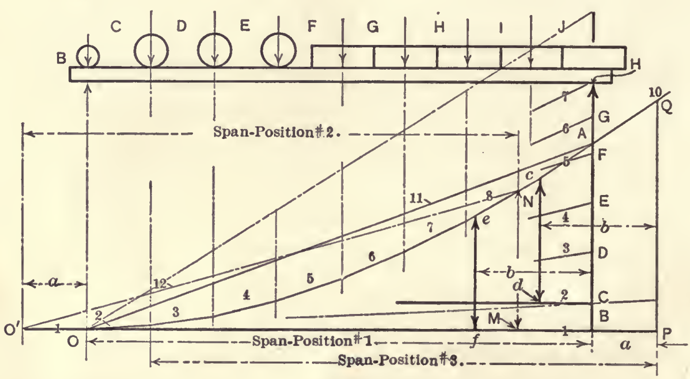


FIG. 66.

preceded by a much lighter pilot wheel there may be a question as to whether the greatest shear at any section occurs when the pilot wheel, load BC, is at the section or the first driver, load CD, is there. In Fig. 66, measure the span marked position 3, produce string 2 until it cuts PQ, the line of the right reaction. From the point where string 2 cuts BH draw a heavy horizontal line to the left. To find the maximum shear a distance b from the left support draw the intercept c–d, a distance b from the line QP; this will be the shear when the load CD is at the distance b from the left support. Compare c–d with the intercept e–f drawn between the broken line of the equilibrium polygon and the horizontal line OB and at the distance b from the line of forces

HB. In this way the maximum shear may be found for any point.

This method is applicable to the determination of the maximum stresses in truss members as well as in plate girders.

The following illustrates the method applied to the determination of the stresses in a Pratt truss.

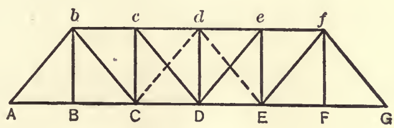


Fig. 67.

Example. — The Pratt truss, Fig. 67, spans 150 ft. It has 6 panels and height of 30 ft. The loading is Cooper's E–60; see page 72, under Influence diagrams, for this loading.

Find the stresses in *BC*, *bc* and diagonal *bC*, locating the positions of loads for maximum stress and shear by influence diagrams and using equilibrium diagrams for determining the maximum bending moments and shears.

CHAPTER IV

ALGEBRAIC DETERMINATION OF STRESSES

THE conditions of equilibrium used in the determination of stresses are:

1. Sum of the horizontal forces = 0.
2. Sum of the vertical forces = 0.
3. Sum of the moments of the forces about any point = 0.

The stresses in the members cut by the section *a–a* can be replaced by forces F_1, F_2 and F_3, equal to the respective stresses

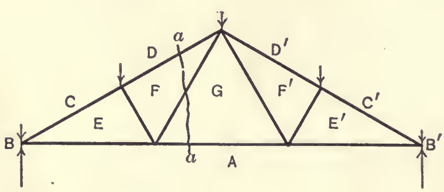


FIG. 68.

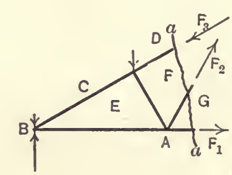


FIG. 69.

in these members. These forces F_1, F_2 and F_3 hold the portion of the truss to the left of the section, Fig. 68, in equilibrium and the stresses in the members may be determined by using conditions 1, 2 and 3.

Referring to conditions 1 and 2, they should be understood to mean that the sum of the components along any line must = 0, and the sum of the components along a line at right angles to the first line must = 0.

Frequently the work may be abridged by using the more general scheme. Forces and components acting to the right or upwards are positive, those acting to the left or down are negative. Clockwise moments are positive; counterclockwise moments are

negative. To find the stresses in CE and EA in Fig. 70, cut the truss at a–a, then, by conditions 2 and 1,

$$(AB - BC) + CE \sin \alpha = 0,$$
$$CE = -(AB - BC) \operatorname{cosec} \alpha,$$

also $$-CE \cos \alpha + AE = 0 \quad \text{or} \quad AE = CE \cos \alpha;$$

here AE is plus, acts to the right and produces tension.

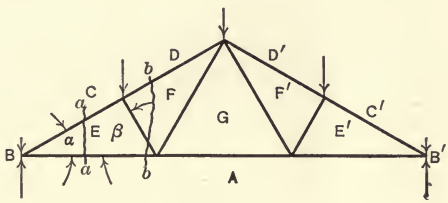


FIG. 70.

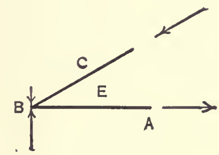


FIG. 71.

In a similar way to find the stress in DF or FE, cut the truss in section b–b. Considering the vertical components we have

$$(AB - BC - CD) - DF \cdot \sin \alpha + EF \sin \beta = 0.$$

Now considering the horizontal components

$$+ EA - DF \cos \alpha - EF \cos \beta = 0.$$

The solution of these equations will give the stresses DF and EF. The forces acting at a point may also be treated by this method. Considering the forces acting at point 1 in Fig. 72, and using condition 2 we have

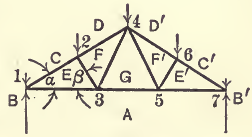


FIG. 72.

$$(AB - BC) + CE \sin \alpha = 0$$

or $$CE = -\frac{(AB - BC)}{\sin \alpha}.$$

The force CE acts down towards point 1. $AE - CE \cos \alpha = 0$; AE, being plus, acts towards the right. The direction and character of the forces can be checked by drawing the force polygon for the forces at the point.

Method of Moments

The stresses in framed structures can be determined algebraically by placing the sum of the external and internal moments equal to zero.

To determine the stress in any member cut the truss by a section passing through the member whose stress is desired. Taking the portion of the truss to the left of the section replace the members by forces acting to the right of the section as shown in Fig. 74. Now select some point about which to take moments. If possible, take a point through which all the cut members pass, excepting the member whose stress is desired. In the example

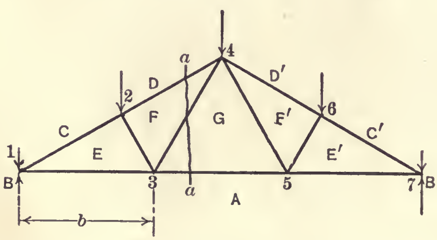


Fig. 73.

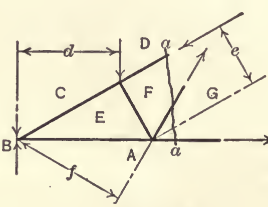


Fig. 74.

given, members *FG* and *GA* pass through point 3 so that moments will be taken about point 3. To determine the direction of the force acting in the member assume that it acts away from the cut section or to the right and that the piece is in tension. If the solution gives a plus value the assumption was correct; if the value is minus the piece is in compression. As before clockwise moments are positive, while counterclockwise moments are negative.

Taking moments about point 3 in Figs. 73 and 74, we have

Moments of external forces + moments of internal forces = 0.

$$[(AB - BC)\,b] - [CD \times (b-d)] + DF \times e = 0,$$

$$DF = \frac{-[(AB - BC)\,b] + [CD \times (b-d)]}{e}.$$

The quantity $(AB - BC)\,b$ will be much greater than $CD\,(b - d)$ and DF will, therefore, be negative, and consequently the piece DF will be in compression instead of in tension as assumed.

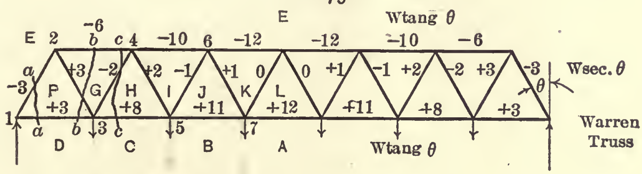
To determine the stress in FG take moments about point 1, then

$$(CD \times d) - (FG \times f) = 0,$$

$$FG = +\frac{CD \times d}{f}.$$

The value of FG, being plus, agrees with the assumption and the member is in tension.

FIG. 75.



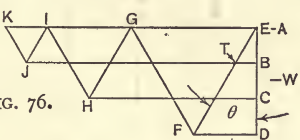


FIG. 76.

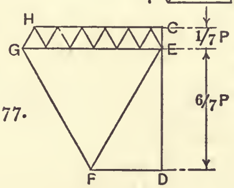


FIG. 77.

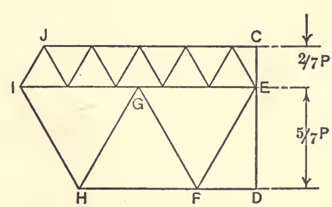


FIG. 78.

In dealing with bridge trusses with parallel chords an abridgment of the method of resolution of forces is possible.

In Fig. 76 it should be noted that all the stresses are multiples of one of the sides of the small triangle ETB, thus $F\text{–}D = 3 \cdot T\text{–}B$; $E\text{–}F = 3 \cdot E\text{–}T$ and $E\text{–}D = 3 \cdot E\text{–}B$. $E\text{–}D$ is the vertical shear in panel D, $E\text{–}C$ that in panel C, and $E\text{–}B$ that in panel B. $E\text{–}B$ = load W.

Expressed in terms of the apex loads W and the angle θ, E–T $= W \cdot \sec\theta$ and T–B $= W \cdot \text{tang}\,\theta$, hence:

Member.	Stress.	Member.	Stress.
E–F and F–G	$3\,W \sec\theta$	F–D	$3\,W\,\text{tang}\,\theta$
G–H and H–I	$2\,W \sec\theta$	E–G	$6\,W\,\text{tang}\,\theta$
I–J and J–K	$1\,W \sec\theta$	H–C	$8\,W\,\text{tang}\,\theta$
K–L	$0\,W \sec\theta$	I–E	$10\,W\,\text{tang}\,\theta$
....	....	B–J	$11\,W\,\text{tang}\,\theta$
....	....	K–E	$12\,W\,\text{tang}\,\theta$

It is seen that the coefficients of the diagonals are the coefficients of the shears in their panels, thus the shear in panel D is $3 \cdot W$, hence 3 is the coefficient on E–F and F–G, being $-$ for compression members and $+$ for tension pieces. In the same way the shear in panel C is $2 \cdot W$, hence the coefficient is 2 for diagonals G–H and H–I.

The next point to be noted is that the sum of the coefficients of the members cut by the sections a–a, b–b and c–c must in each case be zero. Section a–a cuts members E–F and F–D; the coefficients then are $-3 + 3 = 0$. At section b–b the coefficients of D–F and F–G are each $+3$. Now if the coefficient of E–G is x,

then $$+3 + 3 + x = 0,$$

hence $$x = -6.$$

In this way all the coefficients can be found and stresses computed by multiplying these coefficients by $W \cdot \sec\theta$ for diagonal members or by $W \cdot \text{tang}\,\theta$ for horizontal chords.

Live Loads. — To analyze the influence of live loads at the several apexes of the lower chord, construct Fig. 77. With the load at apex 3, the left reaction is $\frac{6}{7}\,P$, the right reaction $\frac{1}{7}\,P$. The coefficients of the web members to the left of the load, apex 3, are $\frac{6}{7}$ while those to the right are $-\frac{1}{7}$.

In the same manner from Fig. 78, which is the stress diagram for the load P at apex 5, it is seen that the coefficients of the web members to the left of apex 5 are $\frac{5}{7}$ while to the right the coefficients are $-\frac{2}{7}$.

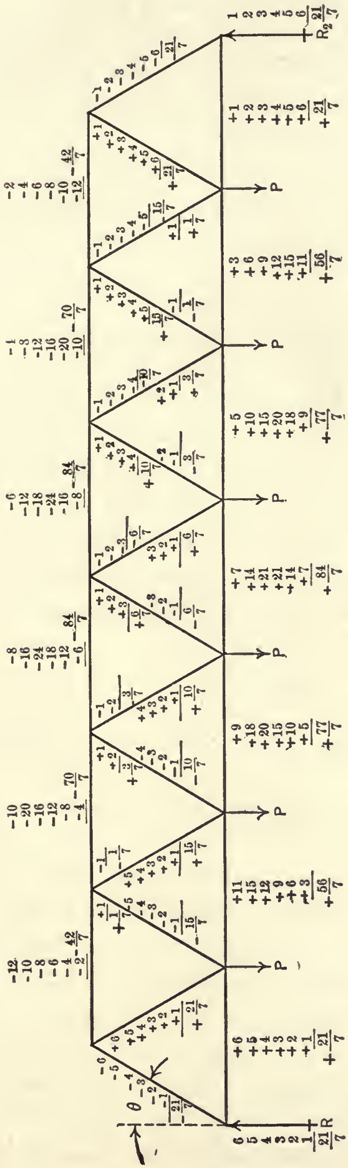


FIG. 79.

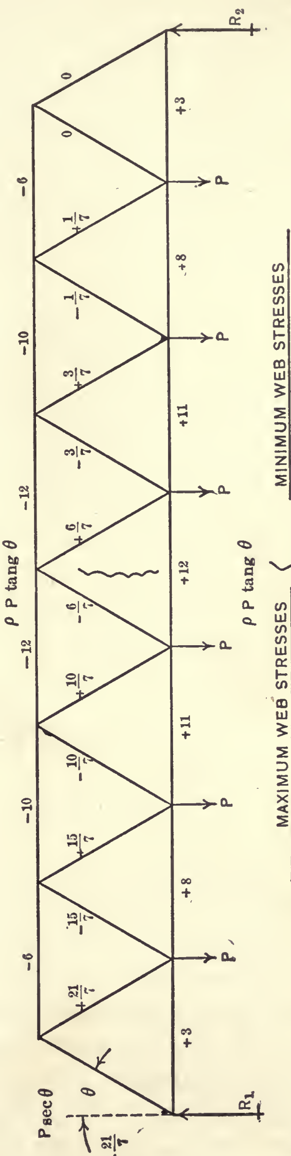

P sec θ
ρ P tang θ
MAXIMUM WEB STRESSES
MINIMUM WEB STRESSES
R_1
R_2

FIG. 80.

It will be seen that for maximum chord stresses the truss should be fully loaded, while for minimum chord stresses there should be no live load on the truss. In this type of truss all loads on upper or lower chords produce tension in the lower chord members and compression in the upper chord members.

The maximum web stress will be produced when the longer segment from that panel to a pier is fully loaded. The minimum web stresses will be created when the shorter segment from panel to pier is fully loaded.

Pratt Truss

The analysis of the stresses in a Pratt truss can be done by the method of coefficients in the same way as the Warren truss just described. If, as is sometimes done, parts of the dead loads are assumed as applied at the apexes of the upper chord the only members whose stresses will be affected are the vertical ones. To find the shear in these verticals cut the truss diagonally through the vertical member whose stress is desired and equate the shears. (See Fig. 83.)

$$I\text{–}J = (C\text{–}D) - (C\text{–}B) - (B\text{–}A) - (D\text{–}E).$$

Had there been no loads upon the upper chords, as in Figs. 81 and 82, then

$$I\text{–}J = (C\text{–}D) - (C\text{–}B) - (B\text{–}A).$$

The maximum chord stresses are equal to the sum of the dead- and live-load chord stresses, while the minimum chord stresses are those due to the dead load only. The maximum and minimum web stresses are found by adding algebraically the corresponding live- and dead-load stresses.

Since in this truss the diagonals can take only tension the necessity for counter diagonals to care for stresses due to unsymmetrical live loading should be noted. The stress in the counter is the same as the compression would have been in the piece for which it acts.

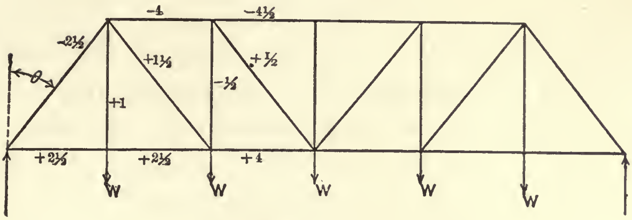


FIG. 81.

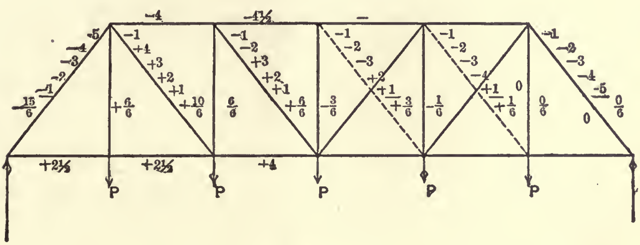


FIG. 82.

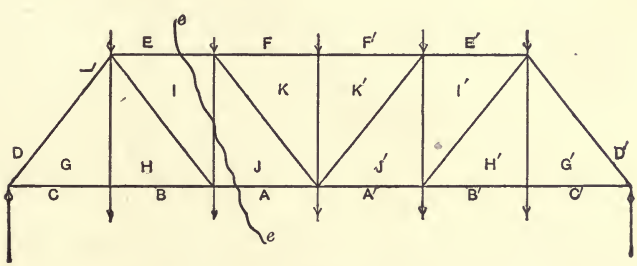


FIG. 83.

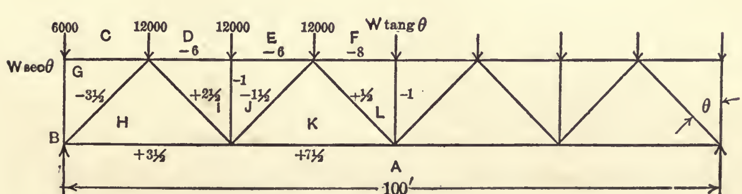


FIG. 84.

Warren Truss

Problem. — Find the dead-load stresses by the method of coefficients in the following deck Warren truss, Fig. 84. Span 100 ft. Load at apexes on the upper chord, 12,000 lbs.

The secant of 45 degrees is 1.414. Tangent of 45 degrees is 1.

Shear in panel $C = 3.5 \times W = 42{,}000$ lbs.
Shear in panel $D = 42{,}000 - 12{,}000 = 30{,}000$ lbs.
Shear in panel $E = 42{,}000 - 12{,}000 - 12{,}000 = 18{,}000$ lbs.
Shear in panel $F = 42{,}000 - 12{,}000 - 12{,}000 - 12{,}000 = 6000$ lbs.

Stress in $HG = -\ 3\frac{1}{2}\,W \sec\theta = -\ 3.5 \times 12{,}000 \times 1.414 = -\ 59{,}400$ lbs.
Stress in $HI = +\ 2\frac{1}{2}\,W \sec\theta = +\ 2.5 \times 12{,}000 \times 1.414 = +\ 42{,}400$ lbs.
Stress in $JK = -\ 1\frac{1}{2}\,W \sec\theta = -\ 1.5 \times 12{,}000 \times 1.414 = -\ 25{,}500$ lbs.
Stress in $KL = +\ \frac{1}{2}\,W \sec\theta = +\ 0.5 \times 12{,}000 \times 1.414 = +\ 8{,}500$ lbs.
Stress in $HA = +\ 3.5\,W \operatorname{tang}\theta = +\ 3.5 \times 12{,}000 \times 1 = +\ 42{,}000$ lbs.
Stress in $KA = +\ 7.5\,W \operatorname{tang}\theta = +\ 7.5 \times 12{,}000 \times 1 = +\ 90{,}000$ lbs.

Stress in DI and $EJ = -\ 6\,W \operatorname{tang}\theta = -\ 6 \times 12{,}000 \times 1 = -\ 72{,}000$ lbs.
Stress in $FL = -\ 8\,W \operatorname{tang}\theta = -\ 8 \times 12{,}000 \times 1 = -\ 96{,}000$ lbs.
Stress in $CG = 0$
Stress in $GB = -\ 6000$ lbs.

CHAPTER V

INFLUENCE DIAGRAMS

WHEN a system of concentrated loads moves across a girder or a trussed bridge the maximum moment or shear at a section, or stress in a given member, will be produced by a certain position of the system of loads relative to that section or member.

Influence lines are used to determine this position of the loads producing maximum moments, shears or stresses.

Influence Diagrams. — An influence diagram shows the variation of the effect at any particular point, or in any particular member, of a system of loads moving over the structure. Influence diagrams are commonly drawn for a load of unity. The moments, shears, or stresses for any system of loads can be computed from the intercepts in this diagram by multiplying them by the given loads.

Influence diagrams to find the position of loading to give maximum moment at a given point in a beam or girder or at a given joint on the loaded chord of a truss.

In Fig. 85,

ΣP_1 is the resultant of moving loads to the left of point 3.

ΣP_2 is the resultant of moving loads to the right of point 3.

To construct the influence diagram, Fig. 86, for the bending moment at 3, compute the bending at 3 due to a unit load at this point

$$R_1 = \frac{1 \times (S - p)}{S} \quad \text{and} \quad M = \frac{(S - p)p}{S}.$$

If p is laid off on the left reaction and $(S - p)$ upon the right reaction and their extremities are joined to the ends of the hori-

zontal line *ea* by *ba* and *fe* then the vertical intercept *cd* represents $\frac{(S-p)p}{S}$ for, let this intercept *cd* be x then by similar triangles *eab* and *dac*, $\frac{x}{p} = \frac{S-p}{S} \therefore x = \frac{(S-p)p}{S}$ and similarly for the triangles *dec* and *aef*.

$$\frac{x}{S-p} = \frac{p}{S} \quad \text{or} \quad x = \frac{(S-p)p}{S}.$$

FIG. 85.

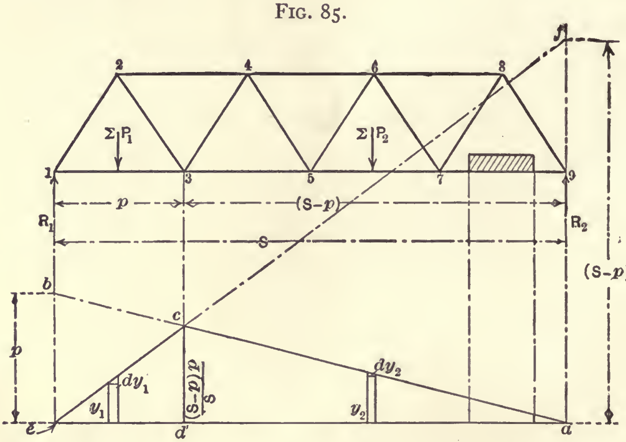


FIG. 86.

To find the moment at a given point 3, due to a system of moving loads, multiply the intercept under each load by that load and take the sum of these products. From Fig. 86

$$M = \Sigma P_1 y_1 + \Sigma P_2 y_2.$$

Now move the system of loads a small distance dx to the right, allowing no load, however, to pass on or off the span or across the given point 3. Then for the new position

$$M + dM = \Sigma P_1(y_1 + dy_1) + \Sigma P_2\,(y_2 - dy_2).$$

Subtracting the preceding equation from this we have

$$dM = \Sigma P_1 dy_1 - \Sigma P_2 dy_2.$$

Now examining for maximum bending by placing this equation $= 0$, we have

$$\Sigma P_1\, dy_1 = \Sigma P_2\, dy_2. \qquad (1)$$

By similar triangles

$$\frac{dy_1}{dx} = \frac{S - p}{S} \quad \text{and} \quad \frac{dy_2}{dx} = \frac{p}{S},$$

from which

$$\frac{dy_1}{dy_2} = \frac{S - p}{p}.$$

Substituting this value in equation (1)

$$\Sigma P_1 (S - p) = \Sigma P_2 p;$$

hence

$$\Sigma P_1 S - \Sigma P_1 p = \Sigma P_2 p,$$

from which

$$\Sigma P_1 S = \Sigma (P_1 + P_2)\, p$$

or

$$\Sigma \frac{P_1}{p} = \frac{\Sigma (P_1 + P_2)}{S}.$$

This may be expressed by stating that the maximum bending will occur at a section when the average load to the left of the section equals the average load on the entire span.

The influence diagram to find the position of the loading to give a maximum moment at a given joint on the unloaded chord of a truss, having either parallel or inclined chords, may be found in a similar way.

Figures 87 and 88 are drawn for joint 4 on upper chord.

If the system of loads is moved a distance dx to the left the rate of change in the bending moment is

$$\frac{dM}{dx} = -\ \Sigma P_1\, dy_1 - \Sigma P_2\, dy_2 + \Sigma P_3\, dy_3. \qquad (1)$$

To be a maximum $\frac{dM}{dx} = 0$.

But by the geometry of the construction

$$\frac{dy_1}{dx} = \frac{S - p}{S}; \quad \frac{dy_3}{dx} = \frac{p}{S} \quad \text{and} \quad \frac{dy_2}{dx} = \frac{fS - pc}{cS}.$$

Fig. 87.

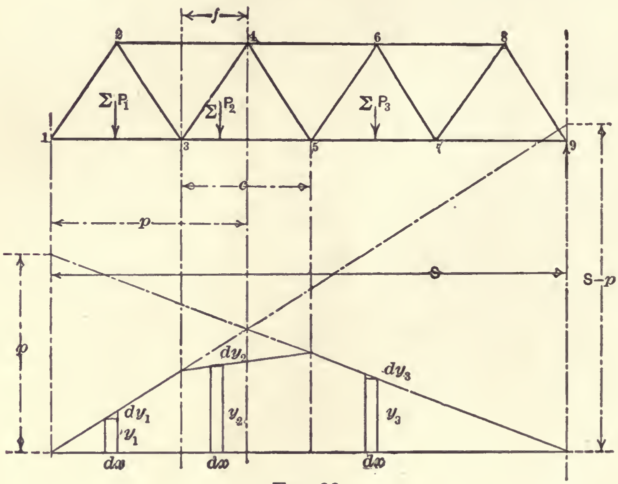


Fig. 88.

These values substituted in equation (1) give the position for maximum moment at joint 4.

$$\frac{\Sigma P}{S} = \frac{\Sigma P_2 \frac{f}{c} + \Sigma P_1}{p}.$$

ΣP is sum of all loads P_1, P_2 and P_3.

Maximum Shear

Reactions and Shears. — By definition, shear is the algebraic sum of the vertical forces to the left of the section. When the unit load has moved a distance x to the left of the right support, Fig. 90, the shear to the left of the load is $\frac{x \times 1}{L}$, and since this value under the load is reduced by unity the shear to the right of the load is $\frac{L - x}{L} \times 1$.

The shear under the load for any position is given by the intercept in the triangle. For a system of loads the shear to the left

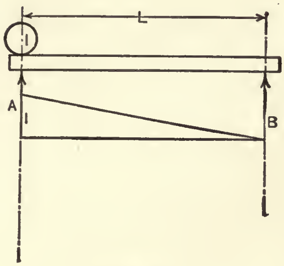


FIG. 89.

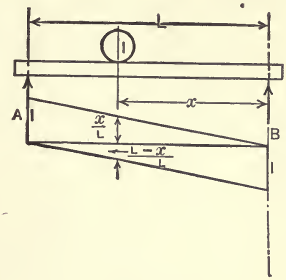


FIG. 90.

of the first load is the sum of the products of the several intercepts in the triangle by their respective loads. In the case illustrated, Fig. 91, the left reaction or shear to the left of load 1 is

$$R_1 = P_1y_1 + P_2y_2 + P_3y_3 + P_4y_4 + P_5y_5.$$

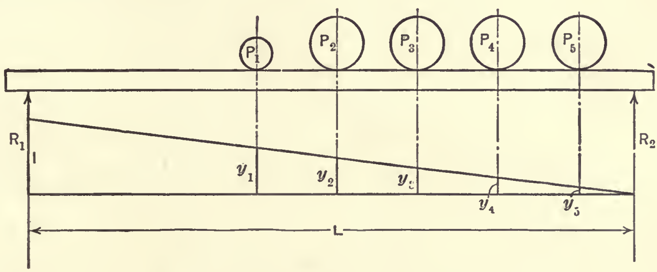


FIG. 91.

If the entire system of loads is moved to the left a distance x, but no loads enter or leave the span, R_1 increases an amount

$$(P_1 + P_2 + P_3 + P_4 + P_5)\ \frac{x}{L},$$

making the reaction

$$R_1' = \Sigma Py + \Sigma P \frac{x}{L}.$$

Maximum Shear at any Point

The vertical shear at a section a distance x from the left reaction, Fig. 92, is $V_{x_1} = \Sigma Py$.

Under the load P_1 this is reduced by the amount P_1 making the shear to the right of P_1, $V_2 = \Sigma\, Py - P_1$. If the loads are moved to the left until P_2 is at section x from the left reaction the

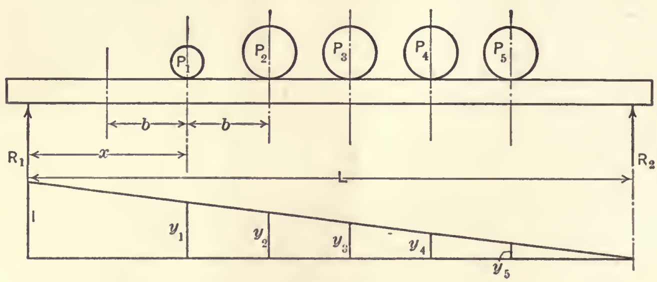


Fig. 92.

shears, providing the same loads only are now on the span, will have increased an amount $\Sigma P \frac{b}{L}$, making

$$V_{x_2} = \Sigma Py - P_1 + \Sigma P \frac{b}{L}.$$

Comparing V_{x_1} and V_{x_2} it is seen that V_{x_1} will be the greater so long as

$$P_1 > \frac{\Sigma Pb}{L} \quad \text{or} \quad \frac{P_1}{b} > \frac{\Sigma P}{L}.$$

Under these circumstances the shear at x will be a maximum with P_1 at x when $\frac{P_1}{b}$ exceeds the sum of all the loads on the span divided by that span. If $\frac{P_1}{b} = \frac{\Sigma P}{L}$ the shears will be the same at x with either P_1 or P_2 at that point, while if $\frac{P_1}{b} < \frac{\Sigma P}{L}$ the maximum shear will occur with P_2 at x.

Had another load P_6 come on the span, Fig. 93, the increase in shear, after the load P_1 had passed x and advanced a distance b to the left of it, would be

$$-P_1 + \Sigma P_{1-5}\frac{b}{L} + \frac{P_6 \cdot c}{L}.$$

Here ΣP_{1-5} represents the sum of the loads P_1 to P_5 inclusive, but not the load P_6 just assumed as coming on the span, while ΣP_{1-6} is the sum of the loads from P_1 to P_6 inclusive. The dis-

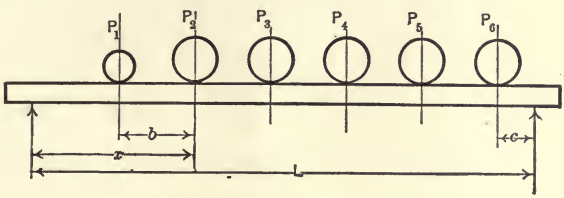


FIG. 93.

tance c can only range in value from zero to $c = b$, hence the increase in shear will be somewhere between

$$\Sigma P_{1-5}\frac{b}{L} - P_1 \quad \text{and} \quad \Sigma P_{1-6}\frac{b}{L} - P_1.$$

Where the first expression is negative and the latter positive both positions of the drivers should be tried.

Load P_1 at section x will give a maximum shear when

$$\frac{P_1}{b} \geqq \frac{\Sigma P_{1-6}}{L}$$

and load P_2 will give a maximum shear when

$$\frac{P_1}{b} \leqq \frac{\Sigma P_{1-5}}{L}.$$

In the case of a uniform load the vertical shear at x is a maximum when the portion of the span to the right of x is fully loaded, while the shear will be a minimum when the portion to the left of x is fully loaded, x being less than one-half the span.

Position of a system of moving loads to give a maximum shear in any panel of a truss with parallel or inclined chords.

In Figs. 94 and 95,

ΣP_1 is the sum of the loads to the left of the panel.

ΣP_2 is the sum of the loads on the panel, here panel 2–3.

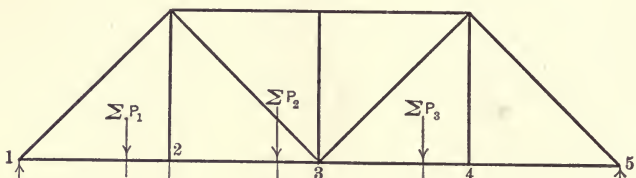
ΣP_3 is the sum of the loads to the right of the panel.

m is the number of panels to the left of panel 2–3.

n is the number of panels in the truss.

p is the length of a panel.

FIG. 94.



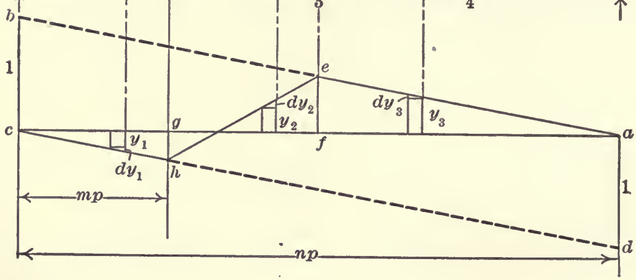


FIG. 95.

The influence lines for loads of unity *ab* and *cd* are drawn as for shear in a beam or girder. When point 3 is reached the intercept in the triangle *abc* is

$$ef = +\frac{1 \times (n - m - 1)}{n}.$$

Now as the load moves across panel 2–3 the shear is reduced gradually until when point 2 is reached the unit load is deducted making the shear

$$gh = \frac{n - m}{n} - 1 = -\frac{m}{n}.$$

The influence diagram for the shear in panel 2–3 is given by the

broken line *chea*. The maximum positive shear in panel 2–3 is

$$V = \Sigma P_3 y_3 + \Sigma P_2 y_2 - \Sigma P_1 y_1.$$

Moving the loads a small distance dx to the left, no load passing a panel point or end reaction, the shear becomes

$$V + dV = \Sigma P_3(y_3 + dy_3) + \Sigma P_2(y_2 - dy_2) - \Sigma P_1(y_1 - dy_1).$$

By subtraction

$$dV = \Sigma P_3\, dy_3 - \Sigma P_2\, dy_2 + \Sigma P_1\, dy_1. \qquad (1)$$

Comparing the differential triangles with the similar triangles *cgh*, *hegf* and *fea*, we have

$$\frac{dy_1}{dx} = -\frac{1}{np}; \quad \frac{dy_2}{dx} = \frac{n-1}{np} \quad \text{and} \quad \frac{dy_3}{dx} = \frac{1}{np}.$$

Now dividing equation (1) by dx and putting $\frac{dV}{dx} = 0$, to examine for a maximum, we have

$$\frac{dV}{dx} = 0 = \Sigma P_3 \frac{1}{np} - \Sigma P_2 \frac{n-1}{np} + \Sigma P_1 \frac{1}{np}$$

and

$$\Sigma P_2 = \frac{\Sigma P_1 + \Sigma P_2 + \Sigma P_3}{n}.$$

This states that the vertical shear in any panel will be a maximum when the load in that panel equals the average panel load for the span. It is necessary that a load near the head of the train be at the panel point to the right of the panel in which the shear is sought.

Maximum Floor-beam Reaction

Figure 96 shows two panels p_1 and p_2 with floor beams a, b and c and accompanying stringers. To find the maximum floor-beam reaction at b construct the influence diagram, Fig. 96, making its ordinate at *ef* equal one. Let ΣP_1 be the sum of the loads in panel p_1 and ΣP_2 the same for panel p_2, then for a maximum floor-beam load $\frac{\Sigma P_1}{p_1} = \frac{\Sigma P_1 + \Sigma P_2}{p_1 + p_2}$. Fig. 97 is the influence line for the bending moment at b.

Ordinates y and y_1 similarly located in Figs. 96 and 97 will be to each other as the corresponding intercepts ef and hi or

$$\frac{y}{y_1} = \frac{p_1 + p_2}{p_1 p_2};$$

hence

$$y = \frac{p_1 + p_2}{p_1 p_2} \cdot y_1.$$

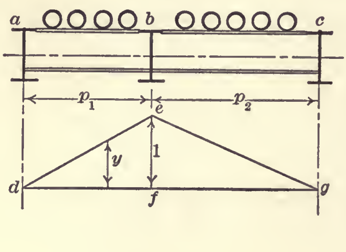


FIG. 96.

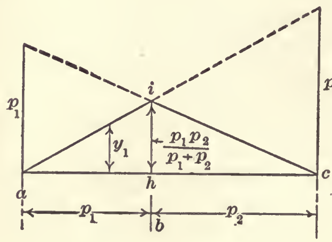


FIG. 97.

To find the maximum floor-beam reaction determine the maximum bending moment on a beam whose span is $l = p_1 + p_2$ at a distance p_1 from the left support a, and multiply it by the sum of the panels p_1 and p_2 and divide this by the product of p_1 and p_2; generally $p_1 = p_2$, so that the maximum reaction will ordinarily equal twice the maximum bending moment divided by the panel width.

Moment Table

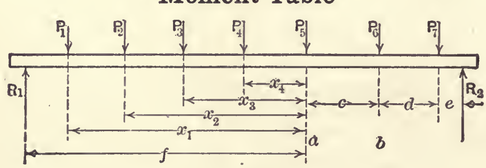


FIG. 98.

When the stresses are determined by calculation, advantage is taken of the following principles in computing.

The moment due to the loads about the section under P_5 in Fig. 98 is

$$M_1 = P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 + P_5x_5.$$

If the moment is desired under P_6, P_6 being a distance c from P_5, then

$$M_2 = P_1(x_1 + c) + P_2(x_2 + c) + \ldots P_5c.$$

In the second instance if ΣP is the sum of all the loads on the span to the left of P_6, then

$$M_2 = M_1 + \Sigma P \times c.$$

In this way, starting at the head of the train, the moments of the loads to the left about a point under each successive load are calculated and noted. Having placed the loads in the position to give maximum bending at the point to be investigated, first find the reaction R_1 and then the bending moment at the point desired.

If M_3 is the bending due to loads P_1 to P_6 about the section under P_7 the bending moment about R_2 of loads P_1 to P_7, inclusive, will be

$$M_4 = M_3 + \Sigma(P_1 - P_7)e$$

and the reaction is

$$R_1 = \frac{M_4}{S}$$

where S is the span.

The bending moment at a section under P_5 then is

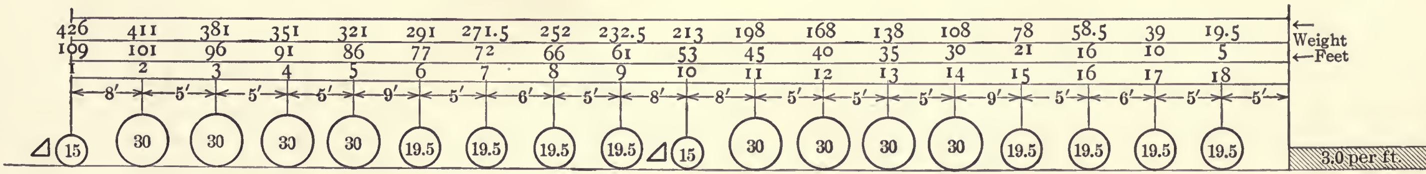
$$M = R_1f - M_1.$$

The work of calculating stresses due to moving locomotive and train loads is facilitated by the use of the following table which has been computed in the manner just described for Cooper's E–60 loading.

In this moment table the consecutive wheel loads are numbered from the left to the right, and the distances given between adjacent wheels, the sum of the distances from the train load to each wheel and the sum of the loads from load 18 to and including each wheel load are also given. In the body of the table above the heavy zigzag line are given the moments of all loads between the heavy vertical line at the right of the horizontal row of moments and any wheel load to the left, about the vertical

MOMENT TABLE FOR COOPER'S CLASS E-60

Two 213-ton engines followed by a 6000 lbs. per ft. uniform train load. Loads are given for one line of wheels.



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24,546	22,911	19,881	17,001	14,271	11,691	10,189.5	8,785.5	7,498.5	6,309	5,514	4,164	2,964	1,914	1,014	604.5	292.5	97.5
22,416	20,856	17,976	15,246	12,666	10,236	8,832	7,525.5	6,336	5,244	4,524	3,324	2,274	1,374	624	312	97.5	....
20,383.5	18,898.5	16,168.5	13,588.5	11,158.5	8,878.5	7,572	6,363	5,271	4,276.5	3,631.5	2,581.5	1,681.5	931.5	331.5	117	....	97.5
18,061.5	16,666.5	14,116.5	11,716.5	9,466.5	7,366.5	6,177	5,085	4,110	3,232.5	2,677.5	1,807.5	1,087.5	517.5	97.5	....	117	331.5
16,224	14,907	12,504	10,254	8,154	6,204	5,112	4,117.5	3,240	2,460	1,980	1,260	690	270	....	97.5	312	624
13,092	11,907	9,777	7,797	5,967	4,287	3,370.5	2,551.5	1,849.5	1,245	900	450	150	....	175.5	448.5	838.5	1,326
11,502	10,392	8,412	6,582	4,902	3,372	2,553	1,831.5	1,227	720	450	150	....	150	423	793.5	1,281	1,866
10,062	9,027	7,197	5,517	3,987	2,607	1,885.5	1,261.5	754.5	345	150	....	150	450	820.5	1,288.5	1,873.5	2,556
8,772	7,812	6,132	4,602	3,222	1,992	1,368	841.5	432	120	....	150	450	900	1,368	1,933.5	2,616	3,396
6,948	6,108	4,668	3,378	2,238	1,248	780	409.5	156	....	240	630	1,170	1,860	2,484	3,205.5	4,044	4,980
5,244	4,524	3,324	2,274	1,374	624	312	97.5	....	120	600	1,230	2,010	2,940	3,720	4,597.5	5,592	6,684
4,276.5	3,631.5	2,581.5	1,681.5	931.5	331.5	117	....	97.5	292.5	922.5	1,702.5	2,632.5	3,712.5	4,590	5,565	6,657	7,846.5
3,232.5	2,677.5	1,807.5	1,087.5	517.5	97.5	....	117	331.5	616.5	1,426.5	2,386.5	3,496.5	4,756.5	5,751	6,843	8,052	9,358.5
2,460	1,980	1,260	690	270	....	97.5	312	624	984	1,944	3,054	4,314	5,724	6,816	8,005.5	9,312	10,716
1,245	900	450	150	....	175.5	448.5	838.5	1,326	1,821	3,051	4,431	5,961	7,641	8,908.5	10,273.5	11,755.5	13,335
720	450	150	....	150	423	793.5	1,281	1,866	2,436	3,816	5,346	7,026	8,856	10,221	11,683.5	13,263	14,940
345	150	....	150	450	820.5	1,288.5	1,873.5	2,556	3,201	4,731	6,411	8,241	10,221	11,683.5	13,243.5	14,920.5	16,695
120	....	150	450	900	1,368	1,933.5	2,616	3,396	4,116	5,796	7,626	9,606	11,736	13,296	14,953.5	16,738	18,610
....	240	630	1,170	1,860	2,484	3,205.5	4,044	4,980	5,820	7,740	9,810	12,030	14,400	16,116	17,929.5	19,860	21,888

Feet →	8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
Weight →	15	45	75	105	135	154.5	174	193.5	213	228	258	288	318	348	367.5	387	406.5	426

line. Thus the moment of loads from 8 to 18 inclusive about the vertical line marking the beginning of the uniform train load is 8,785,500 ft. lbs. (Both pounds and foot pounds are expressed in thousands.)

Below the heavy zigzag line are given the moments of the loads to the right of the heavy vertical line in that row; thus the moment of the loads 9 to 15, inclusive, about load 9 is 3,720,000 ft. lbs. Below the body of the table are given the sum of the distances from wheel 1 to the several wheel loads, and also the sum of the loads from load 1 to any other load including both this latter load and load 1.

A simple example will illustrate one use of this table. In a girder whose span is 80 ft. what is the left reaction when load 1 is over that support?

Wheel 14 being 79 ft. from wheel 1 when 1 is on the left pier, 14 will be 1 ft. from the right pier. The moment of wheels 1 to 14 about wheel 1 is 14,400,000 ft. lbs., read below the zigzag line.

The right reaction is this moment divided by the span or $\frac{14{,}400{,}000}{80} = 180{,}000$ lbs. The left reaction is the sum of the loads on the span minus the right reaction or $348{,}000 - 180{,}000 = 168{,}000$. The reaction can also be determined by using the moments above the zigzag line. The moment of the loads from 1 to 14 is 13,092,000 ft. lbs., but load 14 being 1 ft. from the right pier the moment of the loads as placed upon the girder about the right pier is 13,092,000 + the sum of the loads from 1 to 14 multiplied by 1 ft.

$$M = 13{,}092{,}000 + (348{,}000 \times 1) = 13{,}440{,}000 \text{ ft. lbs.}$$

The left reaction is

$$\frac{13{,}440{,}000}{80} = 168{,}000 \text{ lbs.},$$

the same as found before.

The use of this table and of the rules just derived by means of the influence diagrams will be illustrated by the following problem.

A through Pratt truss, Fig. 99, carries a train load represented by Cooper's E–40 loading.* Find the stresses in the members of the second panel from the left pier.

To find the maximum stress in member *EI* determine the maximum bending at apex 4. The criterion for maximum bending at a joint in a truss is that the average load to the left of the joint shall equal the average load on the entire span.

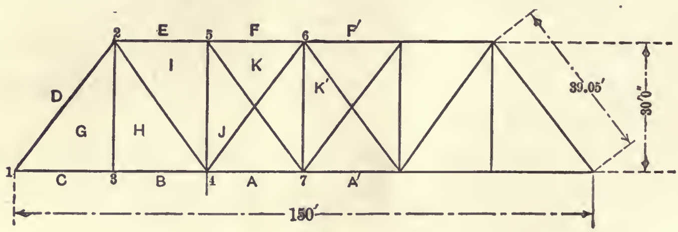


FIG. 99.

Trying load 7 at apex 4:

$$\left\{\begin{array}{l}\text{Panel load to left not}\\ \quad\text{including load 7}\end{array}\right\} = \text{Average panel load on bridge.}$$

$$\frac{103{,}000}{2} = 51{,}500;\quad \frac{340{,}000}{6} = 56{,}670 \text{ lbs.}$$

$$\left\{\begin{array}{l}\text{Panel load to left}\\ \quad\text{including load 7}\end{array}\right\}$$

$$\frac{116{,}000}{2} = 58{,}000;\quad \frac{340{,}000}{6} = 56{,}670 \text{ lbs.}$$

To find reaction at the left, take moments of loads about right support. When load 7 is at apex 4, which is 50 ft. from the left pier, 28 ft. of uniform load will be on the bridge. The moment about the right pier then is

$$M = 16{,}364 + (284 \times 28) + (28 \times 2 \times 14) = 25{,}100 \text{ (1000 ft. lbs.)}$$

from which the reaction at the left is

$$R_1 = \frac{25{,}100{,}000}{150} = 167{,}300 \text{ lbs.}$$

* NOTE. — Loads and moments will be $\frac{40}{60}$ ($\frac{2}{3}$) of Cooper's E–60 loading in the moment-tables, page 72; the wheel spacing will remain the same.

The bending moment under apex 4 is

$$M = \left(\frac{25,100,000 \times 50}{150}\right) - 2,155,000 = 6,211,700 \text{ ft. lbs.}$$

Here 2,155,000 is the bending due to loads 1 to 6, inclusive, about load 7.

The stress in member *EI* is

$$\frac{6,211,700}{30} = 207,060 \text{ lbs.}$$

Stress in member *HI*. To determine the live load stresses in *HI*, first find the maximum shear in panel 3–4. For the shear in any panel to be a maximum the load in that panel must be equal to the average panel load on the bridge.

Load 4 at apex 4.	Load in panel.	Average load in panel.
	Lbs.	Lbs.
Load omitting load 4	50,000	302,000/6 = 50,330
Load including load 4	70,000	302,000/6 = 50,330

The maximum shear in panel 3–4 occurs with wheel 4 at apex 4. To find the reaction under these conditions when load 18 is 9 ft. to the left of the right pier we have

$$16,364 + (284 \times 9) + (9 \times 2 \times 4.5) = 19,001 \text{ (1000 ft. lbs.)}$$

$$\text{Reaction} = \frac{19,001,000}{150} = 126,670 \text{ lbs.}$$

The reaction at 3 due to loads in panel 3–4 is

$$R = \frac{\text{moments of loads 1–4 about 4}}{25}.$$

$$R = \frac{480,000}{25} = 19,200 \text{ lbs.}$$

Maximum shear panel 3–4 = 126,670 – 19,200 = 107,470 lbs. If tried for load 3 at apex 4, the maximum shear will be found to be the same.

Using this shear the live-load stress in *HI* is found to be

$$HI = 107,470 \times \frac{39.05}{30} = 139,890 \text{ lbs.}$$

CHAPTER VI

TENSION, COMPRESSION PIECES AND BEAMS

DESIGNING any piece of a structure requires the determination of the resisting forces called forth in it to balance the external forces acting upon it. In the case of a purely tension piece this requires that the minimum or net section multiplied by the allowable working fiber stress shall equal or exceed the total force acting in the piece.

Compression pieces whose lengths do not exceed five times their least diameter can be designed by assuming the total load equal to the allowable working fiber stress in compression times the area of the gross section. In both cases care should be taken to have the load distributed over the section as improper application of the external forces to the piece may result in introducing bending or injurious local concentrations of stress in it.

BEAMS

Upon the following fundamental conditions of static equilibrium determinations of the required forces or moments acting on or in the piece are made. This applies whether the determinations are made algebraically or graphically.

The sum of all the vertical forces = 0.

The sum of all the horizontal forces = 0.

The sum of the moments of all forces about any point = 0.

Reactions. — In a beam acted on by several vertical forces the sum of the reactions or forces at the supports must equal the sum of the loads, and the algebraic sum of the moments of all forces and reactions referred to any point must be zero. In Fig. 100

$$P_1 + P_2 + P_3 = R_1 + R_2. \qquad (1)$$

Taking moments about any point in the reaction R_2 we have

$$M = P_1a_1 + P_2a_2 + P_3a_3 - R_1l = 0. \qquad (2)$$

The solution of equation (2) gives R_1. R_2 will be given by substitution in equation (1).

Vertical Shear. — The vertical shear at any section is the algebraic sum of all the external forces on the left of that section. Thus a shear diagram of the beam is shown by Fig. 101.

At the left support the vertical shear equals the reaction R_1, and this value of the shear continues toward the right until under the load P_3 the shear becomes $R_1 - P_3$; similarly, under P_2 the vertical shear is $R_1 - (P_3 + P_2)$ and under P_1 the shear is $R_1 - (P_3 + P_2 + P_1) = - R_2$.

FIG. 100.

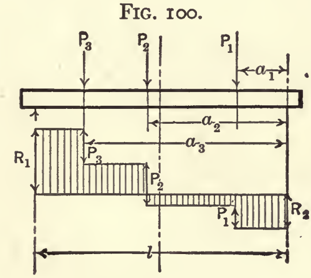


FIG. 101.

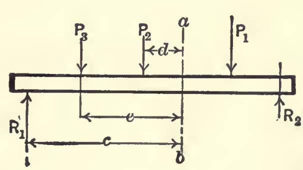


FIG. 102.

Bending Moment. — The bending moment at any section is the algebraic sum of the moments of the external forces on the left of that section referred to a point in that section. In the beam, Fig. 102, the bending at the section *ab* is

$$M = R_1c - P_3e - P_2d.$$

Where rolled beams are used generally only the maximum bending is required; this will occur where the vertical shear passes through zero. (See any book on Mechanics of Materials for the proof of this.) In the case illustrated in Fig. 101, the maximum bending occurs under load P_2. Having found the maximum bending moment, if the beam is supported laterally, the proper

section can usually be selected from a manufacturer's handbook. In building construction beams will commonly be supported laterally by flooring, roofing or bracing.

The external bending moment induces an equal and opposite moment in the material of the beam called a resisting moment. This is expressed by equation

$$M = \frac{fI}{e},*$$

where

M = the external bending moment at a given section, in. lbs.

$\frac{fI}{e}$ = the resisting moment at the same section.

f = the extreme unit fiber stress, generally pounds per square inch.

I = the moment of inertia of the beam section, in inches^4.

e = the distance from the neutral axis to the extreme fibers in inches.

The expression $\frac{I}{e}$ depends entirely upon the form of the beam section and is sometimes called the section modulus.

An example will illustrate the selection of a floor beam. A floor beam with a span of 12 ft. is to carry a uniform load of 14,400 lbs. Select a suitable beam allowing a working fiber stress of 16,000 lbs. per sq. in.

The maximum bending for a supported beam with a uniform load is $M = \frac{Wl}{8}$, where W is the total uniform load in pounds and l is the length of the span in inches.

Substituting the given values in this formula we have

$$M = \frac{14{,}400 \times (12 \times 12)}{8} = 259{,}200 \text{ in. lbs.}$$

The handbooks usually tabulate the values of $\frac{I}{e}$ of their sections.

* For the derivation of this formula see any book on "Mechanics of Materials."

By transposition of the previously given equation $\frac{I}{c} = \frac{M}{f}$, then by substitution

$$\frac{I}{e} = \frac{259{,}200}{16{,}000} = 16.2.$$

The lightest weight standard I beam providing a sufficient section modulus is a 9-in. I beam, weighing 21 lbs. per ft. Its section modulus is 18.9.

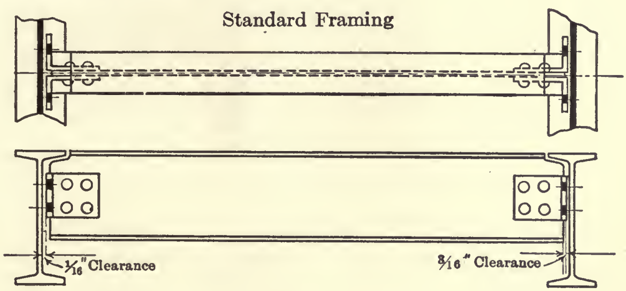


Fig. 103.

Not infrequently the beam must not only be amply strong but it must not deflect excessively under load. When plastered ceilings are carried under the beams this deflection is limited to $\frac{1}{360}$ of the span. The formula for the deflection at the middle of a supported beam carrying a uniform load is

$$\Delta = \frac{5\,Wl^3}{384\,EI} \quad \text{or} \quad I = \frac{5\,Wl^3}{384\,E \times \Delta}.$$

For steel $E = 30{,}000{,}000$ lbs. per sq. in.

If the deflection is $\Delta = \frac{\text{Span}}{360} = \frac{144}{360} = \frac{2}{5}$ in.,

the limiting inertia then will be

$$I = \frac{5\,Wl^3}{384\,E \times \Delta} = \frac{5 \times 14{,}400 \times 144^3}{384 \times 30{,}000{,}000 \times \frac{2}{5}} = 46.7.$$

It is, therefore, evident that the 9-in. I beam weighing 21 lbs. per ft. demanded for strength will be amply stiff, its inertia being 84.9 or almost twice the inertia required for stiffness.

In framing beams into girders, beams or columns each manufacturer has a standard framing. This framing is designed for the shortest span and consequently the greatest load for which the beam is likely to be used.

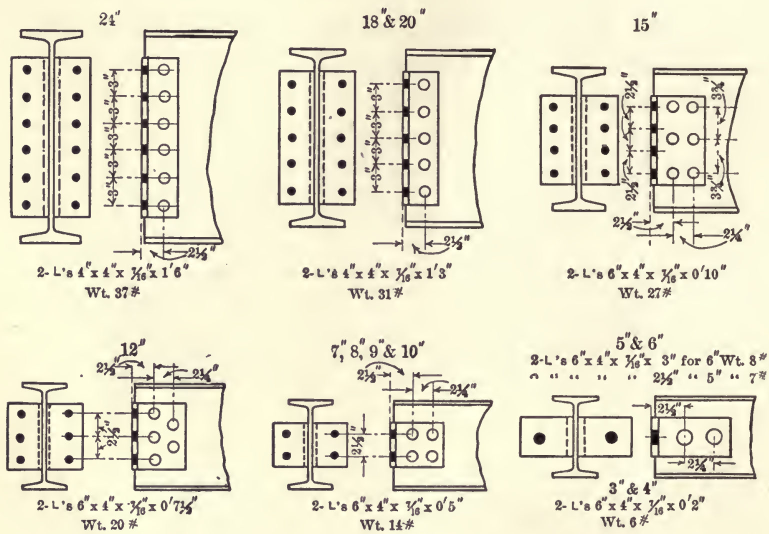


FIG. 104.

The standard framing, Fig. 104, should not be used for spans less than the following:

I.	Lb. Span, ft.	*I.*	Lb. Span, ft.	*I.*	Lb. Span, ft.	*I.*	Lb. Span, ft.
24	80.0–22.0	15	80.0–20.0	10	25.0–9.0	6	12.25–6.0
20	80.0–22.0	15	60.0–15.5	9	21.0–7.0	5	9.75–4.0
20	65.0–18.0	15	42.0–11.0	8	18.0–5.5	4	7.5 –3.0
18	55.0–14.0	12	40.0–11.5	7	15.0–4.0	3	5.5 –2.0
..	....	12	31.5– 9.0	..	......	..	......

All rivets in standard framing are $\frac{3}{4}$ in. in diameter.

As this goes to press the American Bridge Company, in their specifications for steel structures, publishes a revised standard

for framing, in which all beams use 4 in. $\times$ 4-in. angles from 27 ins. to 12 ins. inclusive; smaller beams use 6 in. $\times$ 4-in. angles.

When work is being detailed for production in a particular shop the standards of that shop should be ascertained and adhered to.

It should be noted in Fig. 103 that to facilitate erection the connecting angles extend $\frac{1}{8}$ in. beyond the end of the web of the beam and that the distance back to back of the connecting angles is $\frac{1}{8}$ in. less than the space into which the beam is fitted. When drawings are being made for a shop having standard framing the beam sketch may be like Fig. 103, no dimensions being placed on the standard rivet spaces.

The advantages of the standard framing are:

It simplifies shop work, enabling a large number of angles to be made at one time. The punching of the rivet holes both in beams and angles can be done by means of multiple punches, a group of holes being made at one stroke of the punch. The drawing-room work is also reduced. The total saving in time more than counterbalances any probable waste of material. A criticism of the method used for designing these connections is that there is twisting introduced into the rivet groups increasing the shear on the rivets. Tests made on some full-sized beams with standard framing seemed to justify the usual practice as being ample notwithstanding the above theoretical criticism.

When, as is the case for very short spans or for beams with thin webs, it is necessary to design special connections the calculations should be based upon the shearing and bearing value of the material; see page 85.

Riveting. — There are two types of rivet heads: " button heads," which approximate hemispherical, and "countersunk," which are truncated cones. The button-headed rivets should always be used where clearances will permit. Button heads can be flattened slightly where additional clearance is needed and this requires less work than countersunk rivets. The following illustrations, Figs. 105 and 106, give the dimensions of various

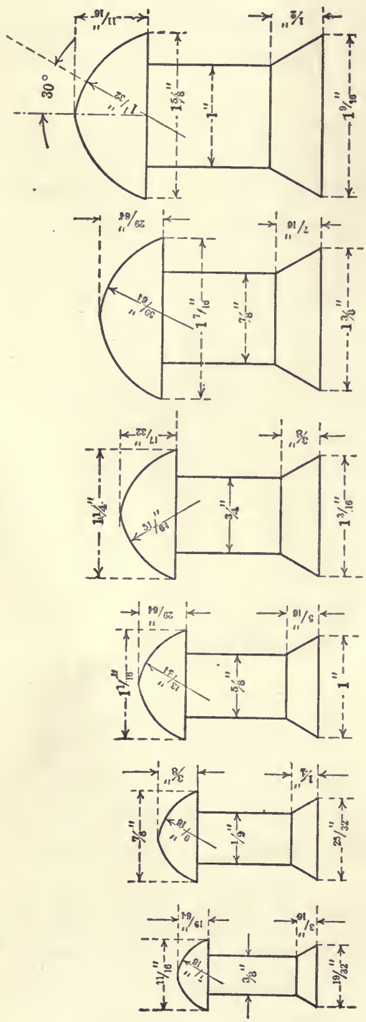


Fig. 105.

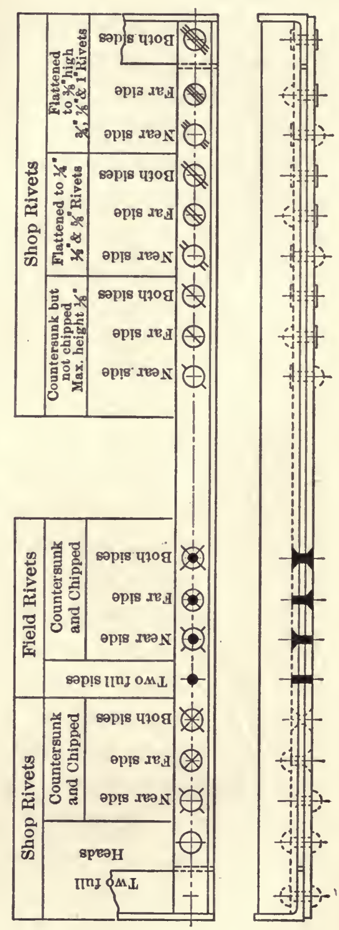


Fig. 106.

rivet sizes and symbols for shop and field driving, and are taken from the American Bridge Company's Book of Standards.

Rivets are designated by their nominal diameter D and the length under the head before driving. This length, Figs. 107 and 108, is made up of the grip, the distance through the material held together plus the length called the upset or the stock required to form the head and bring the body of the rivet up to the size of the rivet hole. It is necessary for the rivet holes to be larger than the rivet stock, otherwise there would be

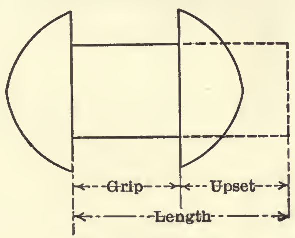


FIG. 107.

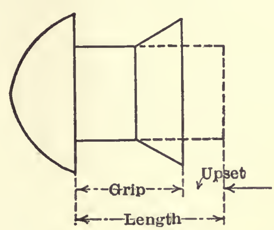


FIG. 108.

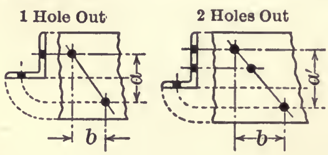
difficulty in placing the rivets in their proper rivet holes and the stock would be cold before they could be driven. The holes are usually $\frac{1}{16}$ in. larger in diameter than the diameter of the rivets.

The number of rivets required in any joint depends upon the forces being transmitted by the rivets, and upon the rivet values in shear and bearing. The value in shear may be that due to either single or double shear as the rivet tends to fail by shearing in one or two cross sections. The bearing value of the rivet is the rivet diameter times the lesser thickness of the materials transmitting the forces to the rivets times the allowable unit bearing value of the rivet material. The following tables give the shearing and bearing values of the usual rivet sizes.

AREA TO BE DEDUCTED FOR VARIOUS SIZES OF RIVETS AND THICKNESSES OF RIVETED METAL

Rivet diam.	Thickness of riveted metal.													
	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
$\frac{3}{8}$	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.31	0.34	0.38	0.41	0.44	0.47	0.50
$\frac{1}{2}$	0.12	0.16	0.20	0.23	0.27	0.31	0.35	0.39	0.43	0.47	0.51	0.55	0.59	0.63
$\frac{5}{8}$	0.14	0.19	0.23	0.28	0.33	0.38	0.42	0.47	0.52	0.56	0.61	0.66	0.70	0.75
$\frac{3}{4}$	0.16	0.22	0.27	0.33	0.38	0.44	0.49	0.55	0.60	0.66	0.71	0.77	0.82	0.88
$\frac{7}{8}$	0.19	0.25	0.31	0.38	0.44	0.50	0.56	0.63	0.69	0.75	0.81	0.88	0.94	1.0
1	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70	0.77	0.84	0.91	0.98	1.05	1.13

NOTE. — The size of the rivet hole has been assumed as $\frac{1}{8}$ inch larger than the nominal diameter of the rivet.

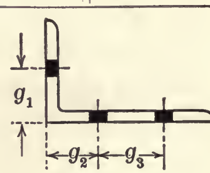


STAGGER OF RIVETS TO MAINTAIN NET SECTION

Dimensions in inches.

a	$\frac{3}{4}$-in. rivet.	$\frac{7}{8}$-in. rivet.	a'	$\frac{3}{4}$-in. rivet.	$\frac{7}{8}$-in. rivet.
	b	b		b	b
1	$1\frac{5}{8}$	$1\frac{3}{4}$	5	$3\frac{1}{16}$	$3\frac{5}{16}$
$1\frac{1}{2}$	$1\frac{7}{8}$	2	$5\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{2}$
2	$2\frac{1}{16}$	$2\frac{1}{4}$	6	$3\frac{3}{8}$	$3\frac{5}{8}$
$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{7}{16}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{3}{4}$
3	$2\frac{7}{16}$	$2\frac{5}{8}$	7	$3\frac{5}{8}$	$3\frac{7}{8}$
$3\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{13}{16}$	$7\frac{1}{2}$	$3\frac{3}{4}$	4
4	$2\frac{13}{16}$	3	8	$3\frac{7}{8}$	$4\frac{1}{8}$
$4\frac{1}{2}$	$2\frac{15}{16}$	$3\frac{3}{16}$	$8\frac{1}{2}$	4	$4\frac{1}{4}$

GAUGES FOR ANGLES, INCHES



Leg	8	7	6	5	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{4}$	1	$\frac{3}{4}$
g_1	$4\frac{1}{2}$	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	2	$1\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$
g_2	3	$2\frac{1}{2}$	$2\frac{1}{2}$	2	...	...	...	...	...	...	...	...	...	...	...
g_3	3	3	$2\frac{1}{4}$	$1\frac{3}{4}$	...	...	...	...	...	...	...	...	...	...	...
Max. rivet	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

NOTE. — Special gauges may be used where advisable.

SHEARING AND BEARING VALUE OF RIVETS

Diam. of rivet, inches.		Area in sq. ins.	Single shear at 10,000 lbs.	Bearing value for different thicknesses of plate, in inches, at 20,000 lbs. per sq. in.												
Fraction.	Decimal.			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	0.375	0.1104	1,100	1,880	2,340	2,810	......	......	......	......	......	......	......	......	......	......
1/2	0.500	0.1963	1,960	2,500	3,130	3,750	4,380	5,000	......	......	......	......	......	......	......	......
5/8	0.625	0.3068	3,070	3,130	3,910	4,690	5,470	6,250	7,030	7,810	......	......	......	......	......	......
3/4	0.750	0.4418	4,420	3,750	4,690	5,630	6,560	7,500	8,440	9,380	10,310	11,250	......	......	......	......
7/8	0.875	0.6013	6,010	4,380	5,470	6,560	7,660	8,750	9,840	10,940	12,030	13,130	14,220	15,310	......	......
1	1.000	0.7854	7,850	5,000	6,250	7,500	8,750	10,000	11,250	12,500	13,750	15,000	16,250	17,500	18,750	20,000

Diam. of rivets, inches.		Area in sq. ins.	Single shear at 12,000 lbs.	Bearing value for different thicknesses of plate, in inches, at 24,000 lbs. per sq. in.												
Fraction.	Decimal.			1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	0.375	0.1104	1,320	2,250	2,810	3,380	......	......	......	......	......	......	......	......	......	......
1/2	0.500	0.1963	2,360	3,000	3,750	4,500	5,250	6,000	......	......	......	......	......	......	......	......
5/8	0.625	0.3068	3,680	3,750	4,690	5,630	6,560	7,500	8,440	9,380	......	......	......	......	......	......
3/4	0.750	0.4418	5,300	4,500	5,630	6,750	7,880	9,000	10,130	11,250	12,380	13,500	......	......	......	......
7/8	0.875	0.6013	7,220	5,250	6,560	7,880	9,190	10,500	11,810	13,130	14,400	15,750	17,060	18,380	......	......
1	1.000	0.7854	9,420	6,000	7,500	9,000	10,500	12,000	13,500	15,000	16,500	18,000	19,500	21,000	22,500	24,000

A few examples will illustrate the application of the above principles.

f_c = the unit bearing value, pounds per square inch.

f_s = the unit shearing value, pounds per square inch.

Evidently the rivets in Fig. 109 would tend to shear in but one place, along *ab*, and are, therefore, in single shear.

$$\text{Shearing value} = \frac{\pi d^2}{4} \times f_s. \qquad \text{Bearing value} = d \times t_1 \times f_c.$$

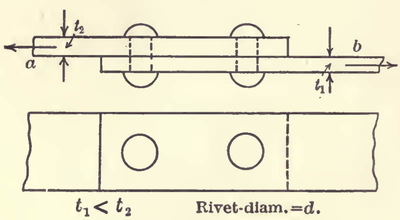


FIG. 109.

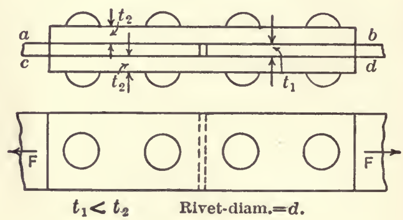


FIG. 110.

In Fig. 110 the rivets are in double shear as they tend to fail along the sections *ab* and *cd*.

$$\text{Shearing value} = \frac{\pi d^2}{2} \cdot f_s. \qquad \text{Bearing value} = d \times t_1 \times f_c.$$

The following numerical example will illustrate the design of a joint like this one.

Example. — A joint of the type shown in Fig. 110 is to transmit 15,000 lbs.; $t_1 = \frac{1}{2}$ in., $t_2 = \frac{3}{8}$ in., rivets $\frac{3}{4}$ in. in diameter, the allowable shearing stress 10,000 lbs. per sq. in., allowable bearing stress 20,000 lbs. per sq. in. The rivet is in double shear while the minimum bearing value will be that of the rivet against the $\frac{1}{2}$-in. plate as in the other direction two plates each $\frac{3}{8}$ in. thick act. From the tables the rivet value of $\frac{3}{4}$-in. rivets in double shear at 10,000 lbs. is $4420 \times 2 = 8840$ lbs. The rivet value in bearing on a $\frac{1}{2}$-in. plate is 7500 lbs. As the joint is to transmit 15,000 lbs. the number of rivets is found by dividing 15,000 lbs. by the smaller of the two rivet values, *i.e.*, $\frac{15,000}{7500} = 2.$

Riveted joints should be designed to avoid eccentric stress if possible. Where this is unavoidable such eccentricity should be reduced to a minimum and the resultant forces acting upon the individual rivets be determined. An instance of such a joint, frequently met with, is the connection of the foot of a knee brace with the column in a light steel building frame. The force F_1, 24,000 lbs., Fig. 111, is transferred through the rivets 1, 2 and 3 to the sketch plate and in turn from the plate by the rivets 4 to 11, inclusive, to the column angles. The center of gravity of the rivet group, 4 to 11, is easily located as its center of symmetry.* Rivets 4, 7, 8 and 11 are each $7\frac{1}{2}$ ins. from the center of gravity and rivets 5, 6, 9 and 10 are each 6.18 ins. from the same point.

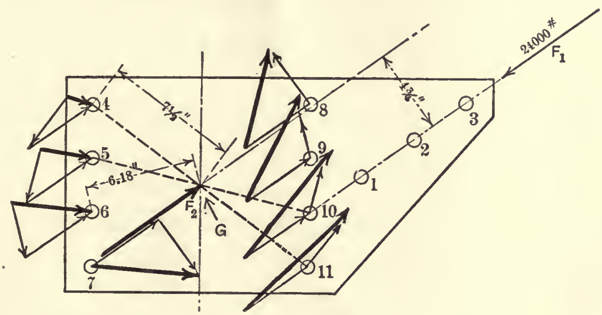


FIG. 111.

The rivets must offer a resistance F_2 equal, opposite and parallel to F_1. This resultant force F_2 acts through the center of gravity of the rivet group, and the direct force F_2 is shared equally by the rivets of the group so that each rivet carries $\frac{24,000}{8} = 3000$ lbs. The forces F_1 and F_2 are 4.75 ins. apart so that the moment opposed by the group is $24,000 \times 4.75 = 114,000$ in. lbs.

The force opposed by a rivet will be proportional to its distance from the center of gravity of the rivet group. The rivets

* When the center of gravity of a rivet group cannot be seen by inspection it can be readily calculated. If the group has any axis of symmetry the center of gravity must lie on that axis.

4, 7, 8 and 11 being equally distant from G will oppose equal forces and similarly of the rivets 5, 6, 9 and 10. Let S be the force acting at 4, 7, 8 and 11, then $\left[\frac{6.18}{7.50} \times S\right]$ is the force acting on rivets 5, 6, 9 and 10. The moment of each of the rivets 4, 7, 8 and 11 will be $S \times 7.50$ and the moment for each of the rivets 5, 6, 9 and 10 will be $\left(S \times \frac{6.18}{7.50}\right) \times 6.18$.

The total moment for the eight rivets is

$$(4 \times S \times 7.50) + \left(4 \times S \times \frac{6.18^2}{7.50}\right) = 114{,}000 \text{ in. lbs.}$$

$$30 S + 20.4 S = 114{,}000 \text{ in. lbs.}$$

$$S = \frac{114{,}000}{50.4} = 2262 \text{ in. lbs.}$$

The force acting on each of the rivets 5, 6, 9 and 10 will be

$$\left(S \times \frac{6.18}{7.50}\right) = 2262 \times \frac{6.18}{7.50} = 1864 \text{ lbs.}$$

The forces F_1 and F_2 create a clockwise moment so that the resisting moment must be counterclockwise and the forces act to produce a counterclockwise moment about G. The forces act at right angles to the arms or lines joining the rivet centers with G.

The resultant shear on any rivet is found by completing the triangle of forces at each rivet, and is indicated in the heavy line. This is done on the figure for the several rivets and the maximum shear comes on rivet 11 and is 5200 lbs., while the minimum shear acts on rivet 4 and is 1000 lbs.

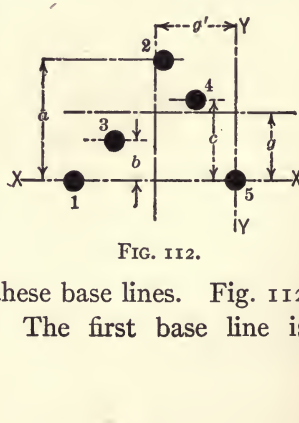


FIG. 112.

Should there be no axis of symmetry take any two base lines, preferably at right angles, and find the distance of the center of gravity from each of these base lines. Fig. 112 represents an irregular rivet group. The first base line is

taken, passing through rivets 1 and 5. The rivets are assumed as being of the same diameter and the cross section of each rivet being A; then the statical moment about axis x–x will be:

Rivet.	Area.	Arm.	Moment.
1	A	0	0
2	A	a	$A \cdot a$
3	A	b	$A \cdot b$
4	A	c	$A \cdot c$
5	A	d	$A \cdot 0$

Total area $= 5A$. Total Moment $= A\ (a + b + c)$.

The distance g from the axis x–x to the center of gravity will equal the statical moment divided by the total area or

$$g = \frac{A\ (a + b + c)}{5A}.$$

It is evident from this expression that the distance g equals the sum of the arms a, b and c, etc., divided by the number of rivets. In the same way the distance g' from the axis y–y can be found and the center of gravity is then located.

In designing riveted joints the following well-established rules are used. (These do not apply to joints in boilers or cylinders where maximum efficiencies are desired.)

1. PREFERABLE MINIMUM DIMENSIONS, INCHES

Rivet diameters.	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$
Center to center of rivets *	3	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{3}{4}$
Rivet center to sheared edge	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{8}$	1
Rivet center to rolled edge	$1\frac{1}{4}$	$1\frac{1}{8}$	1	$\frac{7}{8}$

* Sometimes taken three diameters of the rivet.

2. The maximum pitch in the line of the stress for members composed of plates and shapes should be 6 ins. for $\frac{7}{8}$-in. rivets; 6 ins. for $\frac{3}{4}$-in. rivets; $4\frac{1}{2}$ ins. for $\frac{5}{8}$-in. rivets; and 4 ins. for $\frac{1}{2}$-in. rivets. Where angles have two gauge lines and the rivets are staggered the maximum pitch in each gauge line should be twice the above dimensions.

3. Where two or more plates are used in contact the rivets holding them together should not be spaced farther apart than 12 ins. in either direction.

4. The pitch of rivets in the direction of the stress should not be greater than 6 ins., nor exceed 16 times the thinnest outside plate connected, and not more than 50 times that thickness at right angles to the stress.

5. The maximum distance from any edge should be 8 times the thickness of the plate.

6. The diameter of the rivets in any angle carrying calculated stresses should not exceed one-fourth of the width of the leg in which they are driven. In minor parts rivet diameters may be $\frac{1}{8}$ in. larger.

7. The pitch of rivets at the ends of built compression members shall not exceed four diameters of the rivets for a length equal to one and one-half times the maximum width of the member.

8. Two pieces riveted together should always be secured by at least two rivets.

9. Joints with field-driven rivets should have from 25 to 50 per cent more rivets than would have been required for shop-driven rivets.

CHAPTER VII

COLUMNS

STRUCTURAL engineers generally use either Rankine's formula or Johnson's straight-line formula in designing columns. The latter is a modification of Rankine's formula and the results approximate those given by Rankine's formula within the usual working limits of $\frac{l}{r}$ which will range from 50 to 150. The straight-line formula is preferred as the calculations with it are simpler.

The following give these formulæ in their usual form:

Gordon's or Rankine's formula for soft steel,

$$f = \frac{15{,}000}{1 + \dfrac{l^2}{13{,}500\, r^2}}. \tag{1}$$

Gordon's or Rankine's formula for medium steel,

$$f = \frac{17{,}000}{1 + \dfrac{l^2}{11{,}000\, r^2}}. \tag{2}$$

Johnson's straight-line formula for structural steel,

$$f = 16{,}000 - 70\frac{l}{r}. \tag{3}$$

f = the allowable unit compression on gross section of column in pounds per square inch.

l = the effective length of the column in inches.*

* The effective length of l will have the following relations to L the total length of the column:

Both ends hinged or butting..............	$l = L$
Both ends fixed........................	$l = \frac{1}{2} L$
One end fixed and one end hinged.........	$l = \frac{2}{3} L$
One end fixed and other end free..........	$l = 2 L$

r = the least radius of gyration of the column section in inches $= \sqrt{\frac{I}{A}}$.

I = the least moment of inertia of the column section in inches^4.

A = the area of the column section in square inches.

Ritter's formula most nearly meets the theoretical requirements but it is not much used as, like Rankine's formula, it is cumbersome, and the other formulæ are better known. It has the advantage of being applicable wherever the modulus of elasticity and the strength of the material at the elastic limit are known. In the other formulæ the constant for the material 13,500 and 11,000 in the case of Rankine's and 70 in the straight-line formula had to be determined experimentally. Ritter's formula as generally expressed is:

$$f = \frac{S_c}{1 + \frac{S_e}{\pi^2 E}\left(\frac{l}{r}\right)^2}. \tag{4}$$

The symbols have the same significance as just given for formulæ (1), (2) and (3). In addition

S_c = the maximum compressive unit stress desired on the concave side of the column in pounds per square inch.

S_e = the unit strength of the material at its elastic limit in pounds per square inch.

π^2 = approximately 10.

E = modulus of elasticity of the material in pounds per square inch. If for mild steel, the following values may be assumed:

$$E = 30{,}000{,}000, \quad S_e = 30{,}000 \quad \text{and} \quad S_c = 16{,}000,$$

all, pounds per square inch. With these values substituted, formula (4) reduces to

$$f = \frac{16{,}000}{1 + \frac{30{,}000}{10 \times 30{,}000{,}000} \times \left(\frac{l}{r}\right)^2} = \frac{16{,}000}{1 + \frac{1}{10{,}000}\left(\frac{l}{r}\right)^2}. \tag{5}$$

In this approximate form it is not especially cumbersome but it gives values slightly under those calculated by formula (3). The

straight-line formula will be used for all calculations throughout the text.

The American Bridge Company have issued under date of Dec. 1, 1912, new specifications for structural steel work. These specifications recommend for columns the use of two formulæ, the first, $f = 19{,}000 - 100\frac{l}{r}$, to be applied to values of $\frac{l}{r}$ up to 120; the second formula applies only to secondary members, those permitted to have $\frac{l}{r}$ values ranging from 120 to 200, and is $f = 13{,}000 - 50\frac{l}{r}$. The maximum value of f is placed at 13,000 lbs. per sq. in.

NOTE. — There is probably no section of Mechanics of Materials about which students have a less clear conception than that of columns.

It is here assumed that the student has this fundamental conception clearly and thoroughly his own. Such a conception is essential to the intelligent design of columns and, in fact, to beams also, as will be shown later. Should the student's idea of the subject be in any way vague he is urged to review the subject in any Mechanics of Materials with which he is familiar.

Problem. — Plot the values of f given by formulæ (1), (2), (3) and (5) upon cross-section paper for values of $\frac{l}{r}$ between 50 and 150.

It should be noted in the several formulæ that r is the least radius of gyration, hence that section will theoretically make the best column where the radius of gyration of the section is a constant for all axes. This requires a circular section, which, for economy of material, would generally be a hollow cylinder. On this account rolled-steel and cast-iron pipe would make good columns, but that cast iron is being but infrequently used in important structures while, owing to the difficulty of making connections to them by brackets or framing, round-steel columns are seldom used.

One of the simplest columns that approaches the theoretically ideal section is made by using two channels spaced so as to make the radii of gyration of the column section referred to its principal axes approximately equal. The columns, Fig. 113, are then laced with flat bars riveted to the channel flanges so that the two channels are made to act as a unit throughout the column's length.

I_1 is the moment of inertia of one channel axis 1–1.

I_2 is the moment of inertia of one channel axis 2–2.

a is the area of one channel. $A = 2a$.

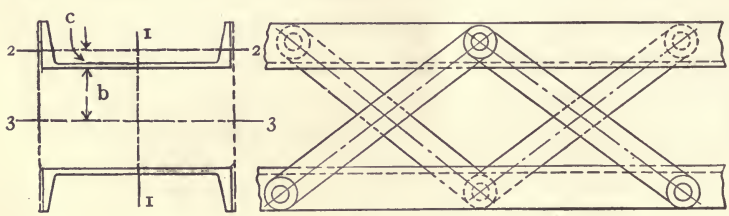


FIG. 113.

The inertia of the total column section about axis 1–1 is $I_c = 2 I_1$.

Since $r = \sqrt{\frac{I}{A}}$ it follows that if the radii of gyration are to be equal about any two axes of a section the inertias must be equal; hence

$$I_c = I_3,$$
$$I_3 = 2\,[I_2 + a\,(b + c)^2] = 2\,I_1,$$

from which

$$b = \sqrt{\frac{I_1 - I_2}{a}} - c.$$

It should not be understood that columns must always approach equal radii of gyration about both axes. In the case of columns of comparatively short lengths it may be cheaper, owing to the reduction in the work of manufacturing, to use a rolled section than a built-up column, even if the material is not used to such good advantage.

It therefore frequently happens that single sections, usually I beams or angles, make good columns. In some cases the base is merely an iron casting of some depth for stiffness, containing a pocket cored in it into which the section fits. When the column is brought into position some molten soft metal is poured around it to hold column and base together. In more important cases, angle and plate bases similar to those shown in Figs. 116 and 117 are used. As previously stated the channel column is one

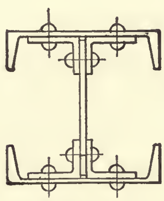


FIG. 114.

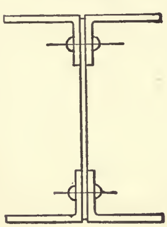


FIG. 115.

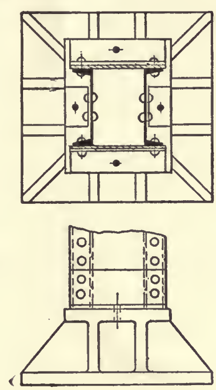


FIG. 116.

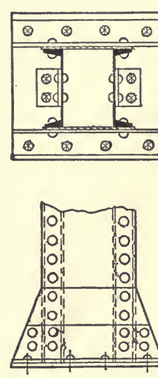


FIG. 117.

of the commonest types. These are illustrated in Fig. 113 and the bases for such a column are shown in Figs. 116 and 117. The flanges of the channels may be turned in instead of out, making a square column, but, owing to the difficulty of riveting, it costs more to manufacture. With the flanges turned out as in Fig. 113 the lacing may be replaced by plates. Another design uses the plate on the outside and is laced on the inside, thus permitting of inspection and painting. The sections shown in Figs. 114 and 115 are also accessible for painting. In Fig. 115, the web may be either lacing or a solid plate. This section, when deep, makes a good column section to resist combined

compression and bending. Its strength may be increased by riveting flange plates outside the angles.

A couple of the simplest forms of column bases are shown in Figs. 116 and 117. The proper distribution of the load carried by the column section to the foundation demands not only an enlargement of the foot of the column but some depth to the bed plate.

Consult paragraphs 90, 92, 96, 115, 126, 128 and 129 of the specifications.

The destruction of the Quebec Bridge due to the failure of a compression member has quickened the interest in the safe designing of columns. A few full-sized tests, but still an insufficient number, have been made on latticed columns. See Bulletin number 44 of the University of Illinois and tests upon columns by Howard, in the A.S.C.E., Vol. 73, with the discussions following the article. All the columns under test showed greater stress than would be indicated by the usual formulas for column design. The suggestions for remedying the difficulty range from the suggestion of using fiber stresses on columns only 60 per cent of those used in tension to the revision of the column formulas. Mr. F. C. Kunz draws attention to the fact that two things, ordinarily neglected, may under certain conditions become of considerable importance. The first of these is that it is usually considered ample if the $\frac{l'}{r'}$ ratio of one of the members of a column between its latticed supports does not exceed the $\frac{l}{r}$ of the entire column. It should be noted that the member may tend to buckle between the points of lattice support at the same time that the entire column buckles throughout its full length.

Secondly, although shearing stress may have a negligible effect upon columns with solid webs, where the web and flange areas are of the usual proportions, and the columns are not too short, in other cases it should be considered.

Combined Stresses

In a column the force may act parallel to the axis but eccentric to it. Here the column is subjected to a bending moment $P(\delta + \Delta)$ where P is the force acting parallel to the axis and Δ is the maximum deflection due to such eccentric loading. The following formula by Merriman gives the maximum resulting fiber stress:

$$f_T = \frac{P}{A}\left[1 + \left(\frac{\delta e}{r^2} \times \frac{1 + v}{1 - 5v}\right)\right],$$

f_T = maximum combined fiber stress in pounds per square inch.
P = eccentric force on column in pounds.
A = area of the column section in square inches.
δ = eccentricity of the load P in inches.
e = distance from the neutral axis to the extreme fibers.
r = radius of gyration of column section referred to the axis about which the bending occurs.

$$v = \frac{Pl^2}{48\,AEr^2} = \frac{Pl^2}{48\,EI}.$$

l = length of the column in inches.
E = modulus of elasticity in pounds per square inch.

Where a piece is subjected to either compression or tension, together with transverse bending, the following formula, due to Johnson, is commonly used:

$$f_b = \frac{Me}{I \mp \dfrac{Pl^2}{10\,E}},$$

f_b = flexural fiber stress in pounds per square inch.
M = transverse bending moment in inch pounds.
e = the distance from the neutral axis to the extreme fibers in inches.
I = moment of inertia of the section referred to the axis about which the bending occurs.

l = length of the piece in inches.

E = modulus of elasticity in pounds per square inch.

The sign (+) is used when P puts the piece in tension, the sign (−) when compression is produced.

To this flexural fiber stress must be added the direct compression (−) or the tension (+). The factor given as 10 varies with the character of the loads and the ends, being 9.6 for a simple beam uniformly loaded, but 12 for a similar beam with a central load. Owing to its simplicity this formula is the more frequently used.

When an approximation only is desired the stress due to bending may be found by solving the formula $f = \frac{Me}{I}$ for the extreme fiber stress due to bending and adding to it algebraically the direct stress due to either tension or compression. This method will be satisfactory when the longitudinal tension or compression is not large. The total fiber stress should not exceed that permitted on the piece. An example will illustrate the use of these formulæ.

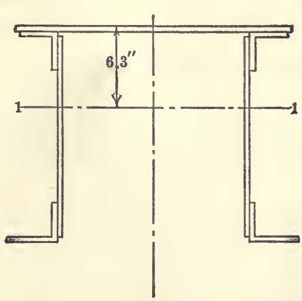


Fig. 118.

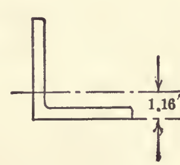


Fig. 119.

The member of the upper chord of a bridge is 25 feet long. It weighs 110 lbs. per foot, and has the section shown in Fig. 118.

The moment of inertia axis 1–1 is 1003.8. P = 238,100 lbs.

The distance from the top of the upper plate to the center of gravity of the built-up section is 6.30 ins.

The bending moment due to the dead load is

$$M = \frac{WL}{8} = \frac{(110 \times 25) \times 300}{8} = 103{,}125 \text{ in. lbs.}$$

The fiber stress due to combined bending and direct stress as given by the approximate method is

$$f = \frac{Me}{I} = \frac{103{,}125 \times 6.30}{1003.8} = 650 \text{ lbs.}$$

Total combined stress

$$650 + \frac{238{,}100}{31.6} = 8190 \text{ lbs.}$$

The following result is given by Johnson's formula,

$$f = \frac{Me}{I - \dfrac{Pl^2}{10\,E}} = \frac{103{,}125 \times 6.3}{1003.8 - \dfrac{238{,}100 \times 300^2}{10 \times 30{,}000{,}000}} = 697 \text{ lbs.}$$

Total compression

$$697 + \frac{238{,}100}{31.6} = 8237 \text{ lbs.}$$

The following example will illustrate the application of the formulæ when the load is parallel to the longitudinal axis of the piece but eccentric to it. A 4-in. $\times$ 4-in. $\times$ $\frac{7}{16}$-in. angle 120 ins. long has a load of 8800 lbs. applied to one leg. The piece is in compression. What is the total extreme fiber stress in compression?

$I = 4.97$; area $= 3.31$ sq. ins.; radius of gyration $= 1.23$.

By Merriman's formula

$$v = \frac{Pl^2}{48\,EI} = \frac{8800 \times 120^2}{48 \times 30{,}000{,}000 \times 4.97} = 0.0177,$$

$$f_T = \frac{P}{A}\left[1 + \left(\frac{\delta e}{r^2} \times \frac{1 + v}{1 - 5\,v}\right)\right]$$

$$= \frac{8800}{3.31}\left[1 + \frac{1.16 \times 1.16}{1.23^2} \times \frac{1 + 0.0177}{1 - (5 \times 0.0177)}\right].$$

$$f_T = 5300 \text{ lbs.}$$

By Johnson's formula

The fiber stress in bending is

$$f_b = \frac{Me}{I - \dfrac{Pl^2}{10\,E}}.$$

$$f_b = \frac{8800 \times 1.16 \times 1.16}{4.97 - \dfrac{8800 \times 120^2}{10 \times 30{,}000{,}000}} = 2610 \text{ lbs.}$$

The fiber stress due to compression is

$$p_c = \frac{8800}{3.31} = 2660 \text{ lbs.}$$

$$f_T = f_b + f_c = 2610 + 2660 = 5270 \text{ lbs.}$$

In this case the maximum stress should be kept under that allowed on the strut as given by a column formula; hence,

$$f = 16{,}000 - \left(70 \times \frac{120}{0.79}\right) = 5360 \text{ lbs.}$$

Long Beams Unsupported Laterally

When a beam of long span is unsupported laterally the upper flange being in compression is liable to fail as a column by buckling sideways. There is no very satisfactory theoretical treatment of this subject. The compressive fiber stress in the upper flange is usually limited by formulæ or rules that have been found safe. Mr. Christie, basing his conclusions upon tests made on full-size beams for the Pencoyd Iron Works, decided it was safe to use a desired limiting fiber stress up to a span of twenty times the flange width, and that from this point the working fiber stress should be uniformly decreased until, at a span of 70 times the flange width, the working fiber stress should be one-half the maximum desired stress.

The handbook of the Cambria Steel Company suggests the following formula,

$$p_1 = \frac{18{,}000}{1 + \dfrac{l^2}{3000\,b^2}},$$

p = allowable compressive fiber stress in pounds per square inch.
l = length of span in inches.
b = width of flange of beam in inches.

Mr. C. C. Schneider in his structural specifications limits the allowable compression in flanges of girders to

$$p_1 = 16{,}000 - 200\,\frac{l}{b} \text{ when flange is composed of plates.}$$

$$p_1 = 16{,}000 - 150\,\frac{l}{b} \text{ when flange is a channel.}$$

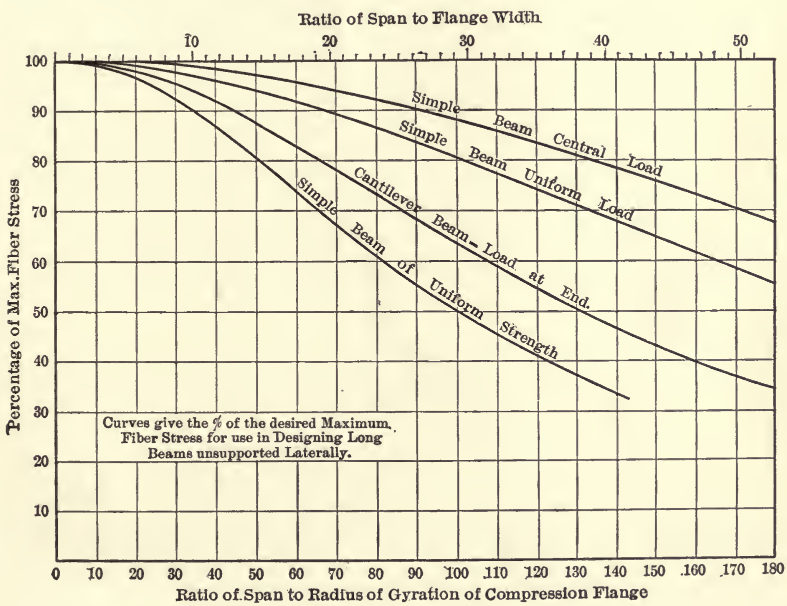


FIG. 120.

In this case the fiber stress is given somewhat lower than for beams, which is consistent, as the shallower the beam the more the lower flange, which is in tension, reinforces the compression flange.

The 1912 specification of the American Bridge Company limits

the span unsupported laterally to forty times the flange width, and when such span exceeds ten flange widths the fiber stress is to be reduced to that given by the formula $19{,}000 - 300\frac{l}{b}$.

The preceding curves, Fig. 120, are based on formulæ derived by the author in an attempt to explain the necessity for reducing the fiber stress in beams where the beam was unsupported laterally. The derivation of these formulæ is fully explained in the Proceedings of the Engineers' Club of Philadelphia, for April, 1909.

CHAPTER VIII

GIRDERS FOR CONVEYORS

THE half plan and section of this girder are given in Fig. 121, the elevation in Fig. 122 and the end view in Fig. 123.

Assumed Loading. —

Metal at 105 lbs. per lineal foot....................	5,360 lbs.
Corrugated steel covering........................	2,000 lbs.
Foot-walk and sheathing........................	2,420 lbs.
Snow..	6,120 lbs.
Total dead load..............	15,900 lbs.
Conveyor load:	
Uniform load on conveyor.......................	3,900 lbs.
Weight of conveyor.............................	3,700 lbs.
Total moving load on conveyor. .	7,600 lbs.
Allowing 25 per cent additional for impact.........	1,900 lbs.
Total..........................	9,500 lbs.

As the conveyor loading is not carried equally by both girders the maximum load on either girder must be calculated.

The total maximum load on one girder $7950 + 6500 = 14{,}450$.

The apex load is $\frac{14{,}450}{7} \sim 2050$ lbs.

Wind Load. — The horizontal wind load is assumed at 20 lbs. per sq. ft. of vertical projection. The apex wind load on the upper horizontal girder is $20 \times 4 \times 7\frac{1}{3} \sim 590$ lbs.

The stresses must now be determined for the truss under the given loading. This may be done graphically or the stresses may be found very readily by the method of coefficients. To illustrate the procedure both methods will be used.

The method of coefficients will be used first. For the explanation of this method see page 55. These calculations are made

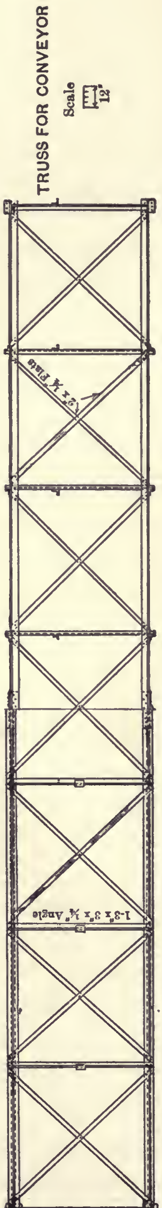


FIG. 121.

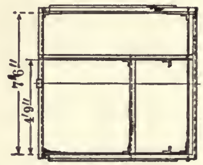


FIG. 123.

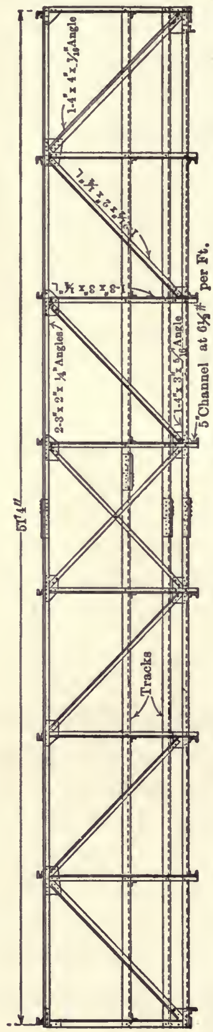


FIG. 122.

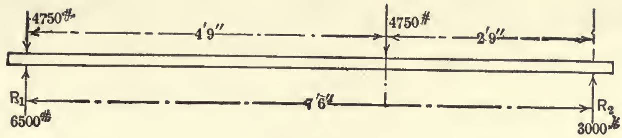


FIG. 124.

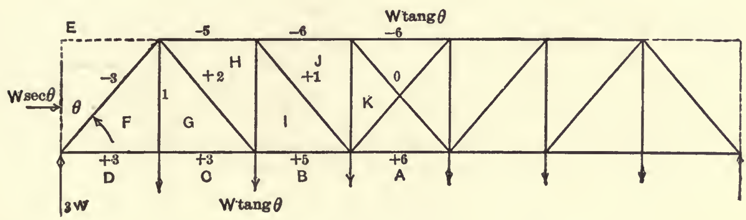
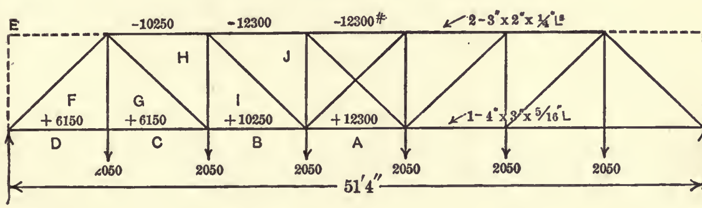


FIG. 125.

FIG. 126.



DEAD LOAD
AND LIVE LOAD
DIAGRAMS.

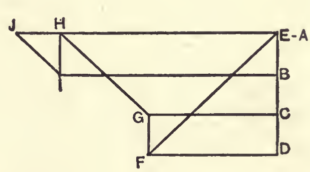


FIG. 127.

for Fig. 125. The graphical solution for the main truss is made in Figs. 126 and 127.

The wind bracing and graphical analysis are given in Figs. 128 and 129.

$\theta = 45°$ tang $45° = 1$ sec. $45° = 1.41$

Stress $EF = - 3 \times 2050 \times 1.41 = - 8670$ lbs.
Stress $EH = - 5 \times 2050 \times 1 = - 10{,}250$ lbs.
Stress $EJ = EK = - 6 \times 2050 \times 1 = - 12{,}300$ lbs.
Stress $FD = + 3 \times 2050 \times 1 = + 6150$ lbs. = stress GC.
Stress $IB = + 5 \times 2050 \times 1 = + 10{,}250$ lbs.
Stress $KA = + 6 \times 2050 \times 1 = + 12{,}300$ lbs.
Stress $GH = + 2 \times 2050 \times 1.41 = + 5780$ lbs.
Stress $IJ = + 1 \times 2050 \times 1.41 = + 2890$ lbs.

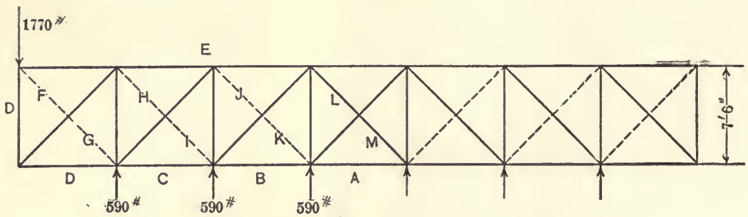


FIG. 128.

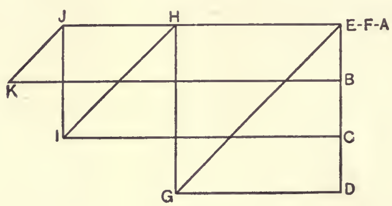


FIG. 129.

Selection of Members. — The upper chord.

Maximum compressive stress, dead and live load...........	12,300 lbs.
Maximum compressive stress, wind load..................	3,540 lbs.
Total....................................	15,840 lbs.

Try two 3 in. $\times$ 2 in. $\times$ $\frac{1}{4}$-in. angles spaced $\frac{1}{4}$ in. back to back, with their long legs parallel. $r_2 = 0.84$. Their length is 88 ins.

$$\frac{l}{r} = \frac{88}{0.84} = 105.$$

The allowable fiber stress according to the straight-line formula is

$$f_1 = 16{,}000 - 70\frac{l}{r} = 16{,}000 - (70 \times 105) = 8650 \text{ lbs.}$$

The load these angles will carry is $2 \times 1.19 \times 8650 = 20{,}600$ lbs.

End Strut *EF*, Fig. 125. — Its length is 120 ins. and its load 8670 lbs. Try one 4 in. × 4-in. angle. Its radius of gyration about its diagonal axis is 0.79. $\frac{l}{r} = \frac{120}{0.79} = 152$. The allowable fiber stress according to the straight-line formula is

$$f_1 = 16{,}000 - (70 \times 152) = 5360 \text{ lbs.}$$

The bending moment due to the eccentric loading, when the angle is assumed as $\frac{7}{16}$ in. thick, is $8670 \times 1.16 = 10{,}060$ in. lbs.

The fiber stress resulting from combined bending and compression is given by the formula

$$f = \frac{Me}{I - \frac{Pl^2}{10E}},$$

see page 97.

$$f = \frac{10{,}060 \times 1.16}{4.97 - \frac{8670 \times 120^2}{10 \times 30{,}000{,}000}} = 2560 \text{ lbs.}$$

The allowable stress according to the straight-line formula being 5360 lbs., deducting 2560 lbs. leaves 2800 lbs. per sq. in. The total allowable direct stress on the angle then is $3.31 \times 2800 = 9270$ lbs.; this, exceeding the 8670 lbs. acting on it, the angle is satisfactory.

Lower Chord. — The maximum stress is 15,840 lbs. Trying one 4 in. × 3 in. × $\frac{5}{16}$ in., and placing the long leg horizontally, its inertia about its axis parallel to the short leg is 3.38; the distance from the back of the short leg to the axis is 1.26 ins.

The bending moment $15{,}840 \times 1.26 = 19{,}960$ in. lbs.

The fiber stress due to bending is

$$f = \frac{Me}{I + \frac{Pl^2}{10E}} = \frac{19{,}960 \times 1.26}{3.38 + \frac{15{,}840 \times 88^2}{10 \times 30{,}000{,}000}} = 6650 \text{ lbs.}$$

The allowable stress on the net section is $f = 16{,}000 - 6650 = 9350$ lbs.

The total allowable force on the piece then is $(2.09 - 0.27) \times 9350 = 17{,}020$ lbs.

Vertical *HI*. — This is a compression piece carrying 2050 lbs. One 3 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angle will be tried.

The bending due to eccentric loading is $M = 2050 \times 0.84 = 1720$ in. lbs.

The fiber stress due to this bending is

$$f = \frac{Me}{I - \dfrac{Pl^2}{10E}} = \frac{1720 \times 0.84}{1.24 - \dfrac{2050 \times 88^2}{10 \times 30{,}000{,}000}} = 1220 \text{ lbs.}$$

The allowable compression, according to the straight-line formula, is

$$f = 16{,}000 - 70\frac{l}{r} = 16{,}000 - \left(70 \times \frac{88}{0.59}\right) = 5600 \text{ lbs.}$$

The net compression allowed per square inch of the section is $5600 - 1220 = 4380$ lbs.

The area of the angle multiplied by 4380 is $1.44 \times 4380 = 6300$ lbs. Hence, the 3 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angle is more than ample.

Diagonals. — The maximum tension in the diagonals is 5780 lbs. Try one $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$-in. angle. Its inertia about an axis through its center of gravity and parallel to its long leg is 0.37. The distance from the back of the angle to the above axis is 0.54 in.

The bending moment due to the eccentric application of the load to the angle is $5780 \times 0.54 = 3120$ in. lbs.

The extreme fiber stress due to this bending is

$$f = \frac{Me}{I + \dfrac{Pl^2}{10E}} = \frac{3120 \times 0.54}{0.37 + \dfrac{5780 \times 120^2}{10 \times 30{,}000{,}000}} = 2590 \text{ lbs.}$$

The allowable stress per square inch of the net section is $16{,}000 - 2590 = 13{,}410$ lbs. Net section $\times$ $13{,}410 = (1.07 - 0.22) \times$

13,410 = 11,400 lbs. This is satisfactory, being the smallest angle we can use.

Wind Bracing. — The maximum stress in the diagonals is 2550 lbs. The smallest flat allowed being $2 \times \frac{1}{4}$ in., and as this will carry $[(2 \times 0.25) - 0.22] \times 16{,}000 = 4480$ lbs. (the net area times the allowable fiber stress), it is ample.

Strut in Upper Horizontal Bracing

$$R_2 = \frac{500 \times 45}{57} \sim 400 \text{ lbs.}$$

The bending moment under the load between R_1 and R_2 is

$$M = 400 \times 12 = 4800 \text{ in. lbs.}$$

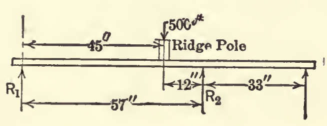


Fig. 130.

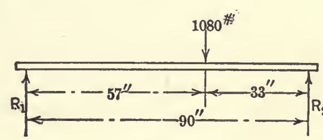


Fig. 131.

Trying 3 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angles, their inertia about an axis through their center of gravity and parallel to a leg is 1.24, the distance from the back of the angle to this axis is 0.84 in., the radius of gyration referred to this axis is 0.93, while the radius of gyration referred to the diagonal axis is 0.59.

$$\frac{l}{r} \text{ for the full length } = \frac{90}{0.93} = 97.$$

$$\frac{l}{r} \text{ for the length of 57 ins. } = \frac{57}{0.59} = 97.$$

The allowable stress $f = 16{,}000 - \left(70 \times \frac{l}{r}\right) = 16{,}000 - (70 \times 97) = 9210$ lbs.

The fiber stresses due to the bending of 4800 in. lbs. in 57-in. span are,

$$f_t = \frac{Me}{I} = \frac{4800 \times 0.84}{1.24} = 3250 \text{ lbs.} \quad f_c = \frac{4800 \times 2.16}{1.24} = 8350 \text{ lbs.}$$

The bending due to the eccentric loading as a column on member *GH*, Fig. 128, will produce a fiber stress of

$$f_c = \frac{Me}{I - \dfrac{Pl^2}{10\,E}} = \frac{(1180 \times 0.84) \times 0.84}{1.24 - \dfrac{1180 \times 57^2}{10 \times 30{,}000{,}000}} = \frac{835}{1.23} = 680 \text{ lbs.}$$

$$f_t = \frac{680 \times 2.16}{0.84} = 1750 \text{ lbs.}$$

The direct compression = force divided by the area of the section $= \dfrac{1180}{1.44} = 820$ lbs.

The maximum compression will be 8490 lbs., which, being under the 9210 lbs. allowed the section, is satisfactory.

The beams serving as struts in the lower horizontal bracing carry a total load of 1080 lbs.,

$$\frac{4750}{7} + 400 \text{ (snow load)} = 1080 \text{ lbs.}$$

$$R_2 = \frac{1080 \times 57}{90} = 685 \text{ lbs.}$$

$$M = 685 \times 33 = 22{,}605 \text{ in. lbs.}$$

Assuming the channel unsupported laterally for its length of 88 ins., and trying a 6-in. channel,

$$\text{length} \div \text{flange width} = \frac{88}{1.92} = 45.$$

Allowable fiber stress about 11,000 lbs. per sq. in.,

$$\frac{M}{f} = \frac{I}{e} = \frac{22{,}605}{11{,}000} = 2.06.$$

This suggests about a 5-in. channel.

The direct compression being 1180 lbs. and applied at the upper flange the moment due to eccentric loading is $M = 1180 \times 2.5 = 2950$ lbs.

$$f = \frac{Me}{I - \dfrac{Pl^2}{10\,E}} = \frac{2950 \times 2.5}{7.4 - \dfrac{1180 \times 88^2}{10 \times 30{,}000{,}000}} = 1000 \text{ lbs.}$$

Direct stress $\frac{1180}{1.95} = 605$ lbs.

The extreme compressive stress due to bending of a 5-in. ⊔ is

$$f = \frac{Me}{I} = \frac{22,605}{3.0} = 7535 \text{ lbs. per sq. in.}$$

Total compressive stress $7535 + 1000 + 605 = 9140$ lbs., being below 11,000, is satisfactory.

CHAPTER IX

TRUSSES, BENTS AND TOWERS

It is frequently necessary around works to carry large gas or air mains overhead. These mains are sometimes lined with brick to prevent radiation loss from the hot gas or air. There is consequently considerable load per foot, and as long spans are preferable, thus avoiding too large a number of posts or bents

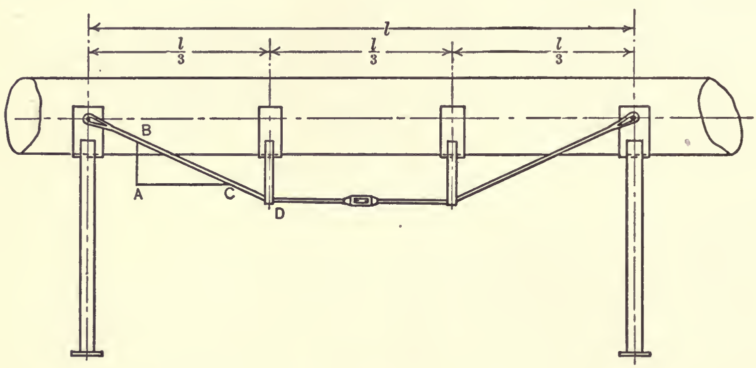


FIG. 132.

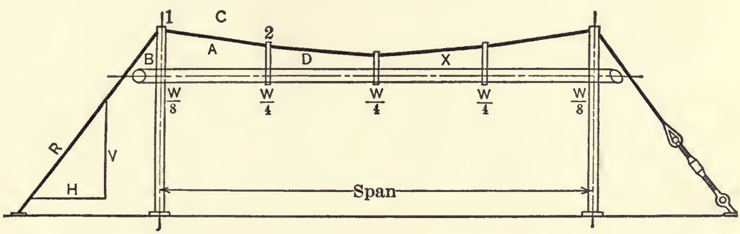
that would block up the yard, the pipe must be trussed. Fig. 132 represents such a trussed pipe. Care should be taken that any load transferred to or from the pipe should be distributed over the pipe by a saddle and if necessary the pipe should be reinforced at such points. In Fig. 132, *AB* is the load in the vertical member, "*D*," *AC* the compression in the pipe, and *BC* the tension in the rods.

The span over which the pipe section acts as a beam is $\frac{l}{3}$. Steam, air, gas and water pipes may be carried upon a light

trussed bridge somewhat resembling the conveyor truss (see page 104), or may be hung from a cable, as shown in Fig. 133.

The stress in the wire rope depends upon the load carried and upon the sag permitted in it. As the weight of the rope will be small compared with the load, the effect of the weight of the rope may be neglected. In Fig. 133, having assumed the rope diameter, lay off R representing the maximum desired pull on the rope, then draw the horizontal and vertical components H and V. Taking the forces acting about apex 1, the horizontal

FIG. 133.



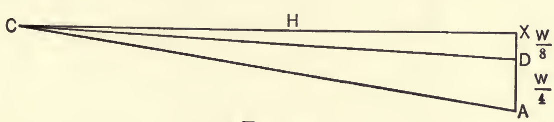


FIG. 134.

component of BC must equal the horizontal component of CA. Similarly, the horizontal components of the forces acting at apex 2 are equal. Now in Fig. 134, $XD = \dfrac{W}{8}$ and $DA = \dfrac{W}{4}$. H is the horizontal component of the force BC acting in the guy rope. CA and CD give the magnitudes and directions of the rope stresses and the sag of the rope may be found by drawing the strings in Fig. 133, corresponding to these rays in Fig. 134.

Should the sag prove objectionable a heavier rope with greater stress or a greater stress may be used in the first rope if permissible.

The vertical load on the post, Fig. 133, is $V + \dfrac{W}{2}$.

Bents, Fig. 135, may be used to support both pipes and wires. If the bent is an end one and guyed the vertical load will be determined as in the preceding example. If it is an intermediate bent it will not carry the vertical component of the guy rope but merely its proportion of the weight of the wires and pipes. It may also be subjected to wind load, in which event the stresses in the bracing can be determined and these pieces selected for their respective loads, see Fig. 137. The wind acting on cylindrical surfaces is commonly assumed as $\frac{6}{10}$ of the pressure that would act on the vertical projection of that surface.

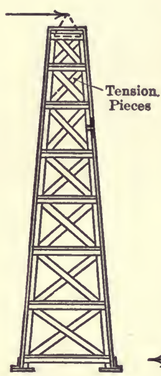


FIG. 135.

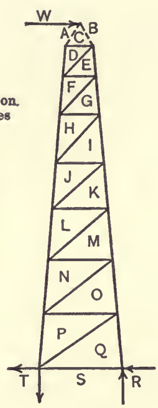


FIG. 136.

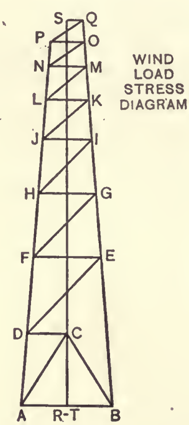


FIG. 137.

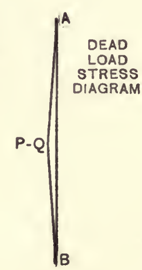


FIG. 138.

As the loads are generally very light and not subjected to shock, the values of $\frac{l}{r}$ are permitted to reach 180 or 200.

As the wind load is transmitted to the truss proper by pieces subjected to bending, the diagram, Fig. 136, has been started by connecting the wind force to the truss through the dotted or substituted members. The dead-load stress diagram is shown in Fig. 138.

Towers for Transmission Lines. — There is no standard practice but the four-angle type is the most common. The

loading upon these towers consists of dead load of towers and weight of wire, also ice, which may add materially to this direct load in localities visited by sleet. A live load due to the wind acts at right angles to the dead load, and in addition there may be a direct pull along the line caused by a wire or wires breaking. The ice load is generally assumed as a ½-in. coating around the wire.

Wind loads assumed as acting on wires vary considerably. Mr. R. Fleming in an article in *Engineering News*, Nov. 28, 1912, recommends 30 lbs. per sq. ft. of exposed surface upon the tower, 15 lbs. per sq. ft. of projected area of the bare wire, reduced to 10 lbs. for ice-coated wires. He further recommends that poles carrying three wires be designed to resist the unbalanced pull, due to one wire breaking, while for six wires two of them shall be considered as breaking. The unbalanced load is the force that breaks the wire and the following percentages of the ultimate strengths of the wires are recommended: the full ultimate strength up to and including a No. 4 wire; a No. 3 wire, 90 per cent; a No. 2 wire, 80 per cent; a No. 1 wire, 70 per cent; a No. 0 wire, 60 per cent; and for all larger wires, 50 per cent. The ultimate strength per square inch of hard-drawn copper wire is from 50,000 to 65,000 lbs. The towers are designed to meet the requirements of the worst combination of these conditions.

Using ordinary structural steel, Mr. Fleming recommends the following formulæ for columns:

$$f_c = \frac{22,500}{1 + \frac{l^2}{18,000\, r^2}}, \text{ for the most exacting city service.}$$

$$f_c = \frac{27,000}{1 + \frac{l^2}{18,000\, r^2}}, \text{ for less exacting service.}$$

In estimating the loading upon the poles the additional loading due to the line running on an incline or rounding a curve should not be overlooked. This is generally provided for by

placing the poles closer together in these places rather than changing the tower design.

In estimating the tension in the line and length of the wire between supports it is common practice to assume the curve of the line a parabola which differs but slightly from the catenary and is much more readily computed.

S = distance between poles in feet.
L = length of wire in feet, measured along the curve.
d = sag or drop in wire in feet, measured midway between poles.
H = tension in lowest point of curve in pounds.
c = tension constant.
w = resultant load, measured in pounds per foot of wire. In winter the weight of wire and ice acting vertically and the wind acting horizontally. In summer the weight of wire acting vertically and the wind on the bare wire acting horizontally.

$$L = S + \frac{S^3}{24\,c^2}, \tag{1}$$

$$d = \frac{S^2}{8\,c}, \tag{2}$$

$$L = S + \frac{8\,d^2}{3\,S}, \tag{3}$$

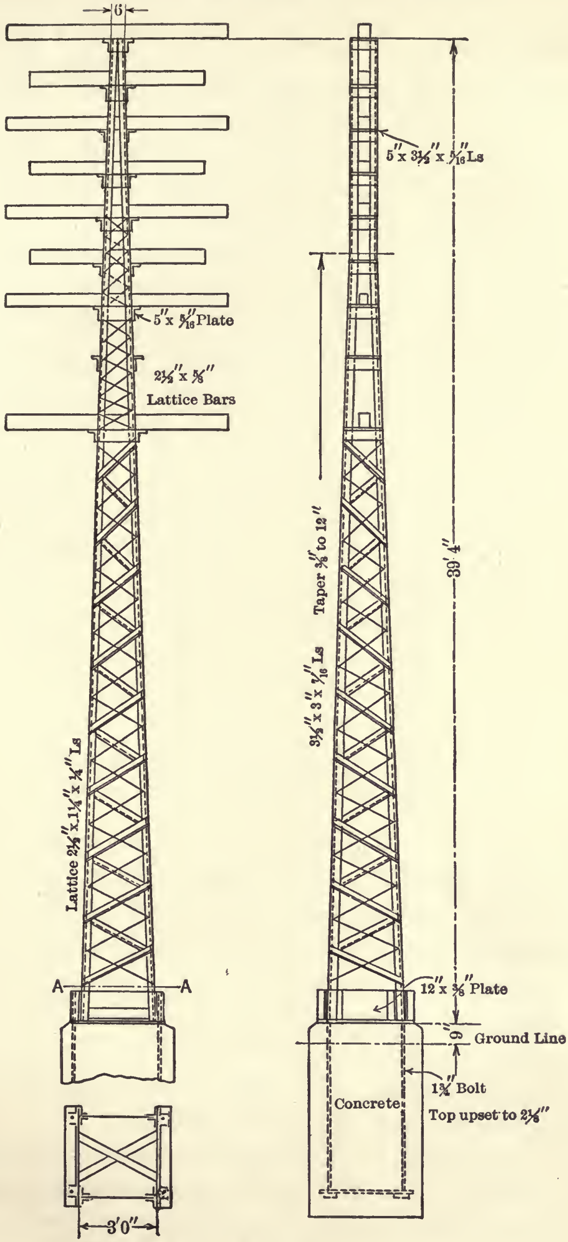
$$H = \frac{S^2 w}{8\,d}. \tag{4}$$

For a more complete mathematical discussion of this subject, see Bulletin No. 54 of the University of Illinois. "Mechanical Stresses in Transmission Lines," by A. Guell. Jan. 22, 1912.

The length of the wire at the summer temperature will increase due to the temperature but will have less extension due to the loading, as the vertical load will not include the weight of the ice, and the wind will not act on so great a surface. Mr. Guell gives the following formula:

$$L_1 = S + \frac{8\,d^2}{3\,S} + \alpha L t - \frac{L\,(H - H_1)}{AE}. \tag{5}$$

NOTE. — This discussion applies only to spans where the points of attachment to the poles are at the same elevation. The pull in the wire at the pole is $T = \sqrt{H^2 + \frac{L^2 w^2}{4}}$ lbs.



SECTION A-A

FIG. 139.

Here

L_1 = length of the wire along the curve, measured in feet, at the summer temperature.

L = length of the wire for winter temperature and loading.

α = coefficient of expansion per degree Fahr. = 0.00000956.

t = temperature range, degrees Fahr.

H = tension at low point in curve in winter, pounds.

H_1 = tension at low point in curve in summer, pounds.

A = area of wire, square inches.

E = modulus of elasticity, pounds per square inch. For hard-drawn copper wire, 12,000,000 to 16,000,000 lbs. per sq. in.

The stresses in the wire will be lower in summer than in winter, but it is necessary to estimate the sag for the summer. It may be approximated as follows. In equation (5) estimate the value of L_1, omitting the last term, as H_1 is not known. A preliminary estimate of H_1 may be made by assuming

$$\frac{H_1}{H} = \frac{w_1}{w}.$$

Here

w_1 = resultant load per foot in summer.

w = resultant load per foot in winter.

This value of H_1 would then be substituted in equation (5) and the last term need not be discarded. Having now found L_1 substitute this value for L in equation (1) and solve for c. This value of c used in equation (2) gives an approximate value of d. Now in equation (4), w is known from the loading and H can be found. This is an approximation to H_1 required in equation (5) and the value of L_1 may now be revised by placing this value of H_1 in this equation. The trials may be repeated until the desired degree of approximation is reached.

Cross arms should be designed for a minimum vertical load of from 1000 to 1200 lbs., also for an unbalanced horizontal pull due to the wires breaking and estimated as previously given.

The twisting of the tower due to such unbalanced loading should not be overlooked.

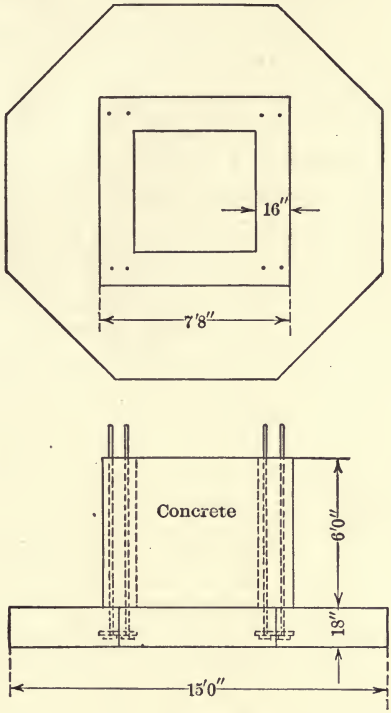


Fig. 140.

Figure 139 shows one type of tower and foundation and follows a design by Westinghouse, Church, Kerr & Co., as does also Fig. 140, which illustrates a type of foundation with greater base area.

CHAPTER X

DESIGN OF A STEEL MILL BUILDING

THE design of a steel mill building as generally carried out requires considerable experience and judgment on the part of the designer. The stresses in simple trusses carried on walls are usually determined for an equivalent load rather than by the separate determination of the dead-, snow- and wind-load stresses and their combination for maximum effect. The following equivalent loads may be used on spans under 80 ft.

Kind of roof.	Lbs. per sq. ft. exposed surface.
Gravel or composition roofing on boards	45–50
Gravel or composition roofing on 3-in. concrete	60
Corrugated steel sheeting	40
Slate on boards	50
Slate on 3-in. concrete, flat	65

Where no snow need be considered these figures can be reduced by 10 lbs., excepting that no roof shall be designed for less than 40 lbs. per sq. ft.

Stiffening the Structure. — The structure is stiffened first where possible by introducing a knee brace, running from a panel point in the lower or upper chord of the truss to a point as low as convenient on the column. Where the column carries a crane runway this knee brace must generally be omitted and is usually replaced by a knuckle brace which stiffens the connection between the column and the truss.

The structure is also braced in the plane of the lower chords of the truss to hold the tops of the columns at constant distances apart, and in some cases to carry the wind forces acting along the side of the building to the transverse bracing at its ends.

NOTE. — See Specifications § 66 to § 135.

To prevent the trusses rotating about their lower chords, bracing is placed in the plane of the upper chords. This bracing is commonly omitted, in alternate bays.

To resist the wind pressure on the ends of the buildings and the cumulative pressure along the roof, together with any crane thrust, lateral bracing is placed in the plane of the building columns. This bracing usually includes an eave strut which is frequently a light latticed girder composed of four angles, the web being composed of diagonal lattice bars.

Where possible the diagonal bracing should extend to the ends of the columns. Where the column height is considerable an additional strut is placed about half way up the column. Both eave strut and intermediate strut run the entire length of the building and fasten to each column. The diagonal braces are commonly placed at the end bays and then at intervals along the building as deemed necessary by the designer. Where the bracing cannot be extended to the column bases the columns must be designed to resist the bending due to the horizontal forces resulting from the wind acting on the end of the building and any crane thrust. This bending will be influenced by the way the column bases are secured and may be treated similarly to the transverse bent, page 45.

The transverse bracing in the ends of the building is intended to resist whatever of the forces acting on the side of the building from wind pressure, or thrust on the columns from the crane, may be transferred to the ends by the structure.

In the simpler buildings it is possible to assume certain external forces resisted by a particular bracing truss and then make the design accordingly. In the more complicated buildings the designer modifies his calculations by what has been previously found to be satisfactory.

The following sketches illustrate one general plan of steel mill bracing, Figs. 141 to 143. See also Figs. 156 to 158.

A number of types of roof trusses are shown in Figs. 144 to 151.

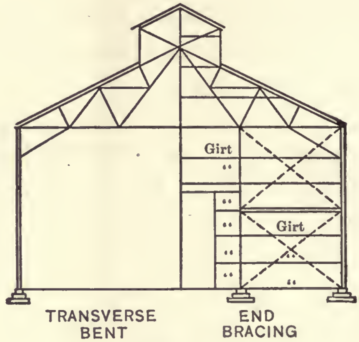


FIG. 141.

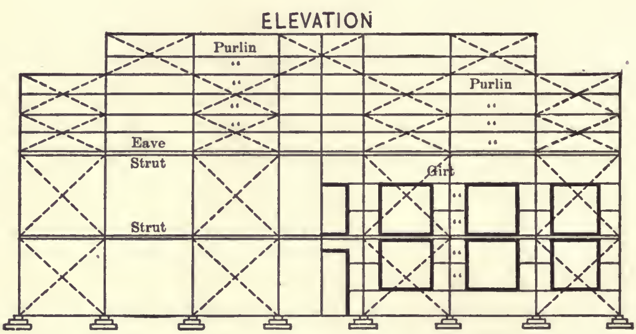


FIG. 142.

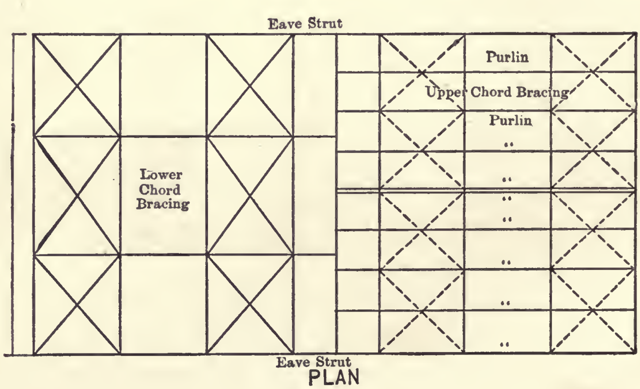


FIG. 143.

The number of panels into which the upper chord should be divided will depend upon the span of the truss and upon the roof covering or sheathing. In the case of corrugated-steel roofing, it is desirable that the sheets should extend over three purlins. That the roof covering be amply stiff it is necessary with the ordinary gauges to limit the distance between purlins to from

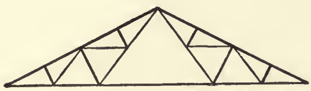


FIG. 144.

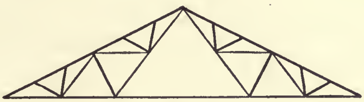


FIG. 145.

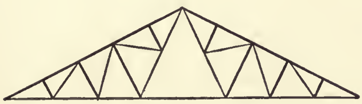


FIG. 146.

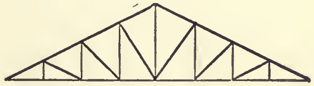


FIG. 147.

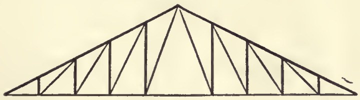


FIG. 148.

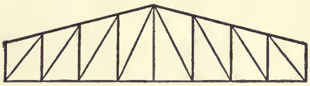


FIG. 149.

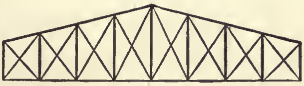


FIG. 150.

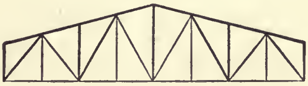


FIG. 151.

4 to 6 ft.; consequently, the purlins are generally spaced about 4 ft. In the case of other roof coverings stiffness is also required, but as most of these coverings are laid upon sheathing the purlins can be spaced farther apart providing the sheathing is made correspondingly thicker. If the upper chord is not intended to take bending it is necessary to have a member of the truss connected to the upper chord under each purlin.

Frequently, however, the upper chord is designed of a plate and two angles forming a tee-section. In this case the section is calculated to resist bending and the purlins can, therefore, be spaced as desired, and a less complicated truss used.

The pitch of the roof will vary with the character of the roof covering. The following table gives the usual pitches.

Kind of Roofing.	Minimum pitch.	Usual pitch.
Corrugated steel	$\frac{1}{5}$	$\frac{1}{4}$
Slate or tile	$\frac{1}{4}$	$\frac{1}{4}-\frac{1}{3}$
Shingles	$\frac{1}{4}$	$\frac{1}{2}$
	Maximum pitch	
Gravel	$\frac{1}{5}$	$\frac{3}{4}''$ in 12″
Asphalt	$\frac{1}{6}$	$\frac{3}{4}''$ in 12″
Patent compositions	....	$\frac{1}{4}-\frac{1}{5}$

Determination of Stresses. — The methods of combining the stresses due to the several classes of loads vary with the designer. In the *Engineering News* of Feb. 4, 1915, Mr. R. Fleming of the American Bridge Co. recommends the following procedure.

In the design of simple trusses the stresses allowed to resist wind loadings may be increased 50 per cent over those used to resist ordinary live and dead load stresses, thus making a wind load of 15 lbs. equivalent to 10 lbs. using the working stresses for other loads. The snow load per square foot horizontally will range from 20 lbs. in New York to 30 lbs. in parts of New England. On a roof with 6-in. pitch this will range from 16.6 to 25 lbs. per sq. ft. over the entire surface. For the combined wind and snow, a load of 25 lbs. per sq. ft., acting vertically over the entire surface, is ample for roofs in the latitude of New York City. To this should be added the weight of the trusses, purlins and roof covering, reduced to the square foot of roof surface. The total load for which a roof should be designed, however, should not be less than 40 lbs. per sq. ft. of exposed surface, excepting in tropical climates with no snow where 30 lbs. per sq. ft. may be the minimum, or where snows are severe the minimum

FIG. 15

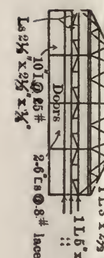
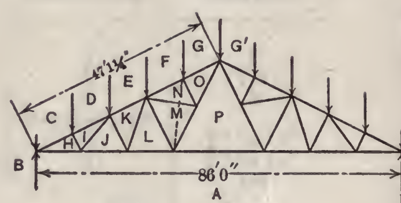


FIG. 152.



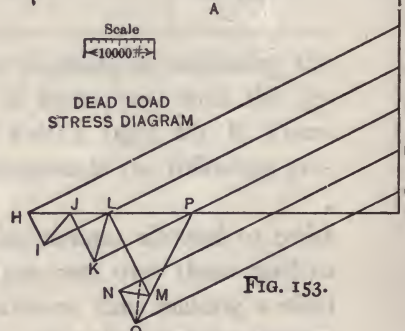


FIG. 153.

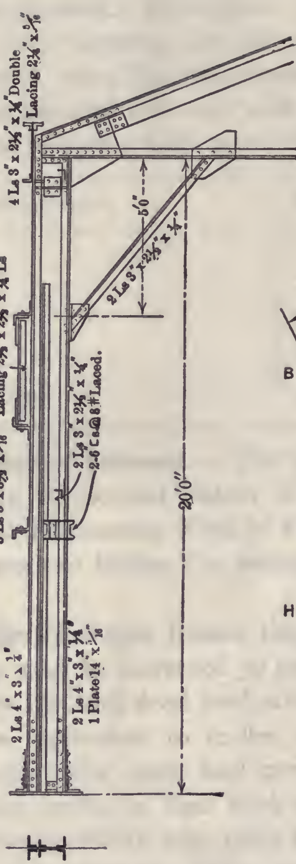


FIG. 155.

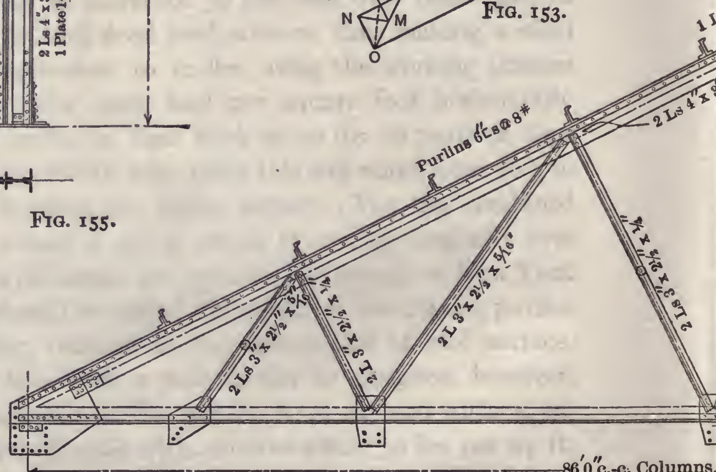


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FIG. 157.

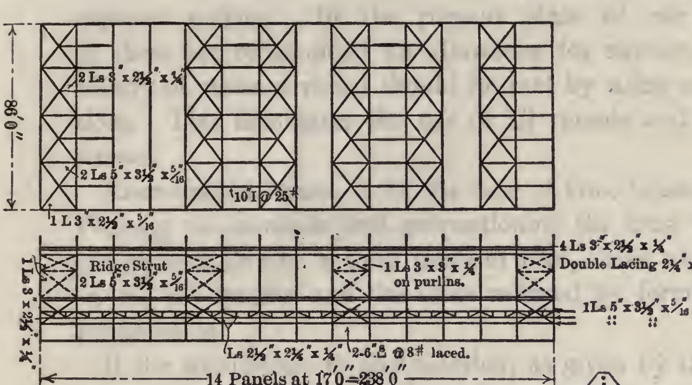
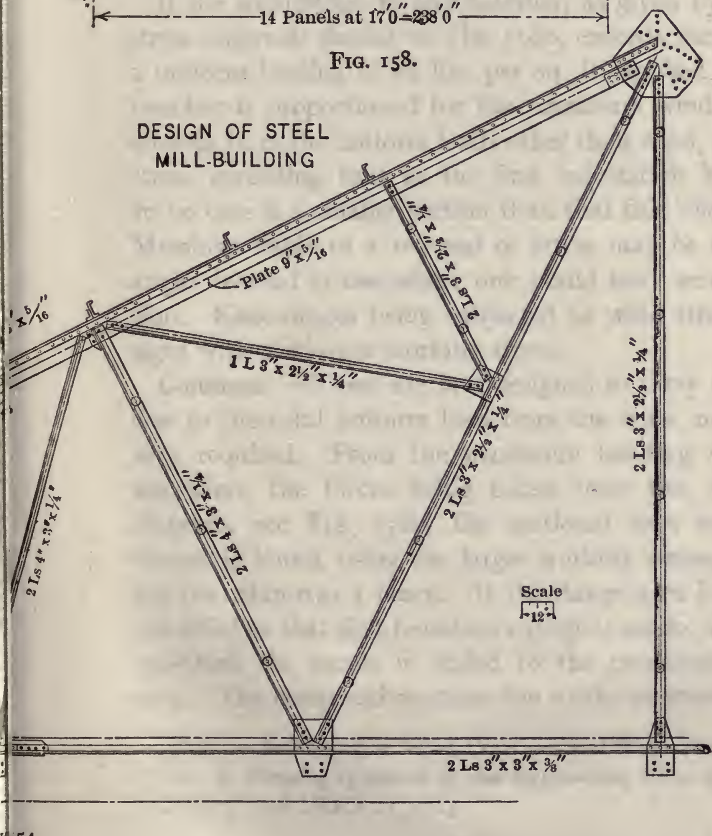


FIG. 158.



54.

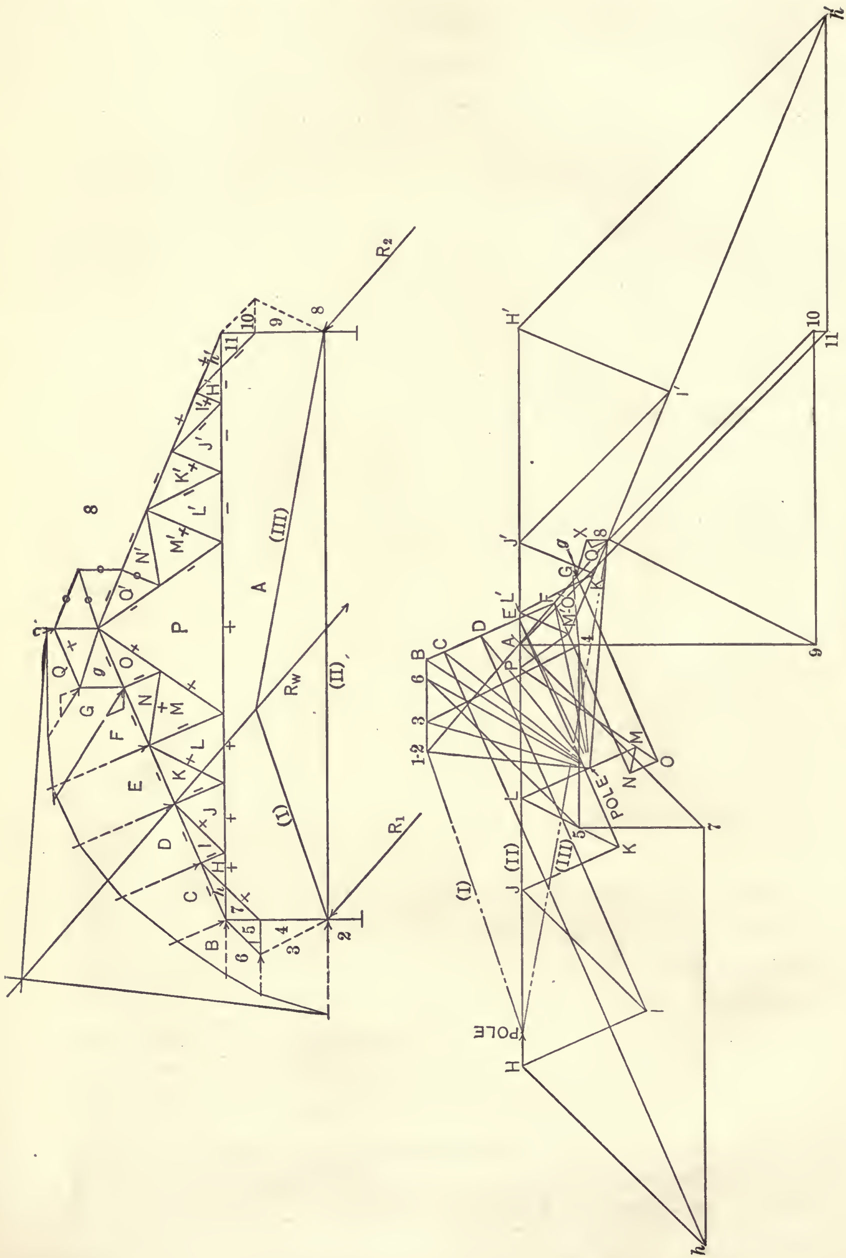
load should be increased to from 45 to 50 lbs. per sq. ft. of exposed surface. In the present state of our information he does not recommend an allowance for suction. The possibility of stress reversal should be met by using only stiff sections. This eliminates the use of all rounds and flats in roof trusses.

Knee-braced Bents. — In the case of knee-braced bents, Mr. Fleming recommends first proportioning the truss members for the stresses due to a total uniform load, using 16,000 lbs. per sq. in. net tension and the same reduced by formula for gross compression.

If the wind stress in any member, as given by the wind load stress diagram, similar to Fig. 158b, exceeds the stresses due to a uniform loading of 10 lbs. per sq. ft. applied vertically, that member is proportioned for the maximum wind stress plus the stresses from the uniform loads other than wind, using a working stress exceeding that in the first calculation by 50 per cent. In no case is a smaller section than that first chosen to be used. Members liable to a reversal of stress may be designed of two angles instead of one where one would have served for the tension. Knee-braces being subjected to wind stress only, he designs with the larger working stress.

Columns. — These are first designed to carry the direct stress due to the total uniform load from the truss, noting the flange area required. From the maximum bending moment due to the wind, the forces being taken from the wind-load stress diagram, see Fig. 158b, the sectional area required for the flanges is found, using the larger working stresses and considering the column as a beam. If this flange area is not more than one-third of that first found no change is made, if it is more than one-third the excess is added to the previously found flange area. The compressive stress due to the overturning (the verti-

NOTE. — Articles upon Wind Pressures and Wind Stresses in Structures by Mr. R. Fleming appeared in the *Engineering News* of Jan. 28; Feb. 4, 11 and 25; and March 11, 1915.



FIGS. 158a and 158b.

cal component of the wind reaction R_2 in Fig. 158a) need not be considered unless it exceeds the stress from the wind portion of the uniform roof load.

Design of a Steel Mill Building, Figs. 152 to 158. — The truss, Fig. 159, will be designed according to the specifications para-

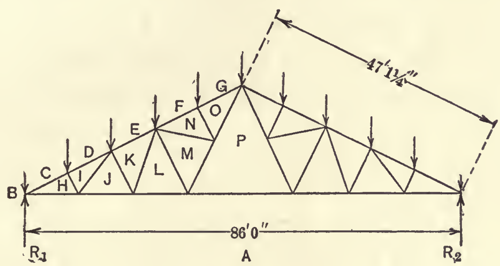


FIG. 159.

graphs numbers 66 to 135 inclusive. The span is 86 ft. and the rise 19.22 ft., thus making the angle of slope 24 degrees 5 minutes.

The several members have the following calculated lengths,

CH, *DI*, *EK*, *FN*, *GO*, 9.42 feet.
HI, 4.21 feet. *LM*, 12.63 ft. *JK*, 8.42 ft.
IJ, *HA*, *JA*, *LA*, 10.32 ft. *MP*, *OP*, *NM*, 11.34 ft.
NO, 6.31 ft. Length of rafter, 47.1 ft.

The normal wind pressure by Duchemin's formula is 14 lbs. per sq. ft. of roof surface.

The dead load is estimated at the following pounds, per square foot of horizontal projection of roof, trusses, 3.7; purlins and bracing, 2.8; roof covering, 3.5, making a total of 10.0 lbs. per sq. ft.

The trusses are spaced 17 ft. center to center.

The apex dead loads are $(86 \times 17 \times 10) \div 10 = 1462$, say 1460 lbs.

The apex wind load normal to the roof is $14 \times 9.42 \times 17 = 2240$ lbs.

The apex snow load being due to a load of 20 lbs. per sq. ft. of horizontal roof projection will be twice the apex dead load,

and the stresses due to the maximum snow load may be taken as twice those due to the dead load.

The dead load stress diagram is given in Fig. 153. The wind load stress diagram is shown in Fig. 158b. In this diagram the columns are assumed as partially fixed, the points of contra-flexure being considered as at a distance from the base of the column equal to one-third the distance from this base to the foot of the knee-brace. It is then treated similarly to Case II, Chapter III, page 46. The diagram is most readily drawn by starting it from the right-hand column. The table on the following page gives the stresses read from these diagrams. All stresses are in pounds.

Design of Members. — Considering the members HI, JK and NO, JK is the longest and carries a load nearly equal to that carried by HI; it will require the heaviest section. Its loads are $-$ 9900 and $+$ 2500 lbs. Its length is 8.42 ft.

Trying two 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{1}{4}$ in.-angles with their long legs placed parallel, their least radius of gyration is 0.95; this makes

$$\frac{l}{r} = \frac{(8.42 \times 12)}{0.95} = 106.$$

The allowable fiber stress then is $f = 16{,}000 - \left(70 \times \frac{l}{r}\right) =$ 8580 lbs. per sq. in.

The load the two angles will carry is $2.64 \times 8580 = 22{,}600$ lbs. These angles are therefore ample.

Member *IJ*. — The length of this piece is 123 ins. and its loads are $-$ 10,000 and $+$ 12,800 lbs. Trying two 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angles, $\frac{l}{r} = \frac{123}{0.95} = 129$. Considering the character of the loads and the method of attachment this is satisfactory. The load it will carry is $f = 16{,}000 - \left(70 \times \frac{l}{r}\right) = 16{,}000 - (70 \times 129) =$ 6970 lbs. per sq. in.

The total load they will carry is $2 \times 1.63 \times 6970 = 22{,}700$ lbs.

Member.	Dead load.	Max. snow load.	Min. snow load.	Wind load.		Dead load and max. snow load.	Dead load, min. snow load and wind load.		Dead load and wind load.		Basis for design.		Uniform load, 40 lbs. per sq. ft. of exp. surface.
				Left.	Right.		Left.	Right.	Left.	Right.			
CH	−15,900	−31,800	−15,900	−35,900	+31,000	−47,700	−67,700	− 800	−51,800	+15,100	−67,700	+15,100	−69,800
DI	−15,300	−30,600	−15,300	−22,600	+ 8,600	−45,900	−53,200	−22,000	−37,900	− 6,700	−53,200	....	−67,200
EK	−13,250	−26,500	−13,250	−13,700	− 2,000	−39,750	−40,200	−28,500	−26,950	−15,250	−40,200	....	−58,100
FN	−12,150	−24,300	−12,150	−10,300	− 5,500	−36,450	−34,600	−29,800	−22,450	−17,650	−36,450	....	−53,300
GO	−11,500	−23,000	−11,500	−11,200	− 5,500	−34,500	−34,200	−28,500	−22,700	−17,000	−34,500	....	−50,500
HI	− 1,350	− 2,700	− 1,350	− 7,500	+ 9,000	− 4,050	−10,200	+ 6,300	− 8,850	+ 7,650	−10,200	+ 7,650	− 5,930
JK	− 2,000	− 4,000	− 2,000	− 5,900	+ 4,500	− 6,000	− 9,900	+ 500	− 7,900	+ 2,500	− 9,900	+ 2,500	− 8,800
LM	− 3,300	− 6,600	− 3,300	− 7,000	+ 2,000	− 9,900	−13,600	− 4,600	−10,300	− 1,300	−13,600	....	−14,480
NO	− 1,350	− 2,700	− 1,350	− 1,700	0	− 4,050	− 4,400	− 2,700	− 3,050	− 1,350	− 4,400	....	− 5,930
IJ	+ 1,600	+ 3,200	+ 1,600	+ 9,600	−11,600	+ 4,800	+12,800	− 8,400	+11,200	−10,000	−10,000	+12,800	+ 7,030
KL	+ 2,000	+ 4,000	+ 2,000	+ 6,000	− 4,600	+ 6,000	+10,000	− 600	+ 8,000	− 2,600	− 2,600	+10,000	+ 8,800
MN	+ 1,200	+ 2,400	+ 1,200	+ 1,500	0	+ 3,600	+ 3,900	+ 2,400	+ 2,700	+ 1,200	....	+ 3,900	+ 5,270
OP	+ 4,800	+ 9,600	+ 4,800	+ 9,200	− 3,400	+14,400	+18,800	+ 6,200	+14,000	+ 1,400	....	+18,800	+21,050
MP	+ 3,500	+ 7,000	+ 3,500	+ 7,700	− 3,400	+10,500	+14,700	+ 3,600	+11,200	+ 100	....	+14,700	+15,350
HA	+14,500	+29,000	+14,500	+23,200	−17,200	+43,500	+52,200	+11,800	+37,700	− 2,700	+52,000	− 2,700	+63,600
JA	+12,900	+25,800	+12,900	+13,500	− 5,700	+38,700	+39,300	+20,100	+26,400	+ 7,200	+39,000	....	+56,600
LA	+11,200	+22,400	+11,200	+ 8,500	− 1,800	+33,600	+30,900	+20,600	+19,700	+ 9,400	+33,600	....	+49,100
PA	+ 8,000	+16,000	+ 8,000	+ 1,250	+ 1,250	+24,000	+17,250	+17,250	+ 9,250	+ 9,250	+24,000	....	+35,000
	Knee-brace.			+14,300	−24,200	....	+14,300	−24,200	....	....	−24,200	+14,300	....

+ indicates tension. − indicates compression.

Members *KL* and *LM*. — These are each 151 ins. long and both are liable to be in compression. *LM* receives the greater load which is − 13,600 lbs. Trying two 4 in. × 3 in. × $\frac{1}{4}$-in. angles, we have $\frac{l}{r} = \frac{151}{1.28} = 118$, $f = 16{,}000 - \left(70 \times \frac{l}{r}\right) = 7740$ lbs.

The total load then is 3.38 × 7740 = 26,160 lbs.

Members *OP* and *MP*. — These are tension pieces and will require two 3 in. × $2\frac{1}{2}$ in. × $\frac{1}{4}$-in. angles with their long legs placed parallel. They will carry in tension (2.64 − 0.44) × 16,000 = 35,200 lbs.

These angles are heavy but will be used, being the lightest allowed.

Member *MN* is a tension piece carrying + 3900 or + 5270 on uniform load basis and one 3 in. × $2\frac{1}{2}$ in. × $\frac{1}{4}$-in. angle will be sufficient. Specifications frequently specify that where single angles are used as tension pieces and fastened by one leg only, such leg shall be able to carry the full load.

Upon this basis the net area of the 3-in. leg is 0.75 − 0.22 = 0.53 sq. in. It will carry 0.53 × 16,000 = 8480 lbs.

Lower chord members, *PA*, *LA*, *JA* and *HA*. Excepting *HA* these are all purely tension pieces. The maximum force acts in *HA* and is + 63,600 lbs. on the uniform load basis. Trying two 3 in. × 3 in. × $\frac{3}{8}$-in. angles, their net section is found to be 2 (2.11 − 0.33) = 3.56 sq. ins. These angles will carry a tensile load of 3.56 × 16,000 = 57,000 lbs., and since this may be increased 25 per cent they will be ample.

This member is restrained by the knee-brace and its unsupported length is about 60 ins. The minimum radius of gyration of these angles is 0.91, so that $\frac{l}{r} = \frac{60}{0.91} = 66$.

The allowable unit compression is $f = 16{,}000 - (70 \times 66) = 11{,}380$ lbs. per sq. in. The total load allowed on the section then is 2 × 2.11 × 11,380 = 48,000 lbs. The estimated compression being 2700 lbs. these angles are satisfactory.

As the lower chord must be made in three pieces for shipment, a lighter section might be used for the member *PA*.

Upper Chord. — The upper chord will be made of the section shown in Fig. 160. It is first necessary to find the center of gravity and then the moment of inertia of this section.

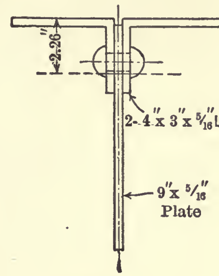


FIG. 160.

Area, sq. ins.		Moment.
2×2.09	$= 4.18 \times 0.76 =$	3.18
9×0.313	$= 2.82 \times 4.50 =$	12.65
	7.00	15.83

$$x = \frac{15.83}{7.00} = 2.26 \text{ ins.}$$

The inertia of the combined section then follows:

$$\begin{aligned}
&\text{Angles, } 2 \times 1.65 && = 3.30\\
&\text{Angles, } Ah^2,\ 2 \times 2.09 \times (2.26 - 0.76)^2 && = 9.42\\
&\text{Plate, } \frac{bd^3}{12} = \frac{0.3125 \times 9^3}{12} && = 19.00\\
&\text{Plate, } Ah^2 = 2.81 \times (4.50 - 2.26)^2 && = 14.08\\
& && I = 45.80
\end{aligned}$$

The span of CH is 9.42 ft. When two purlins are used in each span of 9.42 ft. the purlin load will be $\frac{5930}{2} = 2965$ lbs. Assuming the purlins centrally located on the above span the bending moment is

$$M = \frac{5}{8}\left(\frac{WL}{4}\right) = \frac{5}{8} \times 2965 \times \frac{9.42 \times 12}{4} = 52{,}370 \text{ in. lbs.}$$

The extreme fiber stress in compression is

$$f_c = \frac{Me}{I} = 52{,}370 \times \frac{2.26}{45.80} = 2580 \text{ lbs.},$$

while the extreme fiber stress in tension is

$$f_t = \frac{Me}{I} = 52{,}370 \times \frac{6.74}{45.80} = 7700 \text{ lbs.}$$

Combining these fiber stresses we have,

Total fiber stress in compression side,

$$-\,2580 - \frac{69{,}800}{7.0} = -\,12{,}550 \text{ lbs.}$$

Total fiber stress in tension side, $+\,7700 - \frac{69{,}800}{7.0} = -\,2270$ lbs.

The radius of gyration of this section must now be found about the axis through the plate and parallel to the short legs of the angles.

Inertia of two angles referred to axis 2–2, $2 \times 3.38 = 6.76$
Ah^2 of the angles, $2 \times 2.09 \times (1.26 + 0.155)^2 = 8.36$
$I = 15.12$

The radius of gyration $= r = \sqrt{\frac{I}{A}} = \sqrt{\frac{15.12}{7.0}} = 1.47,$

$$f = 16{,}000 - 70 \times \frac{l}{r} = 16{,}000 - \left(70 \times \frac{9.42 \times 12}{1.47}\right) = 10{,}640 \text{ lbs.}$$

The maximum allowable stress may be taken $1.25 \times 10{,}640 = 13{,}300$ lbs. per sq. in.

The maximum fiber stress in compression being 12,550 lbs. the section will do.

Columns. — The horizontal reactions at the columns will be assumed equal so that the horizontal components of R_1 and R_2 will be equal and their sum will be equal to the horizontal component of all the wind forces acting on the structure above the points of contraflexure of the columns. The horizontal components of R_1 and R_2 are each equal to 5900 lbs. The distance from the point of contraflexure to the foot of the knee-brace has been assumed 10 ft.; this is two-thirds of the distance from the foot of the knee-brace to the base of the column and is taken as suggested by Mr. R. Fleming, the writer believing with him that this will be ordinarily nearer the truth than either considering the column hinged or fixed with the point of contraflexure midway between the knee-brace and the column base. Under the assumed conditions the bending moment will be a maximum on the leeward column and will equal $5900 \times 120 = 708{,}000$ inch pounds.

The direct load will be the sum of the dead load, the minimum snow load and the vertical component of the wind load all acting on the leeward column. The direct load on the column then equals $\frac{86 \times 17 \times 20}{2} + 5000 = 19{,}600$ lbs. We will try four

4 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angles with one 14 in. $\times$ $\frac{5}{16}$-in. plate. The area of the section is

4 angles,	$4 \times 1.69 =$	6.76 sq. ins.
1 plate,	$\frac{5}{16} \times 14 =$	4.37 sq. ins.
	Total	11.13 sq. ins.

Moment of Inertia of Section. —

4 angles,	4×1.4	$= 5.6$
Ah^2 of angles,	$4 \times 1.69 \times (7.25 - 0.74)^2$	$= 286.5$
Plate, $\frac{b \cdot d^3}{12}$,	$\frac{5}{16} \times \frac{14^3}{12}$	$= 71.5$
	Total	363.6

The extreme fiber stress in the column then due to bending is

$$f = \frac{M \times e}{I} = \frac{708{,}000 \times 7.25}{363.6} = 14{,}100 \text{ lbs. per sq. in.}$$

The column must now be tested to see if this is under the allowable stress. The radius of gyration is

$$\sqrt{\frac{I}{A}} = \sqrt{\frac{363.6}{11.13}} = 5.72.$$

The direct unit stress is

$$\frac{19{,}600}{11.13} = 1760 \text{ lbs. per sq. in.}$$

The total combined unit stress is

$$14{,}100 + 1760 = 15{,}860 \text{ lbs. per sq. in.}$$

Assuming the column as having one end fixed, the other supported, the allowable fiber stress is

$$\left(16{,}000 - 47 \times \frac{l}{r}\right) \times 1.25 = \left[16{,}000 - \left(47 \times \frac{240}{5.72}\right)\right] \times 1.25$$
$$= 17{,}500 \text{ lbs. per sq. in.}$$

The 25 per cent additional stress is allowed in accordance with paragraph 90 of the specifications. Some designers allow 50 per cent additional here. The 15,860 lbs. per sq. in. is sufficiently close to the 17,500 lbs. per sq. in.

The column should now be examined for its strength referred to its other axis. The inertia about the axis parallel to the

column's web is found to 24.4. From this the radius of gyration is $r = \sqrt{\frac{24.4}{11.13}} = 1.48$. The unsupported length of this column being 8 ft., $\frac{l}{r} = \frac{8 \times 12}{1.48} = 64.8$. This value of $\frac{l}{r}$ being within the desired limit the column is satisfactory.

WIND BRACING

The reactions at the tops of the end columns, Fig. 161, due to wind pressure are

$$\frac{86 \times 20 \times 20}{4} + \frac{86 \times 19.2 \times 20}{2 \times 3} = 14{,}100 \text{ lbs.}$$

The force in the eave strut then is 14,100 lbs. The length of this strut is the distance between columns, or 17 ft.

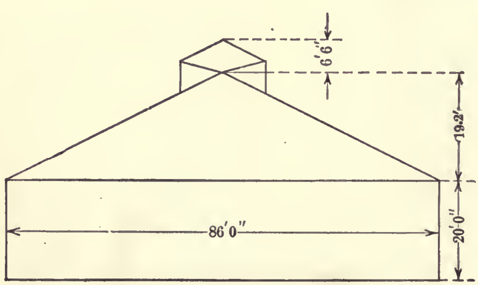


FIG. 161.

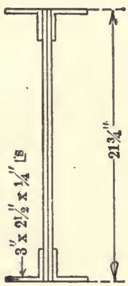


FIG. 162.

For an eave strut try Fig. 162, composed of four 3 in. × $2\frac{1}{2}$ in. × $\frac{1}{4}$-in. angles latticed, the long legs of the angles being turned out,

$$\frac{l}{r} = 17 \times \frac{12}{1.40} = 145;$$

according to the straight-line formula

$$f = \left[16{,}000 - \left(70 \times \frac{l}{r}\right)\right] \times 1.25 = 7300 \text{ lbs. per sq. in.}$$

In addition to acting as a strut the eave strut replaces a purlin so that it is also subjected to bending. The total load on a purlin has been estimated as 2965 lbs.; the corresponding load on the eave strut will be somewhat less than this depending on the amount of overhang of the roof covering.

The bending moment on this strut then is

$$M = \frac{W \cdot L}{8} = 2965 \times 17 \times \frac{12}{8} = 75{,}600 \text{ in. lbs.}$$

The distance between the centers of gravity of the strut flanges is $21.75 - (2 \times 0.66) = 20.43$ ins.

The mean fiber stress in the flanges due to the bending equals the bending moment due to the external loading divided by the product of the area of one flange (two angles) and the distance between the centers of gravity of the flanges.

$$f = \frac{75{,}600}{(2 \times 1.32 \times 20.43)} = 1400 \text{ lbs. per sq. in.}$$

The total resulting fiber stress in the strut then is

$$f = \frac{14{,}100}{4 \times 1.32} + 1400 = 4070 \text{ lbs. per sq. in.}$$

No lighter section can be used as these are the lightest allowed by the specifications, for the $\frac{l}{r}$ value determines the section.

The transverse brace between the first and second trusses will be designed as a simple truss to carry the wind loads at its several apexes.

Having determined the apex loads in Fig. 163, the stresses in the truss members can be found either by the method of coefficients or by making a stress diagram, Fig. 164. The diagonals will be assumed as taking only tension. The maximum stress is in the end diagonal and is found to be 17,600 lbs. Trying a 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angle, and assuming one rivet hole at a section, the net area is $1.63 - 0.27 = 1.36$ sq. ins.

The fiber stress due to direct loading then is $\frac{17{,}600}{1.36} = 13{,}000$ lbs. per sq. in. This piece is approximately 27 ft. long. Cross braces will be used. Their size will be estimated as though but one set were in place. These angles will be subjected to bending due to their own weight and also due to the attachment being made to one leg. The angles weigh 5.6 lbs. per ft. and the bending moment due to its weight is $M = \frac{W \cdot L}{8} = 5.6 \times 27 \times 27 \times \frac{12}{8} = 6124$ in. lbs. The bending due to the tensile force acting in the piece is $17{,}600 \times (0.68 - 0.156) = 9222$ in. lbs.

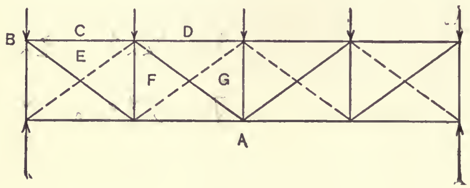


Fig. 163.

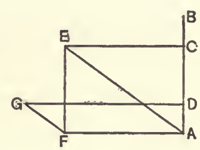


Fig. 164.

When these act in the same direction their sum is 15,346 in. lbs.

The flexural stress resulting is

$$f = \frac{M \cdot e}{I + \frac{P \cdot l^2}{10 \times E}} = \frac{15{,}346 \times 0.68}{0.90 + \frac{17{,}600 \times 324^2}{300{,}000{,}000}} = 1480 \text{ lbs. per sq. in.}$$

As the allowable unit stress in bracing is 20,000 lbs. per sq. in. there is ample margin for initial stress.

As the stresses in the diagonals of the braces in the top chords of the roof trusses and the bracing between the columns are both lower than the stress in the above diagonal the 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{1}{4}$-in. angle will be used in all these places. The strut in the bracing in the upper chord of the trusses will be stiffened by attaching it to the purlin. Here a 3 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angle will be used.

GIRTS

The load on the girts is $4 \times 17 \times 20 = 1360$ lbs.

$$M = \frac{WL}{8} = 1360 \times 17 \times \frac{12}{8} = 34{,}700 \text{ in. lbs.}$$

$$\frac{M}{f} = \frac{I}{e} = \frac{34{,}700}{1.25 \times 16{,}000} = 1.73.$$

A 5 in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angle will be used. The struts in the transverse truss will be made of two 5 in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angles, with the long legs parallel. The radius of gyration here being 1.46 the value of $\frac{l}{r}$ is $17 \times \frac{12}{1.46} = 140$.

Building of Lighter Design. — In the following problem (Fig. 165) the stresses due to dead load, snow load and wind load have been tabulated and their resultants compared with the stresses produced by an equivalent dead load of 40 lbs. per sq. ft. of horizontal projection of the roof.

The building is of somewhat lighter design than the previous one. Span of truss, 86 ft. 0 ins.; rise, $\frac{1}{4}$ span. Truss spacing, 25 ft. 0 ins. center to center. Figures 167 and 168 are the stress diagrams for wind load and dead load. The horizontal wind pressure is assumed at 20 lbs. per sq. ft. and the normal pressure on the roof is taken by Duchemin's formula at 14.9 lbs. per sq. ft. The wind is shown as acting upon the left of the truss, and the truss members in compression are represented by heavy lines.

The dead-load stress diagram is given in Fig. 168 and has been made to serve for both dead load and snow load by changing the scale. The snow load has been assumed at 20 lbs. per sq. ft. of horizontal projection. The weight of the truss is taken as

$$W = \frac{PL}{300 + 6L + \frac{PD}{3}},$$

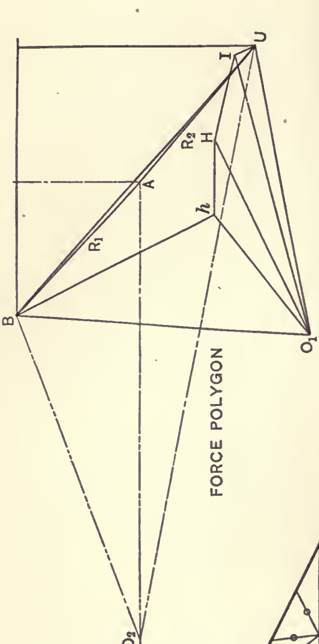


Fig. 166.

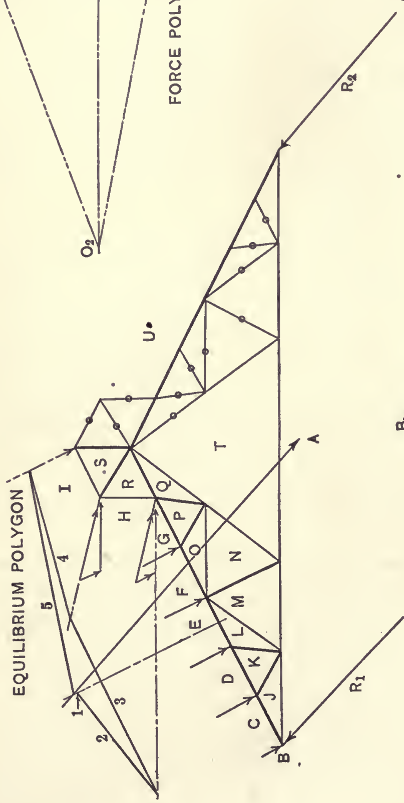


Fig. 165.

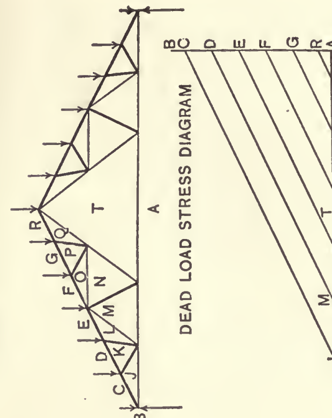


Fig. 168.

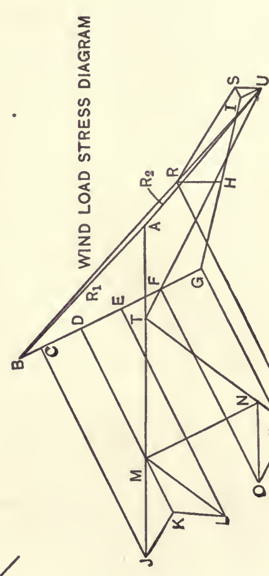


Fig. 167.

where

W = weight of truss per square foot of building.
L = span of truss, in feet.
D = distance center to center of trusses, in feet.
P = load per square foot on truss.

$$W = \frac{40 \times 86}{300 + (6 \times 86) + \frac{40 \times 25}{3}} = 3.$$

The estimated total weight of one truss is $3 \times 25 \times 86 =$ 6450 lbs.

The weight of the purlins is assumed as that given by

$$W_1 = \left(\frac{\sqrt{P_1}}{45} \times D\right) - \frac{1}{4},$$

where

W_1 = weight of the purlins per square foot of building, in pounds.
P_1 = load per square foot on purlins, in pounds.
D = distance center to center of trusses, in feet.

$$W_1 = \frac{\sqrt{40} \times 25}{45} - \frac{1}{4} = 3.25 \text{ lbs.}$$

The roof covering will be No. 20 galvanized corrugated iron, weighing about 2 lbs. per sq. ft. or 2.75 lbs. allowing for laps, etc.

It is commonly specified that roofs of this character shall be designed for a uniform dead load of 40 lbs. per sq. ft. of exposed surface. This is treated as an equivalent loading and replaces the wind, snow and dead loads.

The stresses will now be determined for the estimated dead, snow and wind loadings and their resulting maximum stresses compared with stresses determined from the uniform dead load of 40 lbs. per sq. ft.

The apex dead load on the upper chord of the truss at 9 lbs. per sq. ft. is

$$\frac{86 \times 25 \times 9}{12} = 1610 \text{ lbs.}$$

The apex snow load at 20 lbs. per sq. ft. of horizontal projection is

$$\frac{20 \times 25 \times 86}{12} = 3580 \text{ lbs.}$$

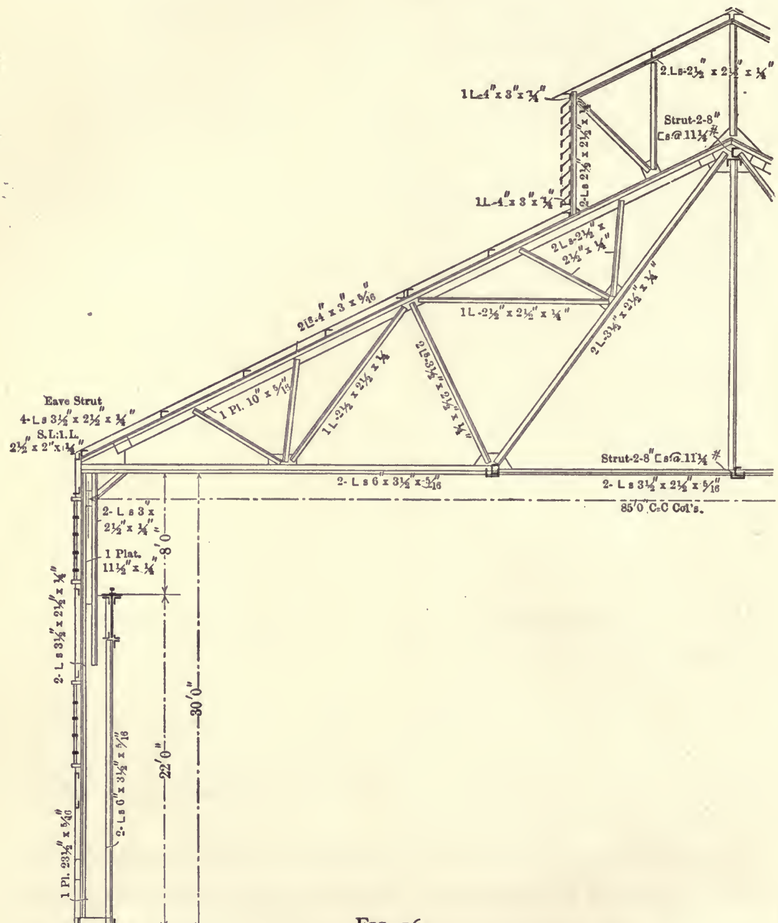


Fig. 169.

The purlins are trussed.

The apex normal wind load at 14.9 lbs. per sq. ft. of roof surface, the distance between the panel points along the upper member of the truss being 8.05 ft., is $8.05 \times 25 \times 14.9 = 3000$ lbs.

Member.	Dead load at 9 lbs.	Min. snow load at 10 lbs.	Wind load at 14.9 lbs. (normal).	Combined.	Equivalent load at 40 lbs. per sq. ft. of exposed surface.
CJ	−20,250	−22,500	−15,500	−58,250	−101,000
JK	− 1,800	− 2,000	− 3,480	− 7,280	− 8,970
LM	+ 3,240	+ 3,600	+ 6,600	+13,440	+ 16,150
QT	+ 8,370	+ 9,300	+13,200	+30,870	+ 41,900
JA	+18,450	+20,500	+22,200	+61,150	+ 92,000
AT	+ 9,900	+11,000	+ 6,350	+27,250	+ 49,400
TU	−20,250	−22,500	−17,100	−59,850	−101,000
MN	− 4,320	− 4,800	− 8,300	−17,420	− 21,500

The above figures indicate the truss members designed upon a basis of a uniform load of 40 lbs. per sq. ft. of exposed roof surface would be considerably on the safe side.

The selection of the truss members may be made as was done in the preceding problem and will not be detailed again.

Columns

When knee braces are omitted the building must be stiffened in some other way. This may be done by designing the columns to withstand the wind loading in each bay on the side of the building, by designing the lateral bracing to carry all wind pressures to the ends or by assuming a division of this loading between the columns and the bracing. Actually the columns and bracing always share such loads, but as the proportion carried by each depends upon their relative stiffness the actual loading becomes a matter of considerable uncertainty. Just what assumptions are best to make will depend upon the length and height of the building, the assumed wind pressure, the manner of securing the tops of the columns to the trusses and the bases of the columns to the foundations. The assumptions for the design will also depend upon the judgment of the designer.

All bracing should be given an initial stress when erected, to insure its acting promptly when the loading comes upon it.

The portion of the column above the crane girder carries the roof load and will be liable to buckle about an axis parallel to

the web. The load is applied concentrically to the column. There may be bending on the column about this axis from wind pressure on the end of the building, and from thrust due to the crane stopping and starting on its runway which should also be considered.

Below the crane girder the inside angles of the column must transfer the crane load, including impact, to the full section of the column. As this transfer will occur in a short distance the $\frac{l}{r}$ value may be neglected, hence the angles on this side of the column would require a minimum section of $\frac{66,000}{16,000} = 4.13$ sq. ins. The two 6 in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angles used give an area of 5.78 sq. ins. and should prove ample.

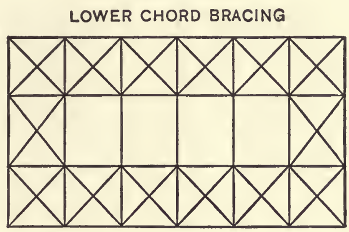
The roof load and the side of the building carried by the columns being both eccentric to the axes of the columns will produce bending on them.

The resultant of these two loads and the crane load will most likely be eccentric to the center of gravity of the columns and will, therefore, produce a bending moment upon them. In addition to these the transverse thrust from the crane and the wind load on the side of the building will produce bending on the columns about an axis at right angles to the web. The effect of these moments will be materially reduced by the restraint at the column bases if the columns are fixed at that point. The maximum fiber stress resulting from the combination of direct and flexural stresses should not exceed that permitted upon the column when the maximum allowable stress has been reduced by a suitable column formula. Considering that the stresses are the resultant of wind load, crane load and dead load this reduced value may be increased 25 per cent. The column section below the crane girder must also be designed for a possible buckling about an axis parallel to the web, and for bending due to the crane thrust and wind load on the end of the building. A very common practice is to place a channel at right angles to the

column axis, riveted to the inside flange angles. The channel carries the crane load directly from the runway girder.

The reactions at the tops of the columns are assumed as carried to the ends of the building by the lateral trusses in the lower

FIG. 170.



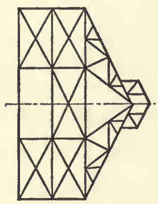


FIG. 170a.

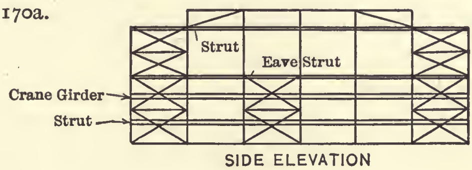


FIG. 171.

chords of the roof trusses. As members of the lower chords of the roof trusses form parts of the lateral bracing such members should be examined to see that they will carry these bracing stresses in addition to those already on them from the truss load. The arrangement of the bracing is shown in Figs. 170 to 171.

CHAPTER XI

DESIGN OF A RAILWAY GIRDER

Specification. — Design the girders for a 68-foot span deck-plate girder bridge to carry a single track for Cooper's E–60 loading, given in the table on page 72. The working fiber stress is to be 16,000 lbs. per sq. in. in tension and this is to be properly reduced by a straight-line formula for compression members. The allowable shearing fiber stress on the rivets will be 12,000 lbs. per sq. in., while 24,000 lbs. per sq. in. will be allowed in bearing. All rivets will be $\frac{7}{8}$ in. in diameter and rivet holes will be reamed to $\frac{15}{16}$ in. in diameter.

The girders will be placed 7 ft. 0 ins. center to center. The unit shearing fiber stress in the web plate shall not exceed 10,000 lbs. per sq. in.

The dead load of the bridge, two girders, shall be assumed of the following weights in pounds per foot of span:

Track	450
Steel, girders, bracing, etc.	1040
Total	1490

Allowable pressure between base plate and pier shall be 400 lbs. per sq. in.

Bending Moments. — The dead-load bending on one girder equals $\frac{WL}{8}$ at the center, at which point it is a maximum.

$$M = \frac{WL}{8} = \frac{1490}{2} \times 68 \times 68 \times \frac{12}{8} = 5{,}167{,}320 \text{ in. lbs.}$$

The maximum live-load bending moment will occur when the greatest load is on the girder, and the heavier loads near the center. The bending will then be a maximum with the loads so

placed on the girder that the center of the span bisects the distance between the center of gravity of the loads on the span and the adjacent wheel. The bending will be a maximum under a wheel and, there being a wheel on each side of the center of gravity, it is necessary to consider each of the two wheels in turn as the adjacent wheel to see under which the bending moment is the greater. Care should also be taken to see if any of the loads in the assumed loading pass from the girder or any additional ones come on it, as the position will not then give a maximum moment.

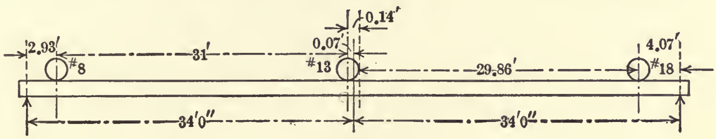


FIG. 172.

With the 68-foot span under consideration to have heavy wheel concentrations near the center when the bridge is fully loaded, it is necessary to have the drivers of the second locomotive in the center of the span. Wheel 13 at the center of the span would bring wheels 8 to 18 inclusive on the span. The center of gravity of this group must now be found, and the table of moments on page 72 will be used for this purpose. The moment of wheels 8 to 18 about wheel 18 from the table is 7,525,500 ft. lbs. and this divided by the sum of these loads, 252,000 lbs., gives 29.86 ft. as the distance of the center of gravity from wheel 18. Now placing these loads on the span in accordance with the previously stated conditions for maximum bending we have the loading shown in Fig. 172.

To determine the bending moment we must first find the left reaction. Take the moment of the loads 8 to 18 about load 18 and add to it the moment of the sum of the loads on the girder by the distance the right pier is to the right of load 18. The sum of these moments divided by the span will give the left reaction.

$$R = [7{,}525{,}500 + (252{,}000 \times 4.07)] \div 68 = 125{,}750 \text{ lbs.}$$

The bending moment under load 13 is then equal to the left reaction multiplied by the distance from the left pier to wheel 13 less the moment of the loads on the girder to the left of load 13. These moments also are taken from the moment table.

$$M = (125{,}750 \times 33.93) - 1{,}831{,}500 = 2{,}435{,}197 \text{ ft. lbs.}$$

or

$$M = 2{,}435{,}200 \times 12 = 29{,}222{,}400 \text{ in. lbs.}$$

The moment must now be found under wheel 14 when the center of the span bisects the distance between the center of gravity of the loads from 8 to 18 and wheel 14. Wheel 18 will then be 6.57 ft. from the right pier and wheel 8 will be 0.43 ft. from the left pier. The moment calculated as in the preceding case gives the bending under wheel 14 as 28,429,000 in. lbs., which is less than that found under wheel 13 in the preceding case.

To the maximum live-load bending must be added the dead-load bending and the bending due to impact. The customary allowance for the latter varies with the designer but will here be taken as that given by the formula

$$I = S\left(\frac{300}{L + 300}\right).$$

I = impact to be added to the live-load stress.
S = calculated maximum live-load stress.
L = length of loaded distance in feet which produces the maximum stress in the member.

The percentage increase in the live-load stress to cover impact in this problem is

$$\frac{300}{68 + 300} = 81.5 \text{ per cent.}$$

The maximum impact allowance then is $29{,}222{,}400 \times 0.815 = 23{,}816{,}256$ in. lbs.

The total maximum bending moment becomes:

Dead-load bending..................	5,167,320 in. lbs.
Live-load bending..................	29,222,400 in. lbs.
Impact allowance..................	23,816,260 in. lbs.
	58,205,980 in. lbs.

In designing a girder it is usually only necessary to determine the maximum bending moment, so the above method is all that is required. If it is desired to determine the bending moments for a number of points along the span this can readily be done by the graphical method explained on page 47; this method has been used to determine the maximum moment and is shown in Fig. 173. This diagram gave the maximum moment as occurring under load 13 when at the center of the span, and the amount of the bending moment was estimated at 29,400,000 in. lbs., this moment differing from that calculated by less than 1 per cent. The calculation gave wheel 13 as 0.07 ft. to the left of the center of the span which is in very close accord with the diagram.

Maximum End Shear. — A diagram of maximum shears will be drawn (see Fig. 175 and explanation on page 49). The end shear will also be calculated. The maximum shear on any section will generally occur when the span to the right of that section is fully loaded. In the case of locomotive wheel loads, owing to the considerable difference in weight between the pilot wheel 1 and the first driver 2, the maximum shear may occur with wheel 2 at the section rather than with wheel 1 there. In this case placing wheel 2 over the left pier will bring wheel 13 on the span 2 feet from the right pier. The reaction then is

$$R = \frac{10{,}392{,}000 + (318{,}000 - 15{,}000) \times 2}{68} = 161{,}735 \text{ lbs.}$$

The diagram of maximum shears checks this value of the end shear and shows that the maximum shear occurs with wheel 2 at the pier instead of with wheel 1 there.

DIAGRAMS FOR DECK PLATE GIRDER

Span 68 ft. Loading Cooper's E 60

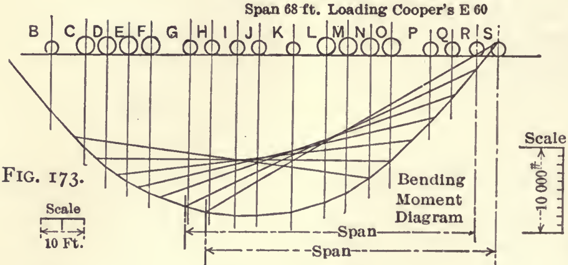


FIG. 173.

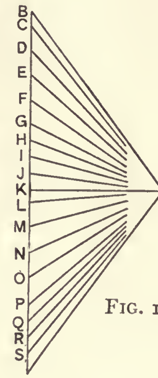


FIG. 174.

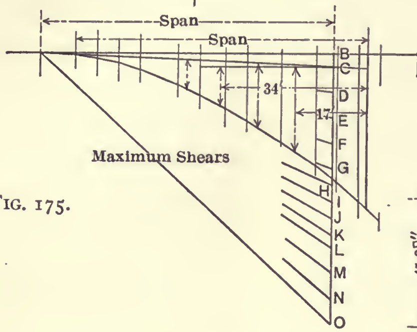
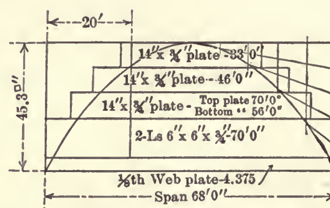


FIG. 175.



Lengths of Flange Plates

FIG. 176.

Wind Bracing

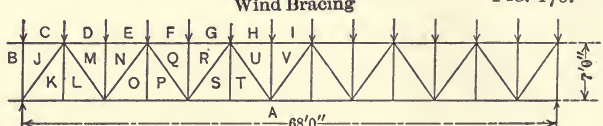


FIG. 177.

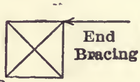


FIG. 178.

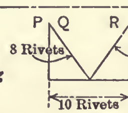
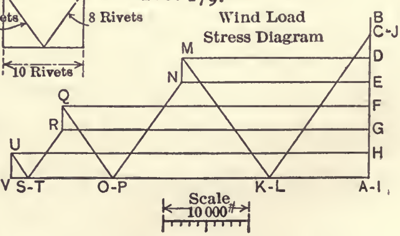


FIG. 179.



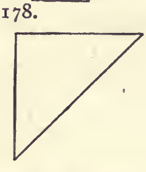


FIG. 181.

FIG. 180.

Dead-load Shears. — The weight per foot of one girder was previously estimated at 745 lbs. The dead-load end shear, being one-half the weight of the girder, is equal to $745 \times \frac{68}{2} = 25{,}330$ lbs. The dead-load shear varies uniformly across the girder, changing from this value at the end of the span to zero at the middle; it will, therefore, be $\frac{25{,}330}{2} = 12{,}665$ lbs. at the quarter point.

The impact shear will be given by the formula that was used for impact bending, $I = S\left(\frac{300}{L + 300}\right)$. This gives the impact as 81.5 per cent of the live-load shear when the girder is fully loaded, and the impact shear becomes $0.815 \times 161{,}735 = 131{,}810$ lbs.

Combining the several end shears we have:

End shear, dead load	25,330 lbs.
End shear, live load	161,735 lbs.
End shear, impact	131,810 lbs.
	318,875 lbs.

Allowing a unit shearing stress of 10,000 lbs. per sq. in. requires a web area of 31.9 sq. ins. A web plate 80 ins. deep and $\frac{7}{16}$ in. thick has an area of 35.0 sq. ins. and is ample.*

Flange Area and Selection of Sections. — It is common practice to assume the effective girder depth, that is, the distance between the centers of gravity of the girder flanges, as approximately the distance back to back of the flange angles. After the flange sections are chosen their centers of gravity are determined and this distance compared with the distance first assumed; if the error is small no correction is made, otherwise the distance is taken as the calculated distance between the centers of gravity of the flanges and new flanges are determined and a new selection of sections is made.

The girder has been assumed as having an 80-in. web plate and the angles will be set out $\frac{1}{4}$ in. on each side from the edge of the plate to avoid the possibility of the plate extending beyond the angles. The effective depth will be taken as 80.5 ins. The

* See Specifications, § 100 to § 154.

allowable working fiber stress being 16,000 lbs. per sq. in. the net flange area is found by dividing the maximum bending moment in inch pounds by the product of the allowable fiber stress and the girder depth.

$$\text{Net flange area} = \frac{58{,}205{,}980}{16{,}000 \times 80.5} = 45.2 \text{ sq. ins.}$$

This area will be made up of the following sections:

One-eighth web area = $\frac{1}{8} \times 35.0$ =	4.4 sq. ins.
Two 6 × 6 × $\frac{3}{4}$-in. angles, less 4 rivet holes..............	13.9 sq. ins.
Three 14 × $\frac{3}{4}$-in. plates, less 2 rivet holes................	27.0 sq. ins.
Total area..................	45.3 sq. ins.

This is sufficiently near the 45.2 sq. ins. required. The rivet holes have been assumed as 1 in. in diameter. As the calculated distance between the centers of gravity of the flanges exceeds that assumed by 0.20 in. the section assumed will be slightly larger than required, about $\frac{1}{4}$ per cent. This is an unimportant difference and will be neglected.

The live-load shears will be taken from the diagram of maximum shears, Fig. 175, and to these shears will be added the dead-load and impact shears.

	Shears in pounds.		
	End.	$\frac{1}{4}$ span.	$\frac{1}{2}$ span.
Dead load................	25,330	12,665	0
Live load.................	162,000	99,000	46,000
Impact..................	131,800	82,500	40,400
Total shear..............	319,130	194,165	86,400

At the quarter point, when wheel 2 is at this section, the loading extends on the girder 59 ft. from the right pier and according to the formula the percentage of live-load shear to be allowed for impact is $\frac{300}{59 + 300} = 83.5$ per cent. The impact shear is $99{,}000 \times 0.835 = 82{,}500$ lbs. In the same way the allowance for impact at the center of the span is found to be 40,400 lbs.

Stiffening Angles. — The end stiffeners act as columns, but as the load is shared by the web and as their length is short they are generally estimated by using the allowable fiber stress but not reducing it by a column formula. The area required in the end stiffeners then is

$$\frac{\text{Max. end shear}}{16{,}000} = \frac{319{,}130}{16{,}000} = 19.93 \text{ sq. ins.}$$

Four 5 in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{11}{16}$-in. angles, having a combined area of 4 $\times$ 5.38 = 21.52 sq. ins., will be used. The intermediate stiffeners are not usually calculated. Angles of the same dimensions but of lighter section are used. The intermediate stiffeners will be made two 5 in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{3}{8}$-in. angles.

The four end stiffeners being over the end bearing plate, the rivets will be assumed as transferring the end shear from the web to the four angles; that is, the load will be assumed as distributed over the entire number of rivets in the two pairs of stiffeners. The rivets will be $\frac{7}{8}$ in. in diameter and will be in double shear and will bear on the $\frac{7}{16}$-in. web plate. Allowing a shearing fiber stress of 12,000 lbs. per sq. in. and a bearing fiber stress of 24,000 lbs. per sq. in. the rivet value in double shear is 14,430 lbs. while the value in bearing is 9190 lbs. The number of rivets required in the two pairs of end stiffeners is $\frac{319{,}130}{9190} = 35$.

Lengths of Flange Plates. — The flange-plate lengths can be approximated as in Fig. 176. Lay off a line representing the span of the girder to scale and at its center erect a perpendicular representing to scale the calculated net flange area, in this case 45.3 sq. ins. Now upon the span as a base and with the 45.3 as an altitude draw a parabola. The parabola construction is shown in the light lines. Starting at the base line lay off in succession distances representing one-eighth the web area, 4.4 sq. ins.; the net area of the two flange angles, 13.9 sq. ins.; and the net area of the three flange plates, 9 sq. ins. each. The distance measured along the lower line of any section between the sides of the

parabola gives the theoretical length on the assumption that the bending moment across the girder varies as the ordinates in the parabola. This is not exact, although approximately true. Were greater accuracy desired the bending moments could be estimated for intervals along the girder and the corresponding moment diagram drawn, which could then be used instead of the parabola. A close approximation for a locomotive and train load may be made by drawing a rectangle for the center of the diagram, Fig. 176, whose width is one-tenth the span and whose ordinate represents the net flange area. On each side of this rectangle a half parabola is then drawn.

It is customary to take lengths of plates slightly exceeding the lengths determined in the way just described. The following are the scaled lengths and the lengths used.

	Scaled length.	Actual length.
	Ft.	Ft.
Outside plate, top and bottom	30	33
Middle plate, top and bottom	43	46
Inside plate, top	53	70
Inside plate, bottom	53	56

Flange Rivets. — First consider the rivets in the vertical legs of the flange angles securing these angles to the web plate. The rivets transfer the change in horizontal shear from the flanges to the web plate. The flange forces acting at intervals along the girder can be determined and the change in this force between adjacent sections used to estimate the number of rivets required. The change in flange force divided by the rivet value will give the number of rivets between the two sections. Besides this change in horizontal shear the rivets also transfer the vertical loads to the web through the upper flange rivets. The maximum wheel concentrations are usually considered as distributed over three ties or approximately 36 ins. along the flange.

These two shears can be combined in the following manner: Find the change in flange force per inch of span by dividing the difference between the flange forces acting at two adjacent

sections by the distance between these sections; this will give the average change of flange force per inch of span between the sections chosen. The vertical shear transferred from the angles to the web plate per inch of span will be the maximum wheel concentration divided by 36 ins. The square root of the sum of the squares of these two quantities will give the resultant shear per inch of span within the given section and the rivet value divided by this resultant shear per inch will give the rivet spacing in this section.

The rivet value referred to is the lower of the two values in bearing or shear.

The following is the commonly used and less tedious method of determining the flange riveting. The rate of change in flange force per inch at a section is given by

$$f = \frac{V}{h},$$

where

V = maximum vertical shear in pounds at the section.

h = the distance between rivet rows in top and bottom flanges, measured in inches. Where there are double rows of rivets in the vertical legs of the flange angles h is the average distance.

Where the web is assumed as resisting bending the value f should be reduced as the rivets are not required to provide strength to secure that part of the flange to the web which is already an integral part of it, that is, the one-eighth web area.

A = total net area of flange; and

a = one-eighth web area, then the reduced force becomes

$$f_1 = \frac{V}{h} \times \frac{A - a}{A}.$$

As before the vertical shear per inch transferred by the rivets due to the maximum wheel concentration is

$$f_2 = \frac{\text{Max. wheel load}}{36}$$

and the resultant shear per inch of span at the section under consideration is

$$f_3 = \sqrt{f_1^2 + f_2^2}.$$

The rivet spacing then is

$$p = \frac{\text{Rivet value}}{f_3}.$$

The rivet spacing is generally made the same in the two flanges.

The rivet spacing will now be estimated for the ends of the girder.

The average distance h in this case is $80.5 - (3 \times 2.25) = 73.75$.

$$f_1 = \frac{V}{h} \times \frac{A - a}{A} = \frac{319{,}130}{73.75} \times \frac{(4.4 + 13.9 + 9) - 4.4}{4.4 + 13.9 + 9} = 3630 \text{ lbs.}$$

$$f_2 = \frac{30{,}000}{36} = 835 \text{ lbs.} \qquad f_3 = \sqrt{3630^2 + 835^2} = 3730 \text{ lbs.}$$

The rivet value in bearing for $\frac{7}{8}$-in. rivets at 24,000 lbs. per sq. in. is 9190 and the rivet spacing then is $p = \frac{9190}{3730} = 2.46$ ins. Use $2\frac{1}{4}$-in. spacing.

Rivet spacing a distance of one-quarter the span from the piers,

$$f_1 = \frac{V}{h} \times \frac{A - a}{A} = \frac{194{,}000 \times 31.9}{73.75 \times 36.3} = 2310 \text{ lbs.}$$

As found before,

$$f_2 = 835 \text{ lbs.}$$
$$f_3 = \sqrt{2310^2 + 835^2} = 2460 \text{ lbs.}$$
$$p = \frac{9190}{2460} = 3.73 \text{ ins.}$$

Rivet spacing at the center,

$$f_1 = \frac{V \times (A - a)}{h \times A} = \frac{86{,}400 \times 40.9}{73.75 \times 45.3} = 1055 \text{ lbs.}$$
$$f_3 = \sqrt{1055^2 + 835^2} = 1350 \text{ lbs.}$$
$$p = \frac{9190}{1350} = 6.8 \text{ ins.}$$

The rivet spacing may be calculated for the several panels by the method just shown or it will be given with sufficient accuracy by laying off, at right angles to a line representing the span, ordinates showing to scale the rivet spacing at the ends, the quarter points and the middle of the girder and then connecting the adjacent points with lines. The spacing at any section may be found by scaling the distance from the axis to the line just drawn at the particular section.

Riveting of the Flange Plates. — From what has been stated concerning riveting the legs of the flange angles to the web plate it will be evident that the rivets connecting the flange plates to the angles are called upon to do much less. A common practice is to put in rivets securing the plates to the angles and have them stagger the rivets in the vertical legs. This prevents the interference of the rivets and furnishes more than sufficient rivets. Where there is no liability of interference this practice need not be followed. The number of rivets required to secure a plate in place and develop its strength can be estimated by multiplying the net area of the plate by its working fiber stress and dividing this by the proper rivet value. In this girder the plates are $14 \times \frac{3}{4}$ in. and the net area is 9 sq. ins.; at 16,000 lbs. per sq. in. the force in the plate is 144,000 lbs. and the rivets being $\frac{7}{8}$ in. in diameter and in single shear their value is 7215 lbs., which is less than the bearing value and must, therefore, be used. The number of $\frac{7}{8}$-in. rivets in single shear to transmit 144,000 lbs. is $\frac{144,000}{7215} = 20.$

The middle plate being 46 ft. long and the top plate 33 ft. long, the middle plate extends 6.5 ft. beyond the top plate at each end and the full strength of this middle plate must be developed in 6.5 ft.

The rivet spacing, considering that the rivets are in a double row, must not exceed

$$\frac{6.5 \times 12}{10} = 7.8 \text{ ins.}$$

The spacing ordinarily does not exceed 6 ins. and in this class of work is frequently specified not to exceed 4 ins. The rivet spacing is also commonly made less at the ends of the plates and opposite open holes left for field rivets.

Bracing. — The lateral horizontal bracing will be designed in accordance with paragraph 144 of the specifications.

Since only one lateral girder will be used the load per foot of girder will be 1000 lbs. The maximum stress will occur in JK when the bridge to the right of KL is loaded. The shear in JK then is

$$\frac{(68 - 5.23) \times 1000 \times 62.77}{68 \times 2} = 29{,}000 \text{ lbs.}$$

The stress in JK is $\dfrac{29{,}000 \times 8.74}{7.00} = 36{,}200$ lbs.

Trying one 6 in. $\times$ 4 in. $\times$ $\frac{1}{2}$-in. angle, the direct stress is

$$\frac{36{,}200}{4.75} = 7600 \text{ lbs. per sq. in.}$$

Riveting by the longer leg the allowable fiber stress is

$$1.25\left(16{,}000 - 70\frac{l}{r}\right) = 1.25\left(16{,}000 - \frac{70 \times 96}{1.15}\right)$$
$$= 12{,}700 \text{ lbs. per sq. in.}$$

The stress due to the bending is

$$f_b = \frac{Me}{I - \dfrac{Pl^2}{10E}} = \frac{36{,}200 \times 0.74 \times 0.99}{6.3 - \dfrac{36{,}200 \times 96^2}{10 \times 30{,}000{,}000}} = \frac{26{,}500}{5.2}$$
$$= 5100 \text{ lbs. per sq. in.}$$

The combined fiber stress is

$$7600 + 5100 = 12{,}700 \text{ lbs. per sq. in.}$$

which agrees with the allowable.

This piece will require at least seven $\frac{7}{8}$-in. diameter rivets, these being in single shear, with a rivet value of 7220 lbs., and

being field driven requiring an allowance of 25 per cent. The calculation being as follows:

$$\frac{36,200 \times 1.25}{7220} = 6.3, \text{ say } 7 \text{ rivets.}$$

End Frames. — The stress in BJ will be assumed as one-half the reaction or $\frac{68 \times 1000}{2 \times 2} = 17,000$ lbs. This piece is about 75 ins. long.

The diagram of the end-bracing is given in Fig. 178, the force triangle is drawn in Fig. 181, and this gives the stress in the diagonal at 23,600 lbs. The diagonals will be fastened together at their point of intersection.

These lengths and forces being less than those for JK the minimum weight 6 in. × 4-in. angles will be more than ample.

Intermediate Cross-Frame. — The estimated load on the upper strut of the cross-frame is $\frac{68 \times 3 \times 800}{13} = 12,500$ lbs. Its length is approximately 75 ins. The least radius of gyration should be $\frac{75}{120} = 0.63$. This suggests a $3\frac{1}{2}$ in. × $3\frac{1}{2}$ in. × $\frac{3}{8}$-in. angle, being the smallest allowed. This angle will carry $1.25\left(16,000 - \frac{70 \times 75}{0.69}\right) \times 2.48 = 26,100$ lbs. These angles should be ample for the intermediate cross-frames.

Riveting. — The number of rivets between the girder and the plate connecting the horizontal bracing members may be determined as shown in Fig. 179.

Lay off on a line parallel to PQ a distance representing the number of rivets in PQ, and similarly for RS; as one of these is in compression, the other in tension, their components will be in the same direction along the main girder flange, and hence the number of rivets between flange and plate can be determined by projecting these lengths upon the horizontal line which here scales 10 rivets.

Web Splice. — When the web is assumed as only resisting shear the splice is calculated to provide for the maximum shear at the

spliced section. When, however, the web is assumed as resisting bending, that is, one-eighth of the web area is taken as acting at the flanges, then the splice plates and rivets must be designed to resist a corresponding bending moment. The depth of the splice plate will be the distance between the edges of the vertical legs of the flange angles. In this case, allowing $\frac{1}{2}$ in. for clearance, the depth of the splice plate will be $80.5 - (12 + \frac{1}{2}) = 68$ ins.

The fiber stress at the center of gravity of the flanges was assumed as 16,000 lbs. per sq. in.* The distance from the neutral axis of the girder to the center of gravity of a flange was assumed as $\frac{80.5}{2} = 40.25$ ins. The fiber stress at the top of the splice plate will be $(16{,}000 \times \frac{68}{2}) \div 40.25 = 13{,}500$ lbs. per sq. in.

The bending moment resisted by the web is $\frac{\text{web area}}{8} \times$ depth of girder $\times$ fiber stress $= M = \frac{35}{8} \times 80.5 \times 16{,}000 = 5{,}635{,}000$ in. lbs. The area of the splice plate required to resist this moment is determined by a calculation similar to the one made for the web plate. This area is

$$A = \frac{8 \times \text{bending moment at section, in. lbs.}}{\text{depth of splice plate, ins.} \times \text{extreme fiber stress, lbs. per sq. in.}}$$

$$A = \frac{8 \times 5{,}635{,}000}{68 \times 13{,}500} = 49.0 \text{ sq. ins.}$$

The thickness of each of the two splice plates then is

$$t = \frac{49}{2 \times 68} = 0.36 \text{ in.}$$

If the rivet farthest from the neutral axis develops its full rivet value the stress upon any other rivet will be proportional to its distance from the neutral axis and the moment due to any rivet being also proportional to this distance the total resisting moment of a single row of rivets similarly arranged about the neutral axis is

$$M = \frac{2 \times \text{rivet value}}{y_1} \Sigma y^2.$$

* Note. — This is for the center of the span, elsewhere the fiber stress should be estimated.

Here

M = resisting moment in inch pounds of one vertical row of rivets.

R = rivet value, in pounds. (The lower of the values in bearing or shear should be used.)

y = distance of the rivet from the neutral axis, in inches.

y_1 = distance from the neutral axis to rivet in splice plate farthest from the neutral axis.

Assuming the following distances of the rivets in one row above the neutral axis, the following values are calculated for y^2 in Fig. 182.

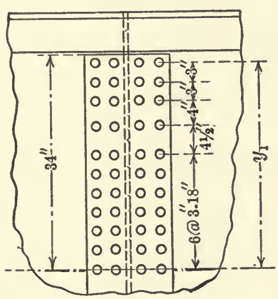


FIG. 182.

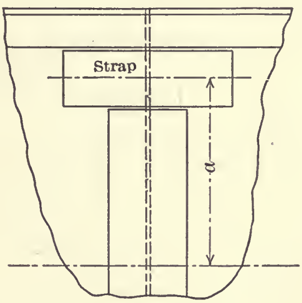


FIG. 183.

Number of rivet.	y	y^2	Number of rivet.	y	y^2
1	0	0	7	18	324
2	3	9	8	22.5	506.25
3	6	36	9	26.5	702.25
4	9	81	10	29.5	870.25
5	12	144	11	32.5	1056.25
6	15	225			3954.00

The rivet value of a $\frac{7}{8}$-in. rivet bearing in a $\frac{7}{16}$-in. plate is 9190 lbs., hence the moment of one row of rivets spaced as assumed is

$$M = \frac{2 \times 9190}{32.5} \times 3954 = 2,240,000 \text{ in. lbs.}$$

The number of rows required on this basis is

$$\frac{5,635,000}{2,240,000} = 2\tfrac{1}{2} \text{ rows.}$$

This would take three rows or a rearrangement of the rivet spacing to increase the number of rivets in the vertical row and at the same time the moment of the rivet group.

There are several other forms of splices, one of which, see Fig. 183, consists in replacing the one-eighth of the web area by straps placed on each side of the flange angles. The area of the strap is to one-eighth web area as the square of the distance between the centers of gravity of the flanges is to the square of the distance from center to center of the straps. If the straps are 9 ins. wide the distance center to center of straps will be 59 ins.

$$\text{Strap area} = \frac{4.4 \times 80.5^2}{59^2} = 8.2 \text{ sq. ins.}$$

The number of rivets on each side of the strap will be

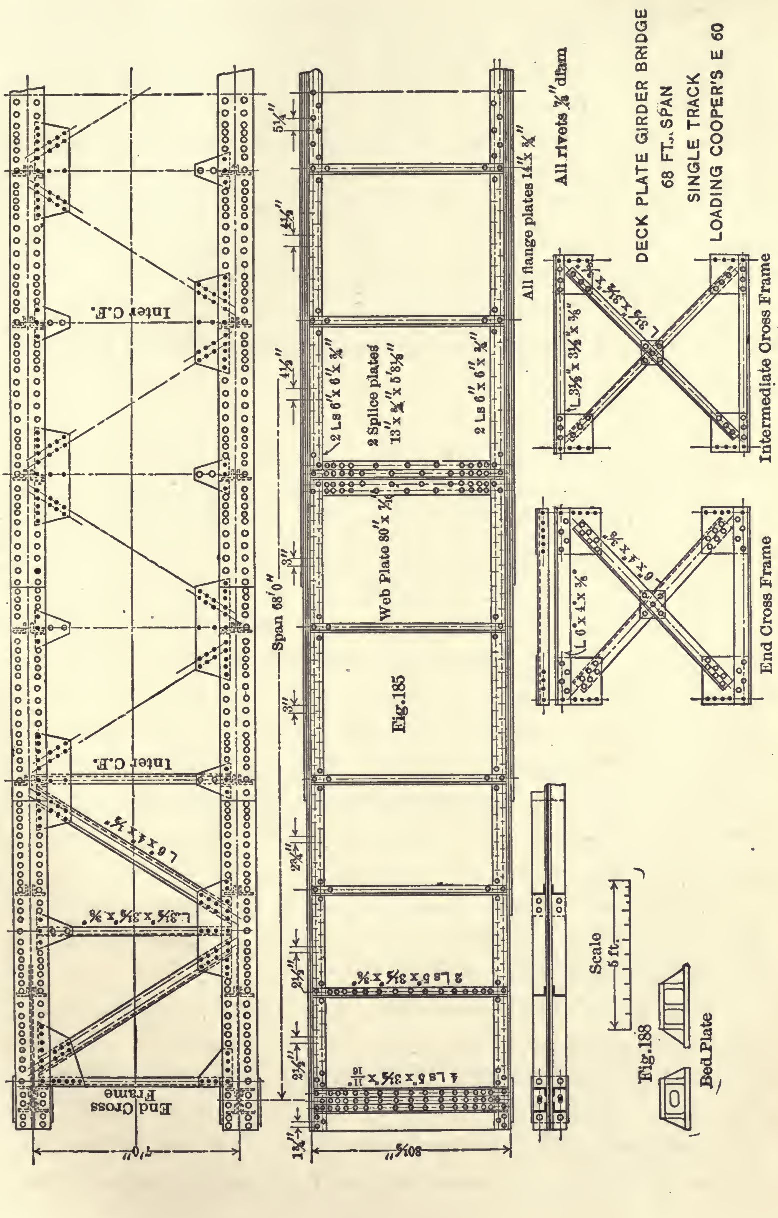
$$\frac{8.2 \times 16,000}{9190} = \text{say } 15 \text{ rivets.}$$

The central plates are designed to resist the maximum vertical shear at the section.

Still another method is to splice the web where excess material in the flange makes provision for all or part of the bending assumed as taken by the web and then design the splice for the maximum shear and any bending not otherwise provided for occurring at the section.

In the problem in hand the web splice is made 20 ft. from the piers, and the shears at this point are dead-load shear 10,300 lbs., live-load shear 90,000 lbs., and the impact allowance 76,000 lbs., making a total shear of 176,300 lbs. This would require twenty $\frac{7}{8}$-in. rivets to provide for the maximum shear at this point. There is more than sufficient surplus material in the flange (see Fig. 176) to care for the bending assumed as coming upon the web, so that a splice designed for shear only would prove ample.

FIG. 184.


DECK PLATE GIRDER BRIDGE
68 FT. SPAN
SINGLE TRACK
LOADING COOPER'S E 60
All flange plates 14" x ¾"
All rivets ⅞" diam
Inter C.F.
Inter C.F.
End Cross Frame
L 6" x 4" x ½"
L 3½" x 3½" x ⅜"
7' 0"
Span 68' 0"
2 Ls 6" x 6" x ¾"
2 Splice plates 13" x ¾" x 5' 8⅛"
2 Ls 6" x 6" x ¾"
Web Plate 80" x 7/16"
Fig.185
2 Ls 5" x 3½" x ⅜"
4 Ls 5" x 3½" x 11/16
80⅜"
Intermediate Cross Frame
Fig.187
End Cross Frame
Fig.186
Scale
5 ft.
Fig.188
Bed Plate

Flange Splices. — For the ordinary girder spans splices in the angles and plates of the flanges are not quite so likely to be required as in the web. When such splices are required the same general principles may be followed as in the case of the web splice. Splice plates may be placed where angles or plates are cut or the cut may be made where the flange plate would otherwise end and then the splice may be made by continuing the flange plate beyond the cut. This affords one of the neatest ways of splicing the angles or flange plates.

Bed Plates. — The area of the bed plate allowing 400 lbs. per sq. in. on the masonry will be the reaction divided by 400 or $\frac{319,130}{400} = 798$ sq. ins.

In small girders carrying light loads the connection between the girder and the masonry may be made by the use of a couple of rolled plates. These should extend but a few inches beyond the flange plates as the usual depth of about 1 in. is not stiff enough to transmit much load to the pier beyond the edges of the flange plate. The bed plate rests upon the masonry while the sole plate lies on the bed plate and carries the girder. At one end the girder is held firmly in place by both sole and bed plates having round holes a little larger than the foundation bolts. At the other end the holes in the sole plates are slotted, permitting the movement of the sole plate across the bed plate due to the expansion and contraction of the girder. The plates at the moving end should be planed.

Where heavy loads are carried upon girders of short span, greater depth of bed plates may be required; this type of cast-iron bed plate is shown in Fig. 188.

In the longer spans, exceeding 75 to 80 feet, greater provision must be made for expansion and also more care used to distribute the load on the masonry. One way of doing this is shown in Fig. 189. Here both ends of each girder are carried on pins, while one end rests on friction rollers. The pin assists the distribution of the pressure uniformly over the rollers, if the webs of

the pedestal are of ample depth. The method of calculation would be as follows: Assuming the rollers 4 ins. in diameter the allowable load per inch of roller length is $L = 600 \times d$, where L = load in pounds and d = diameter of roller in inches. $L = 600 \times 4 = 2400$ lbs.

The combined lengths of all the rollers in the nest is $\frac{319{,}130}{2400} =$ 133 ins. Making the rollers 23.5 ins. long would require $\frac{133}{23.5} \sim$ 6 rollers. The bearing area between pin and web is $\frac{319{,}130}{24{,}000} =$

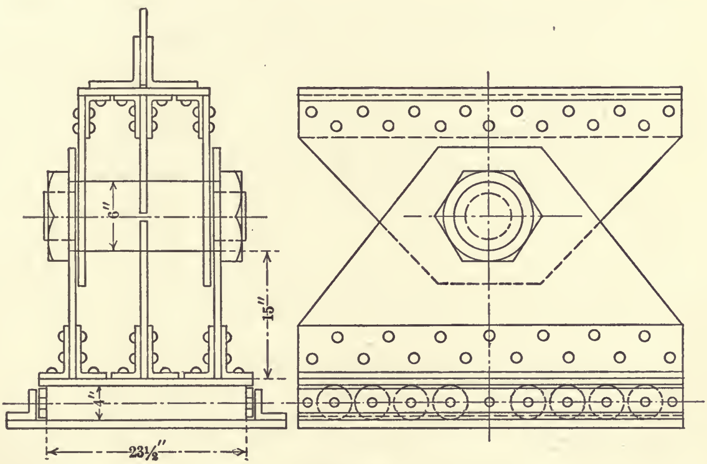


Fig. 189.

13.3 sq. ins. If three 1-in. web plates are used the projected area is 3×1.00 in. $\times$ 6 ins. = 18 sq. ins., and the area will be more than ample.

The web plate is 45 ins. long and the load is to be distributed over it. A rough approximation of the depth of the web from the bottom of the pin to the lower edge of the web plate may be made by considering it a beam section. $M = \frac{W \cdot L}{8} = f \cdot \frac{I}{e}$

$= f \cdot \frac{b \cdot d^2}{6}$; here d = depth of the web in inches, b = combined thickness of the webs in inches.

$M = 319{,}130 \times \frac{45}{8} = 16{,}000 \times 3.00 \times \frac{d^2}{6}$, from which d = 15 ins. In this calculation no account has been taken of the angles and plate secured to the lower edge of the web plates, so that the 15-in. depth used should prove ample. The rollers are kept in place by two flats held by three rods shown at the ends and the middle. Angles at the sides secured to the bed plate prevent the rollers from moving laterally. The bending on the pin is

$$\frac{319{,}130 \times 1.25}{3} = 133{,}000 \text{ in. lbs.}$$

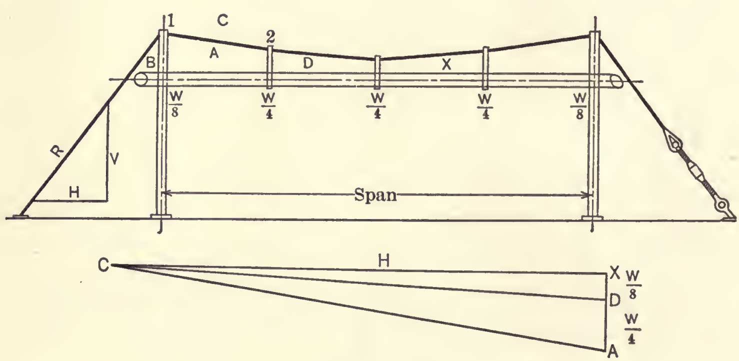


Fig. 189a.

On the basis of an allowable extreme fiber stress in pins of 22,000 lbs. per sq. in. a pin 6 ins. in diameter will resist a bending moment of 466,500 in. lbs., so that in this way the pin is exceedingly strong.

The methods indicated illustrate a couple of the simplest methods used for securing girders in place.

Figure 189a is a very common type made of steel castings, the metal ranging from $1\frac{3}{4}$ to $2\frac{1}{2}$ ins. thick. This design permits a pin of smaller diameter and a shallower bearing.

CHAPTER XII

CRANE FRAMES

FIGURE 190 represents a frame for an underbraced jib crane. The live load it is to carry is 3 tons (6000 lbs.). To this must be added an assumed weight of block, trolley, chain, etc., and this will be taken at 500 lbs. The total live load thus becomes 6500 lbs. The dead load is made up of the weight of the frame and hoisting machinery and will be assumed at 2000 lbs. The line of action of this dead load will be taken at one-quarter of the effective radius from the mast, or $11.75 \times 0.25 \sim 3.0$. The equivalent load at apex 1 will be the load that would produce the same moment on the mast, and is $2000 \times \frac{3}{10} = 600$ lbs.

The chain pull at different points in its length will be estimated upon an assumed efficiency of each sheave of 97 per cent, thus above the jib the chain pull will be $6500 \div (2 \times 0.97^2) = 3460$ lbs., while parallel to the mast it is $6500 \div (2 \times 0.97^3) = 3560$ lbs. The pull on the racking chain will be assumed at 1000 lbs.

The maximum direct stress will occur in the bracing when the load is at its maximum radius. To determine the direct stresses graphically the apex loads must be found. To calculate the equivalent apex load at 1 (Fig. 191), when the load is at its maximum radius, take moments about point 3. The equivalent

apex load at $1 = 6500 \times \frac{11.75}{10} = 7650$ lbs.

The upward reaction at point 3 is $7650 - 6500 = 1150$ lbs.

The diagram (Fig. 192) gives the combined live- and dead-load stresses; these can be determined with greater accuracy if done separately but this diagram is sufficiently accurate in this instance.

The jib will have to be considered for bending both when the load is at its maximum radius and when the trolley is between

Fig. 190.

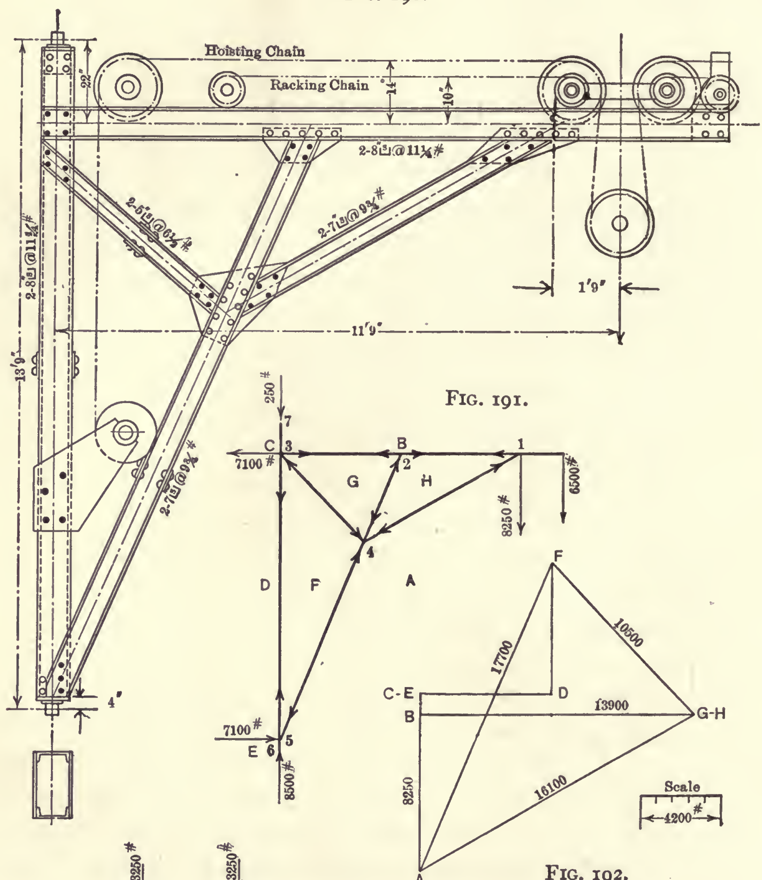
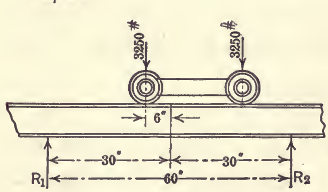
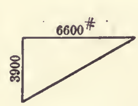


Fig. 191.

Fig. 192.





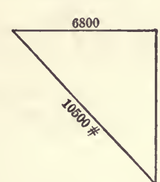


Fig. 193. Fig. 194. Fig. 195.

points 1 and 2. Here the span is 60 ins. and the distance center to center of the trolley wheels is taken as 24 ins. Although commonly discussed as a central load the position of maximum bending will here be assumed accurately; that is, when a wheel is one-quarter the distance between the wheels from the center of the span. Here this distance is $\frac{24}{4} = 6$ ins. (see Fig. 193). The load of 6500 lbs. being carried upon four wheels the load on two wheels or an axle is $\frac{6500}{2} = 3250$ lbs. The reaction

$$R_2 = \frac{(3250 \times 24) + (3250 \times 48)}{60} = 3900 \text{ lbs.}$$

$$R_1 = 6500 - 3900 = 2600 \text{ lbs.}$$

The direct stress in the section of the jib carrying the trolley is given by Fig. 194 as 6600 lbs.

Fiber Stresses. — Crane frames are liable to considerable shock. The usual method of caring for this is to take a lower working fiber stress than is ordinarily used in structures not subjected to shock. These fiber stresses run from 10,000 lbs. to 12,000 lbs. per sq. in. for mild steel, which is the material commonly used for crane frames. The allowable stress in columns is the above properly reduced by a suitable column formula. As it is difficult, if not impossible, to stiffen some parts of crane frames — the jib, for instance, in the present problem — the allowable stress in members acting as beams must be reduced to prevent buckling of the compression flange; see curves, page 102.

Selection of Jib. — The load at the maximum radius creates a direct stress in the jib of 13,900 lbs., in tension. The hoisting-chain pull is 3460 lbs., while the pull in the racking chain is 1000 lbs.; this creates a force of 1000 lbs. acting in the jib when the trolley is pulled back and of 2000 lbs. when the trolley is drawn forward. The average height of the two racking chains from the center of the jib is taken as 8 ins. Calling tension +, and compression −, we have:

Direct stresses in jib.	
Stress due to live and dead loads	+ 13,900 lbs.
Stress due to hoisting rope	− 3,460 lbs.
Stress due to racking rope	− 2,000 lbs.

The greatest stress then is the combination of the stress due to live and dead loads with the stress due to hoisting. The racking force will not be considered as it gives a lower stress than the above combination. The maximum stress is 13,900 − 3460 = 10,440 lbs. in tension.

The bending moment is $(6500 \times 21) - (3460 \times 14) =$ 88,060 in. lbs. The stress must now be considered when the trolley is between points 1 and 2. The maximum bending = $R_1 \times 24 = 2600 \times 24 = 62{,}400$ in. lbs., due to the hook load. To this must be added the bending due to the hoisting- and racking-rope pulls. The former is $M_2 = 3460 \times 14 \times 0.5 =$ 24,200 in. lbs. That due to the latter is $M_3 = 2000 \times 8 \times 0.5$ = 8000 in. lbs. The total bending is 62,400 + 24,200 + 8000 = 94,600 in. lbs. The direct stress is 6600 − 3460 − 2000 = 1140 lbs.

The influence of span to flange width must now be considered. Trying 8-in. channels at $11\frac{1}{4}$ lbs. per foot the flange width is 2.26 ins., and the span being 60 ins. it follows that

$$\frac{\text{Span}}{\text{Flange width}} = \frac{60}{2.26} = 26\tfrac{1}{2}.$$

On the cantilevered section,

$$\frac{\text{Span}}{\text{Flange width}} = \frac{21}{2.26} = 9.3.$$

From the curve, page 102, the allowable stress should not exceed 85 per cent of the maximum desired, so that allowing a maximum working fiber stress of 10,500 lbs. per sq. in. it reduces to $10{,}500 \times 0.85 = 8900$ lbs. per sq. in.

The section modulus for an 8-in. channel at $11\frac{1}{4}$ lbs. is 8.1; hence, since $M = f\frac{I}{e}$ and the bending has been calculated for two channels, we have

$$f = \frac{Me}{I} = \frac{94{,}600}{2 \times 8.1} = 5840 \text{ lbs.}$$

The direct stress per square inch $= \dfrac{1140}{2 \times 3.35} = 170$ lbs. The 8-in. channels at $11\frac{1}{4}$ lbs. will be satisfactory.

Member ***AH.*** — The length of this piece is 7 ft. and the force acting in it is 16,100 lbs. This member is in compression and as it cannot be braced and its $\frac{l}{r}$ value must be limited to 140 the least radius of gyration will have to equal or exceed $\frac{84}{140}$ = 0.60. Seven-inch channels will be required and their radius of gyration is 0.59. According to the abridged form of Ritter's formula for soft or mild steel

$$f^1 = \frac{f}{1 + \frac{1}{10,000} \times \left(\frac{l}{r}\right)^2} = \frac{10,500}{1 + \frac{19,600}{10,000}} = \frac{10,500}{2.96} = 3550 \text{ lbs.}$$

and the total allowable load on two 7-in. channels at $9\frac{3}{4}$ lbs. per foot or 2.85 sq. ins. in the section is $3550 \times 2 \times 2.85 = 20,300$ lbs., which is satisfactory.

Member ***AF.*** — This is a compression piece carrying a load of 17,700 lbs.; its length is approximately 13 ft., but, as it can be braced across, its greatest length need not exceed the length of the piece HA, and as the force acting in it is about the same as in the preceding piece it will be made of the same section, 7-in. channels at $9\frac{3}{4}$ lbs. per ft.

Member ***FG.*** — This is also a compression piece; its length is 60 ins. and the force acting in it is 10,500 lbs. Assuming as before that the $\frac{l}{r}$ value shall not exceed 140, r must not be less than $\frac{60}{140} = 0.43$. Trying 5-in. channels at $6\frac{1}{2}$ lbs., which have $r = 0.50$, we find $\frac{l}{r} = \frac{60}{0.50} = 120$. Substituting, as before, in the column formula gives

$$f^1 = \frac{10,500}{1 + \frac{1}{10,000} \times \left(\frac{l}{r}\right)^2} = \frac{10,500}{2.44} = 4300 \text{ lbs.}$$

The total allowable force on the piece is $4300 \times 2 \times 1.95 =$ 16,700 lbs., and these channels will be used.

Mast. — For most positions of the trolley the mast will be in tension. The position of maximum compression will require the

trolley to be placed as close to the mast as possible. The severest stress on the mast will be due to the bending.

The horizontal reaction at the upper pin can be found by taking moments about the lower pin.

$$R \times 13.75 = (6500 \times 11.75) + (2000 \times 3.0) = 6000 \text{ lbs.}$$

The bending at the upper portion of the mast equals $6000 \times 22 = 132,000$ in. lbs. The mast also resists bending due to the 5-in. channels which fall below the line of intersection of the other

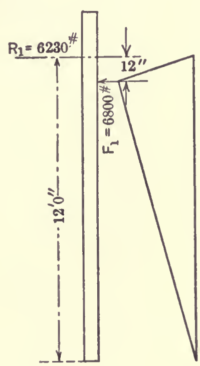


FIG. 196

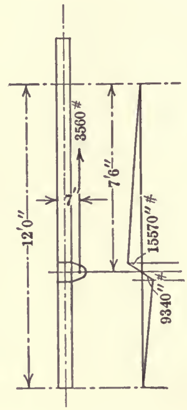


FIG. 197.

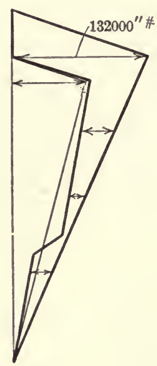


FIG. 198.

pieces at apex 3. The reaction on the mast due to this force in *FG*, the horizontal component of which is 6800 lbs., is

$$R_1 = \frac{6800 \times 11}{12} = 6230 \text{ lbs.}$$

and the bending is

$$M = 6230 \times 12 = 74,760 \text{ in. lbs.}$$

This bending is in the opposite sense to that due to the force on the pin.

The bending due to the hoisting-rope pull must now be determined. The horizontal reaction at apex 3 due to the rope pull on the drum is found by taking moments about apex 5, from which

$$F_1 = \frac{3560 \times 7}{12 \times 12} = 173 \text{ lbs.}$$

The maximum bending due to the rope pull then is

$$M = 173 \times 90 = 15{,}570 \text{ in. lbs.}$$

Fig. 198 shows the maximum bending on the mast to be 132,000 in. lbs.

Since $M = f\frac{I}{e}$, $\quad \frac{I}{e} = \frac{M}{f} = \frac{132{,}000}{10{,}500} = 12.5.$

This is for two channels, or 6.25 for one, and would require 7-in. channels were there no direct stress. Try 8-in. channels, the area of two such channels being $2 \times 3.35 = 6.7$ sq. ins. The direct stress previously found was 9100 lbs. and the unit fiber stress is $\frac{9100}{6.7} = 1350$ lbs. per sq. in. The fiber stress allowed for bending then is $10{,}500 - 1300 = 9150$ lbs. per sq. in. It follows that

$$\frac{I}{e} = \frac{M}{f} = \frac{132{,}000}{9150} = 14.4.$$

This will take two 8-in. channels at $11\frac{1}{4}$ lbs. whose section modulus is $2 \times 8.1 = 16.2$.

The resulting moment at any section is the intercept between the heavy lines in Fig. 198. The small moment at 5 has been omitted as it would not alter the maximum at 3.

Design of Frame for Top-braced Jib Crane

The capacity of the crane is to be 5 tons (10,000), at a maximum radius of 20 ft. The weight of the trolley, hook and chain will be assumed at 600 lbs. The dead load of the frame and machinery will be taken as 3000 lbs., and its center of gravity, or line of action, will be considered as one-fourth of the crane radius from the mast, or $\frac{20}{4} = 5$ ft. The load which, acting at apex 1, would produce the same moment on the mast as the frame weight is $\frac{3000}{4} = 750$ lbs.

The maximum fiber stress in tension is to be 12,000 lbs. per sq. in. In compression the maximum fiber stress will be 12,000 lbs. per sq. in., properly reduced by a column formula. The value of $\frac{l}{r}$ will be limited to 140.

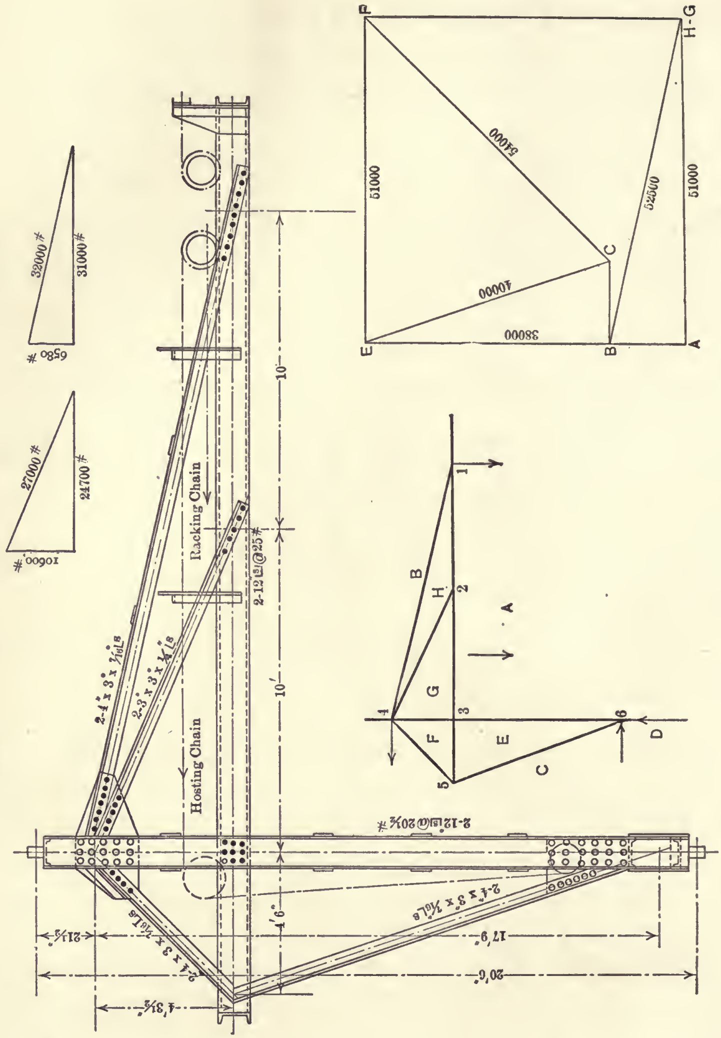


Fig. 203.

Fig. 202.

Fig. 200.

Fig. 201.

Fig. 199.

The skeleton of the frame is shown in Fig. 199, while the stress diagram is represented by Fig. 201. The total equivalent load at apex 1 is the sum of the live load, 10,000 lbs., the weight of the trolley, hook and chain, 600 lbs., and the equivalent frame

load, 750 lbs., making a total of 11,350 lbs. The horizontal reactions at points 4 and 6 can then be found by taking moments about apex 6.

$$R = \frac{11,350 \times 20}{17.75} = 12,800 \text{ lbs.}$$

These several external forces can now be located at the proper points and the stress diagram drawn in accordance with the principles given in Chapter II.

Selection of Members. — The members CE and HB are in tension and having about the same stress acting in them will be made of the same section. The net area required is $\frac{52,500}{12,000} =$ 4.37 sq. ins.

Trying two 4 in. $\times$ 3 in. $\times$ $\frac{7}{16}$-in. angles, and assuming $\frac{7}{8}$-in. diameter holes for $\frac{3}{4}$-in. rivets, the net area afforded by these angles is 5 sq. ins. Owing to their lengths their secondary stresses as given by the formula for combined stresses, Chapter VII, will be low and they will do.

The member CF will also be made of the same section.

Member GH. — The maximum stress for this member occurs when the load is at apex 2. The stress, given by Fig. 202, is 27,000 lbs. The net area required then is $\frac{27,000}{12,000} = 2.25$ sq. ins. Trying two 3 in. $\times$ 3 in. $\times$ $\frac{1}{4}$-in. angles, which have an area of $2 \times 1.44 = 2.88$ sq. ins., and allowing one $\frac{7}{8}$-in. diameter hole for a $\frac{3}{4}$-in. diameter rivet to an angle the net section for two angles is $2.88 - (2 \times 0.22) = 2.44$ sq. ins., and these angles will do.

Members AG and AH. — These members will be subjected to combined compression and flexure, and as it is difficult to brace these members laterally, due consideration must be given to the reduction of the fiber stress to provide for the column action of the compression flange. The axle load is $\frac{10,600}{2} = 5300$ lbs. The maximum bending on the 10-ft. span will occur when one of

the wheels is one-fourth the distance between the wheels from the center of the span.

Finding the reactions we have

$$R_2 = \frac{(5300 \times 54) + (5300 \times 78)}{120} = 5830 \text{ lbs.}$$

Then $R_1 = 10{,}600 - 5830 = 4770$ lbs.

The maximum bending moment on the span then is

$$M = R_1 \times 54 = 4770 \times 54 = 257{,}580 \text{ in. lbs.}$$

The maximum stress will occur in these pieces in member HA when the load is located as shown in Fig. 204, and the direct

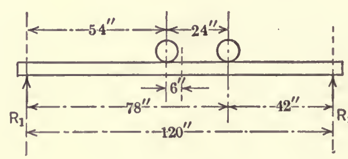


FIG. 204.

stress is given by Fig. 203. To determine this stress the reaction at apex 1 must be found when the load is in the above position.

Under this loading the apex load at 1 is $R_2 = 5830$ lbs. plus the equivalent frame load at 1, which is 750 lbs., these making a total of 6580 lbs.

The direct stress is found to be 31,000 lbs.

The jib will be assumed as braced in two places, so that it will be approximately braced laterally at intervals of about 8 ft.

This bracing can be only a light frame, generally made of a bent angle that must clear the trolley. The total bending on the member AH is due to the vertical loading used in determining the 257,580 in. lbs. just calculated, and in addition to this there is bending due to the hoisting-rope pull and the racking-rope pull, both of which act to produce bending of the same character on the beam and must therefore be added to the bending moment just found.

The bending moment due to hoisting-rope pull = 5650×19 = 107,350 in. lbs.

The bending moment due to the racking-rope pull = $1000 \times 9 = 9000$ in. lbs.

The total bending moment then is $257{,}580 + 107{,}350 + 9000 = 373{,}930$ in. lbs. Assuming that a section will be tried whose flange width is 3 ins., the ratio of unsupported part of span to flange width is $\frac{8 \times 12}{3} = 32$.

From the curve on page 102 the allowable fiber stress in compression for this ratio of length to flange width is about 85 per cent of the maximum desired. The allowable fiber stress is $12{,}000 \times 0.85 = 10{,}200$ lbs. per sq. in. If we try 12-in. channels at 25 lbs. per ft., we find their area to be $2 \times 7.35 = 14.7$ sq. ins. and their section modulus $2 \times 24.0 = 48.0$.

The direct stress per square inch is $\frac{31{,}000}{14.7} = 2100$ lbs.

The stress permitted for flexure becomes $10{,}200 - 2100 = 8100$ lbs. per sq. in. The required section modulus then is

$$\frac{I}{e} = \frac{M}{f} = \frac{373{,}930}{8100} = 46.1.$$

It is evident, therefore, that the channels tried are satisfactory. Before being accepted finally, however, these channels should be examined as columns when the load is at its maximum radius. Under this condition the jib is subjected to a direct stress of 58,650 lbs. The unit stress due to this direct load is $\frac{58{,}650}{14.7} =$ 4000 lbs.

The allowable unit stress according to Ritter's formula is

$$f = \frac{12{,}000}{1 + \frac{1}{10{,}000} \times \left(\frac{l}{r}\right)^2} = \frac{12{,}000}{1 + \frac{120^2}{10{,}000}} = \frac{12{,}000}{2.44}$$

$$= 4900 \text{ lbs. per sq. in.}$$

The $\frac{l}{r}$ value of 120 was found by dividing the unsupported length, 96 ins., by the radius of gyration of the channels about their minor axis, 0.78, or $\frac{96}{0.78} = 120$.

The 12-in. channels were found satisfactory in this respect.

Mast. — The direct stress as given by the diagram, Fig. 201, is approximately 50,000 lbs. The reactions at the pins are

$$R = \frac{11,350 \times 20}{20.5} = 11,120 \text{ lbs.}$$

The bending moment due to the above reaction is

$$M = 11,120 \times 21.5 = 239,080 \text{ in. lbs.}$$

Trying two 12-in. channels at 20.5 lbs. per ft., their area is $2 \times 6.03 = 12.06$ sq. ins., and the direct compression is $\frac{50,000}{12.06}$ $= 4150$ lbs. per sq. in. Assuming the allowable ultimate fiber stress in bending as 12,000 lbs. per sq. in. the balance for bending is $12,000 - 4150 = 7850$ lbs.

The required section modulus is

$$\frac{I}{e} = \frac{M}{f} = \frac{239,080}{7850} = 30.5.$$

An inspection of the tables of the properties of channels in a Manufacturer's handbook or on page 16 will show that either a 10-in. channel at 20 lbs. or 12-in. channels at $20\frac{1}{2}$ lbs. will carry this load. However, since 12-in. channels are used for the member AG and as these channels give considerably greater stiffness they will be used rather than increase the number of sections demanded. As it is possible to brace the channels forming the mast at as frequent intervals as desired no account has been taken of the ratio of span to flange width.

CHAPTER XIII

GIRDERS FOR OVERHEAD ELECTRIC TRAVELING CRANES

Specifications. — Design a bridge for a 10-ton O.E.T. crane whose span is 45 ft. Assume the distance center to center of trolley wheels as 4 ft. 0 ins., that the bridge weighs 13,000 lbs. (two girders), and that the trolley weight is 6000 lbs. Design the girders to resist an additional loading laterally of one-tenth the live load. Cranes operating in the open may be subjected to lateral stress from wind pressure. Considering that during extreme wind storms the crane will probably not be kept in service, the bridge may be considered, firstly, as acted on by a dead load and the maximum wind load; secondly, as acted on by the dead load, the maximum live load with the attendant lateral stresses, and a minimum wind load.

The maximum wind pressure may be taken at 30 lbs. per sq. ft. and is even taken as high as 40 lbs. per sq. ft. The assumed minimum wind pressure will vary with the conditions of the crane's location and with the judgment of the designer from 5 to 15 lbs. per sq. ft. of exposed surface.

Where a vertical bracing girder parallels the main girder the wind is assumed as acting simultaneously upon both girders. Allow a working fiber stress in tension of 10,000 lbs. per sq. in. and the same properly reduced by a column formula for compression pieces. Use $\frac{3}{4}$-in. rivets throughout.

The loading will be reduced to that for a single girder or one-half the bridge. The weight of the bridge will be assumed as a uniform load and, as is usually done, the bending moment for it will be calculated for the middle of the bridge.

Weight of 1 girder $= \frac{13,000}{2} = 6500$ lbs.

$$\text{Dead-load bending} = \frac{W \cdot l}{8} = 6500 \times 45 \times \frac{12}{8} = 438{,}750 \text{ in. lbs.}$$

Assuming the load as hung centrally on the trolley, the load on each trolley wheel will be

$$\tfrac{1}{4}\,(\text{trolley weight} + \text{live load}) = \tfrac{1}{4}\,(6000 + 20{,}000) = 6500 \text{ lbs.}$$

The reaction

$$R_1 = \frac{[(6500 \times 19.5) + (6500 \times 23.5)]}{45} = 6210 \text{ lbs.}$$

The maximum bending due to live load then is

$$M = 6210 \times 21.5 \times 12 = 1{,}602{,}180 \text{ in. lbs.}$$

The total bending due to combined dead and live loads is

$$M = 438{,}750 + 1{,}602{,}180 = 2{,}040{,}930 \text{ in. lbs.}$$

The material will be determined for the two following sections, the depth being taken as 3 feet.

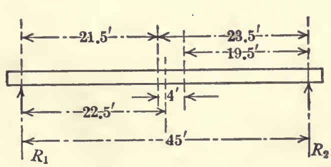


FIG. 205.

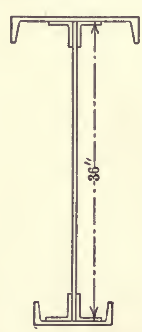


FIG. 206.

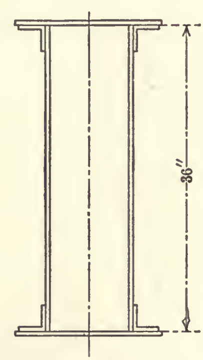


FIG. 207.

The box section (Fig. 207) will be considered first. If the width is made one-thirtieth the span it will be, say, 17 ins. wide. The ends of the girders resting upon the bridge trucks will be assumed 18 ins. deep. The web plates will be made $\frac{1}{4}$ in. thick, making the gross web area at the ends $2 \times \frac{1}{4} \times 18 = 9$ sq. ins.

The maximum end shear is given by the diagram of maximum shears (Fig. 208), and is 15,800 lbs. The unit shearing fiber

stress is $\frac{15,800}{9} = 1760$ lbs. This fiber stress is very low but, notwithstanding that some specifications permit $\frac{3}{16}$-in. material, we will use the $\frac{1}{4}$-in. plates.

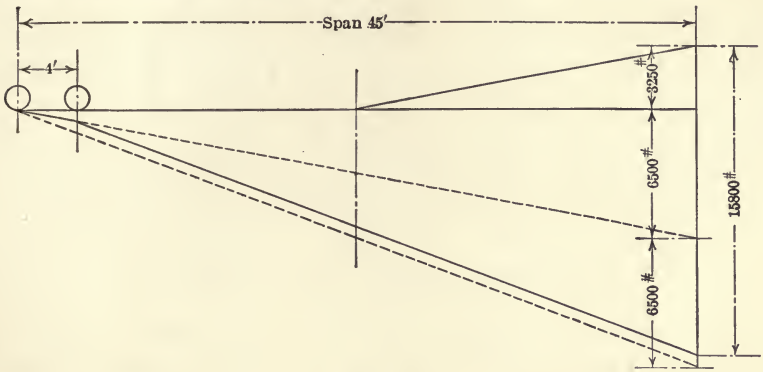


FIG. 208.

Compression Flange. — The distance between the centers of gravity of the flanges will be assumed as approximately the distance back to back of the flange angles, or 36 ins. According to the curve, page 102, the allowable fiber stress in a compression flange when the ratio of the laterally unsupported length of flange to flange width is 30 to 1, is 80 per cent of the maximum desired. The allowable fiber stress becomes $10,000 \times 0.80 = 8000$ lbs. per sq. in.

The net flange area then becomes (see page 149),

$$A = \frac{M}{f \times h} = \frac{2,040,930}{8000 \times 36} = 7.1 \text{ sq. ins.}$$

If the web is considered as taking bending in addition to shear the web will furnish a section equal to one-eighth the web area, or $\frac{1}{8} \times 36 \times \frac{1}{4} \times 2 = 2.25$ sq. ins. In this case the flange plates will work out so light that the web plates will not be assumed as resisting bending. The flange angles will be tried at 3 ins. $\times$ 3 ins. $\times \frac{1}{4}$ in. and they will be considered as having two holes, one in each leg at the same section; this makes it possible to locate

the rivets at any points desired. The plate will be taken as 17 ins. wide. Net area of angles, 3 ins. $\times$ 3 ins. $\times \frac{1}{4}$ in., having two $\frac{7}{8}$-in. diameter holes,

$$(2 \times 1.44) - (4 \times 0.22) = 2.88 - 0.88 = 2.00 \text{ sq. ins.}$$

The net area of the plate then is $7.1 - 2.00 = 5.10$ sq. ins. The net width of the plate is $17 - (2 \times \frac{7}{8}) = 15.25$ ins. The thickness of the plate is $\frac{5.1}{15.25} = 0.334$ in.; use $\frac{5}{16}$ in.

Tension Flange Area. — Here the allowable fiber stress is 10,000 lbs. per sq. in. and the required net area of the flange is

$$A = \frac{M}{f \times h} = \frac{2{,}040{,}930}{10{,}000 \times 36} = 5.67 \text{ sq. ins.}$$

Using two 3 in. $\times$ 3 in. $\times \frac{1}{4}$-in. angles as before, the net area of the plate is $5.67 - 2.00 = 3.67$ sq. ins. and the required thickness of the plate is $\frac{3.67}{15.25} = 0.241$ in., say $\frac{1}{4}$ in.

The girders will be made approximately for uniform strength (see Fig. 209). The depths at points along the girder may be

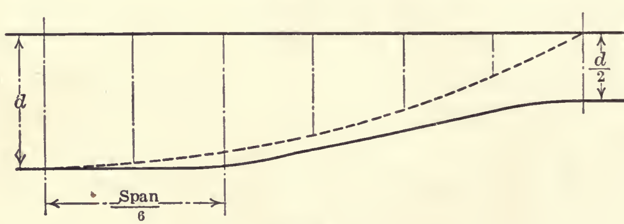


FIG. 209.

checked by drawing the diagram of maximum bending moments similar to that drawn for a railway girder, in Chapter XI, and combining the live-load bending moments thus found with the dead-load bending moments.

Since M = flange area $\times$ mean fiber stress $\times$ girder depth, it follows that the flange area and the mean fiber stress being made constant the depths at any section can be made to vary

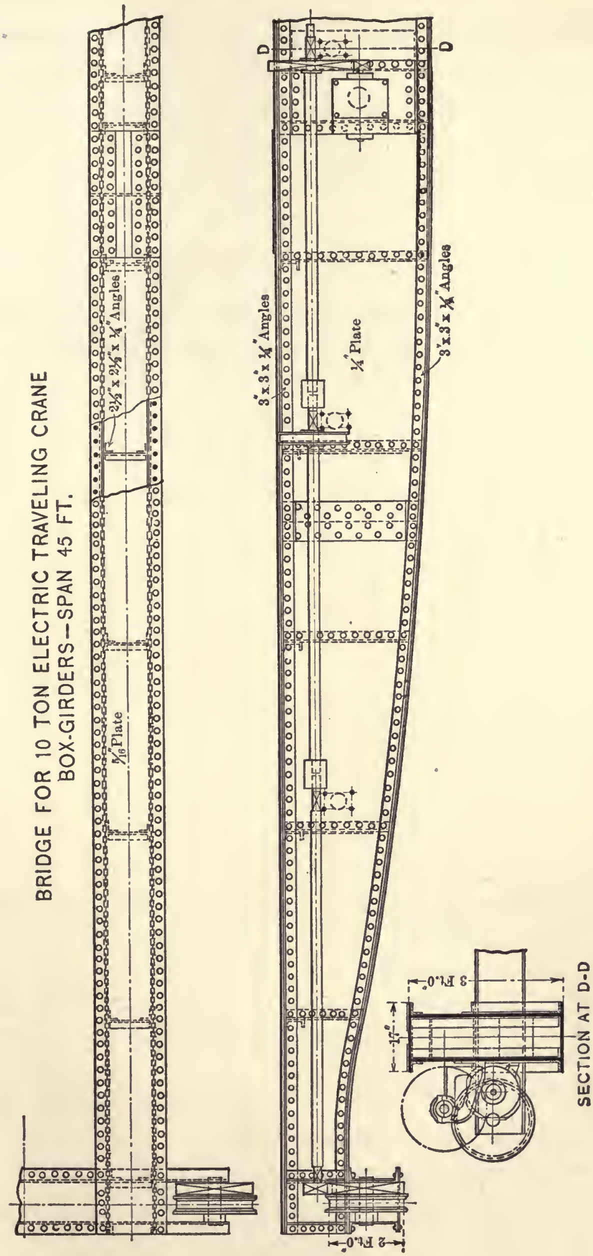

BRIDGE FOR 10 TON ELECTRIC TRAVELING CRANE
BOX-GIRDERS—SPAN 45 FT.
2½"x 2½"x ¼" Angles
5/16" Plate
3"x 3"x ¼" Angles
¼" Plate
3"x 3"x ¼" Angles
D
D
SECTION AT D-D
3 Ft. 0"
2 Ft. 0"

FIG. 210.

with the bending moments. This, of course, is approximately true only so long as the flange area times the square of its distance from the neutral axis of the entire girder section at the point considered is large compared with the inertia of the flange about a parallel axis through its center of gravity. Where greater accuracy is desired the moments of inertia of the several sections may be computed and the formula $M = f\frac{I}{e}$ used.

The section chosen should now be tested when subjected to the lateral moment stated in the specification. This moment being produced by one-tenth of the live load it will equal one-tenth of the live-load moment, or $\frac{1,602,180}{10} = 160,218$ in. lbs. The web plates for a distance of 12 ins. from the top will be considered as

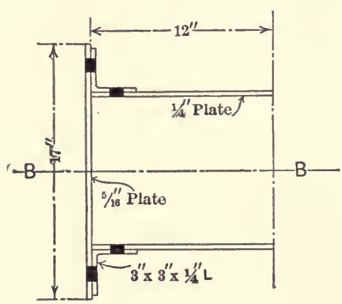


FIG. 211.

resisting this moment. The following calculation gives the moment of inertia about the axis *B–B*, Fig. 211.

One plate, $\frac{bd^3}{12} = 0.31 \times \frac{17^3}{12} =$		126.90
Two plates, $Ah^2 = 2 \times 0.25 \times (12 - 0.88) \times (5.12)^2 =$		145.73
Two angles (about their own axis) = ...		2.48
Two angles, $Ah^2 = 2 \times 1.44 \times 6.09^2 =$		106.82
	Inertia	381.93
Deducting for rivet holes (Ah^2 portion only),		
$2 \times 0.22 \times 5.38^2 =$	12.73	
$2 \times (0.27 + 0.22) \times 6.75^2 =$	44.65	57.38
	Inertia	324.55

The extreme fiber stress is then $f = \frac{M \cdot e}{I}$

or $$f = 160{,}218 \times \frac{8.5}{324.6} = 4200 \text{ lbs. per sq. in.}$$

The mean fiber stress in the compression flange is 8000 lbs. per sq. in.

The total fiber stress is 8000 + 4200 = 12,200 lbs. per sq. in.

This fiber stress, considering that it is the maximum of both vertical and lateral bending, is satisfactory.

Some designers prefer to estimate the forces acting laterally on the crane upon the assumption that the crane when carrying its maximum load is stopped from full speed in an assumed distance, the retardation being considered uniform. In the present instance we will assume a velocity of bridge travel of 300 ft. per minute, and that the crane is stopped in 5 ft.

The crane velocity being $\frac{300}{60} = 5$ ft. per second, the retardation is $f = \frac{v^2}{2 \cdot s}$, where f = acceleration or retardation in feet per second per second.

v = velocity of bridge in feet per second.

s = distance traveled during acceleration or retardation in feet.

In our case the retardation is $f = \frac{5^2}{2 \times 5} = 2.5$ ft. per sec. per sec.

As the force equals mass times retardation, the force required to stop the bridge is $F = \frac{13{,}000}{32.2} \times 2.5 = 1000$ lbs.

This acts as a uniform load laterally along the bridge and will be resisted by both top and bottom flanges of each girder. The force to retard the trolley and live load will act upon the upper flanges of the two girders of the bridge and equals $F = \frac{26{,}000}{32.2} \times 2.5 = 2020$ lbs.

The central load on one girder the equivalent of the above forces is $\frac{1000}{8} + \frac{2020}{2} = 1135$ lbs.

The bending moment due to this central load is

$$M = \frac{W \cdot L}{4} = 1135 \times 45 \times \frac{12}{4} = 153{,}225 \text{ in. lbs.}$$

This moment differs so slightly from the moment found by the previous method that the girder dimensions should prove satisfactory viewed from either estimate.

Flange Rivets. — Considering that the unit fiber stress in tension is 10,000 lbs. per sq. in. the allowable unit shearing fiber stress on the rivets, assuming about the same material, would be $\frac{3}{4} \times 10{,}000 = 7500$ lbs. per sq. in. The bearing value would be taken at twice the shearing value or 15,000 lbs. per sq. in. The value of $\frac{3}{4}$-in. rivets in single shear and bearing in a $\frac{1}{4}$-in. plate is 2813 lbs.

Assuming the girder depth at the supports as 18 ins., the distance center to center of rivets vertically is $18 - (2 \times 1.75) = 14.5$ ins.

The maximum end shear will be assumed as transferred to the girder in a distance equal to its depth, or 18 ins. The vertical shear per inch of span is $s_1 = \frac{15{,}800}{18} = 880$ lbs.

The horizontal shear per inch at this point is

$$s_2 = \frac{\text{Vertical shear}}{\text{Distance center to center of rivets vertically}}.$$

$$s_2 = \frac{15{,}800}{14.5} = 1090 \text{ lbs.}$$

The resulting shear per inch of span is

$$s_3 = \sqrt{s_1^2 + s_2^2} = 1400 \text{ lbs.}$$

Considering that the web being double will necessitate a row of rivets in each web plate the horizontal distance center to center of rivets $= \frac{2813 \times 2}{1400} = 4$ ins.

Owing to the concentration of loading at the ends the spacing will be made $3\frac{1}{2}$ ins.

The net area of the compression flange being 7.1 sq. ins. and the mean fiber stress 8000 lbs. per sq. in., the number of rivets to develop the full flange strength is $\frac{7.1 \times 8000}{2813}$, about 20.

These rivets placed in two rows and spaced $3\frac{1}{2}$ ins. center to center would require a distance of $3.5 \times 11 = 38.5$ ins. This would indicate that for about 48 ins. from the ends of the plate the rivets should be spaced about $3\frac{1}{2}$ ins. center to center and at the other sections the spacing in the compression flange should not exceed from 16 to 20 times the thickness of the outside plate. The spacing in the two flanges is usually made the same. The upper plate being $\frac{5}{16}$ in. thick the rivet spacing should not exceed 5 ins.

Angle Stiffeners. — Allowing 10,000 lbs. per sq. in. the area of the stiffeners $= \frac{\text{Maximum end shear}}{10,000} = \frac{15,800}{10,000} = 1.58$ sq. ins.

Using two angles the smallest permitted would be $2\frac{1}{2}$ ins. $\times$ $2\frac{1}{2}$ ins. $\times$ $\frac{1}{4}$ in., and their section $2 \times 1.19 = 2.38$ sq. ins., which is more than ample.

Stiffeners, however, serve a double purpose, as the opposite inside stiffeners are riveted to a plate forming a diaphragm at the section and greatly stiffening the girder laterally. They frequently also serve to secure short channel or angle sections which pass across the flange and assist the flange plate in carrying the rail. These latter are commonly placed at each pair of vertical stiffeners, while the diaphragms are generally placed at alternate panels.

The web plate should be reinforced where brackets or motors are attached to it and at these points handholes should be cut in the web plate on the opposite side of the girder to facilitate bolting the brackets and motor to the girder.

Girder Design for Fig. 206

Using the same data the girder will now be designed for the section (Fig. 206).

The maximum combined live- and dead-load bending was

found to be 2,040,930 in. lbs. The maximum end reaction previously determined was 15,800 lbs. The girder will be assumed 36 ins. deep, back to back of angles, at the center of the span, and 20 ins. deep at the bridge trucks. The web will be taken $\frac{5}{16}$ in. thick making the unit shearing fiber stress $\frac{15,800}{0.31 \times 20} = 2550$ lbs. per sq. in.

The effective depth will be assumed slightly less than 36 ins. to allow for the centers of gravity of the channels falling towards the center of the girder, the distance being $36 - 1.5 = 34.5$ ins. The net flange area in tension then will be

$$A = \frac{M}{f \times h} = \frac{2,040,930}{10,000 \times 34.5} = 5.92 \text{ sq. ins.}$$

Trying one 9-in. channel at $13\frac{1}{4}$ lbs. per ft., its area is 3.89 sq. ins., and two 3 in. $\times$ 3 in. $\times$ $\frac{5}{16}$-in. angles, we have

	Sq. ins.	Sq. ins.
One 9-in. channel at $13\frac{1}{4}$ lbs. gross area..........	3.89	
Less two rivet holes in $\frac{1}{4}$-in. material, 2×0.22 =.	0.44	3.45
Two $3 \times 3 \times \frac{5}{16}$-in. angles, 1.78×2 = ...	3.56	
Less four rivet holes in $\frac{5}{16}$-in. plate, 4×0.27 =...	1.08	2.48
		5.93

This net area is satisfactory.

In the compression flange the fiber stress must be reduced to provide for lateral strength. The ratio of span to flange width if a 15-in. channel is tried is $45 \times \frac{12}{15} = 36$. Reducing the unit working fiber stress to 75 per cent of the maximum desired permits a fiber stress of 7500 lbs. per sq. in. Again substituting in the formula

$$M = A \times f \times h. \quad A = \frac{M}{f \times h} = \frac{2,040,930}{7500 \times 34.5} = 7.88 \text{ sq. ins.}$$

Trying two 3 in. $\times$ 3 in. $\times$ $\frac{5}{16}$-in. angles and one 15-in. channel at 33 lbs. per ft.,

	Sq. ins.	Sq. ins.
One 15-in. channel at 33 lbs..................	9.90	
Less two rivet holes, $2 \times 0.88 \times 0.40$ = ...	0.70	9.20
Two $3 \times 3 \times \frac{5}{16}$-in. angles, 1.78×2 = ...	3.56	
Less four rivet holes..........................	1.08	2.48
Total area............................		11.68

This seems to be too large but before discarding it the combined stresses due to the lateral and vertical bending should be determined. The flexural fiber stress with this flange is

$$f = \frac{M}{A \times h} = \frac{2{,}040{,}930}{11.68 \times 34.5} = 5100 \text{ lbs. per sq. in.}$$

In the preceding girder section the lateral bending due to stopping the crane in 5 ft. was estimated at 153,225 in. lbs. The fiber stress, assuming that the bending is mainly resisted by the 15-in. channel whose section modulus about its axis 1–1 is 41.7, is

$$f = \frac{M \cdot e}{I} = \frac{153{,}225}{41.7} = 3680 \text{ lbs. per sq. in.}$$

The combined maximum stress then is

$$\left(\frac{5100}{0.75}\right) + 3680 = 10{,}480 \text{ lbs. per sq. in.}$$

This is low considering that everything of importance is taken into account, but will be used, as calculations made for 12-in. channels indicate a fiber stress undesirably high. This section of the girder may now be checked by calculating its moment of inertia and using the formula $f = \dfrac{Me}{I}$.

Flange Riveting. — These calculations are similar to those made for the box girder. The rivet value in a $\frac{5}{16}$-in. web plate for $\frac{3}{4}$-in. rivets is 3316 lbs. As in the other case

$$s_1 = \frac{15{,}800}{20} = 790 \text{ lbs.}$$

$$s_2 = \frac{15{,}800}{16.5} = 960 \text{ lbs.}$$

The resulting shear per inch of span is

$$s_3 = \sqrt{s_1^2 + s_2^2} = \sqrt{790^2 + 960^2} = 1240 \text{ lbs.}$$

and the rivet spacing of $\frac{3}{4}$-in. rivets is $\frac{3316}{1240} = 2.67$ ins.

The stiffeners can, as in the preceding problem, be made the smallest allowable, say, $2\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{1}{4}$-in. angles.

Bridge Girders with Horizontal Stiffening Girders

The following example is intended to illustrate this type of bridge. Span 60 ft. Girder depth $\frac{60}{12} = 5$ ft. back to back of angles. The crane capacity is to be 20 tons (40,000 lbs.). The trolley wheel base is 6 ft. 0 ins. The bridge weight, two girders, is 30,000 lbs. The load on each trolley wheel is

$$\frac{40{,}000 + 12{,}000}{4} = 13{,}000 \text{ lbs.}$$

The live-load bending will be found as in the preceding cases and as follows:

$$R_1 = \frac{(13{,}000 \times 31.5) + (13{,}000 \times 25.5)}{60} = 12{,}350 \text{ lbs.},$$

$$M = R_1 \times 28.5 \times 12 = 12{,}350 \times 28.5 \times 12,$$
$$M = 4{,}223{,}700 \text{ in. lbs.}$$

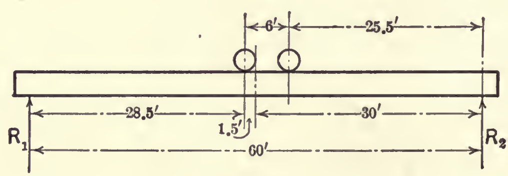


FIG. 212.

In estimating the dead-load bending the girder weight will be assumed as a uniform load. The bending moment is

$$M = \frac{WL}{8} = \frac{15{,}000 \times 60 \times 12}{8} = 1{,}350{,}000 \text{ in. lbs.}$$

The total bending is $4{,}223{,}700 + 1{,}350{,}000 = 5{,}573{,}700$ in. lbs.

Allowing a mean unit fiber stress of 9000 lbs. per sq. in. the flange area will be

$$A = \frac{M}{f \cdot h} = \frac{5{,}573{,}700}{9000 \times 58} = 10.65 \text{ sq. ins.}$$

The distance between the centers of gravity of the flanges has been taken 58 ins. as probably no flange plates will be required.

The web plate will be assumed as $\frac{1}{4}$ in. thick and one-eighth the web area will be considered as contributing to the flange section.

$$\text{Web area} = 60 \times \tfrac{1}{4} = 15 \text{ sq. ins.}$$

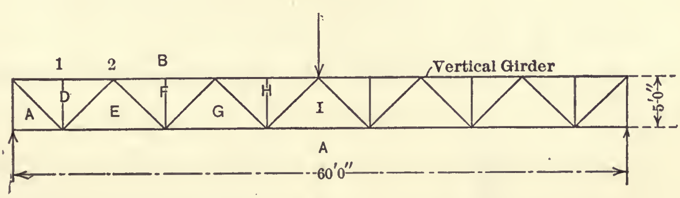
$\frac{1}{8} \times$ web area $= \frac{15}{8} = 1.88$ sq. ins. The balance of the flange area will be made up of the angles, or $10.65 - 1.88 = 8.77$ sq. ins.

The flange having two angles each angle must have a net area of $\dfrac{8.77}{2} = 4.38$ sq. ins. Allowing for two holes for $\frac{3}{4}$-in. diameter rivets, the gross area of each angle must be $4.38 + 0.44 = 4.82$ sq. ins.

Trying 6 in. $\times$ 4 in. $\times$ $\frac{1}{2}$-in. angles, their gross area is 4.75 sq. ins., which is satisfactory. The flange will therefore be made of two 6 in. $\times$ 4 in. $\times$ $\frac{1}{2}$-in. angles.

Horizontal Stiffening Girder (Fig. 213). — First assuming a horizontal loading of one-tenth the vertical loading gives two

FIG. 213.



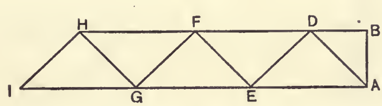


FIG. 214.

concentrated loads of 1300 lbs. each and a uniform load of 1500 lbs. Before using these figures we will check them by estimating the forces acting on the girders when the crane carrying its full load at maximum velocity is stopped within 5 ft.

The velocity of the crane being assumed at 300 ft. per minute, or 5 ft. per second, to stop the crane in 5 ft. the retardation must

be $f = \frac{v^2}{2 \cdot s} = \frac{5^2}{2 \times 5} = 2.5$ ft. per sec. per sec. The force to check the live load and trolley is Force = Mass × Acceleration.

$$F = \frac{52{,}000 \times 2.5}{32.2} = 4040 \text{ lbs.}$$

The force to check the bridge is

$$F = \frac{30{,}000 \times 2.5}{32.2} = 2330 \text{ lbs.}$$

The first $\frac{4040}{4} = 1010$ lbs. compares with a concentrated load of 1300 lbs., while the latter is a uniform load of 1165 lbs. compared with one of 1500 lbs. The first figures, being the greater, will be used.

As this lateral bending is at best only a rough approximation it will be sufficiently accurate to assume a central load of $2 \times 1300 = 2600$ lbs., due to trolley weight and live load, and an equivalent central load of $\frac{1500}{4} = 375$ lbs., due to bridge weight. The reaction then is $\left(\frac{2600 + 375}{2}\right) = 1490$ lbs.

The stress AI, Fig. 214, is 8940 lbs.

A section through the main and bracing girders is shown in Fig. 215.

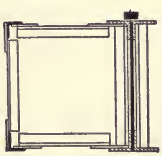


FIG. 215.

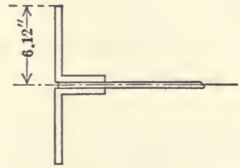


FIG. 216.

As the wheels travel along the flange of the main girder the force of 1300 lbs. transferred to the girder by a wheel may produce bending on this flange; for instance, between the two points of lateral support 1 and 2 this bending will be

$$M = \frac{W \cdot L}{4} = 1300 \times \frac{60}{4} = 19{,}500 \text{ in. lbs.}$$

The moment of inertia of two 6 in. $\times$ 4 in. $\times$ $\frac{1}{2}$-in. angles back to back but separated by a $\frac{1}{4}$-in. plate is

Angles about their own axes, $2 \times 17.40 =$	34.80
$A \cdot h^2 = 2 \times 4.75 \times (1.99 + 0.12)^2 =$	42.27
$I =$	77.07

The fiber stress due to bending is

$$f_1 = \frac{M \cdot e}{I} = \frac{19{,}500 \times 6.12}{77.07} = 1550 \text{ lbs. per sq. in.}$$

Direct fiber stress due to horizontal bracing girder

$$f_2 = \frac{8940}{(2 \times 4.75)} = 940 \text{ lbs. per sq. in.}$$

The total fiber stress then is

$$9000 + 940 + 1550 = 11{,}490 \text{ lbs. per sq. in.}$$

As all the forces acting on the girder both vertically and horizontally have been considered this fiber stress is satisfactory.

The unbraced length of AI being 60 ins. the radius of gyration if $\frac{l}{r} \leqq 120$ then $r \geqq \frac{60}{120} = \frac{1}{2}$. The smallest angles fulfilling these requirements are 3 ins. $\times$ 3 ins. Trying one 3.5 in. $\times$ 3.5 in. $\times$ $\frac{5}{16}$-in. angle, its least radius of gyration is 0.69. $\frac{l}{r} = \frac{60}{0.69} = 87$. According to Ritter's formula,

$$f = \frac{11{,}000}{1 + \frac{1}{10{,}000}\left(\frac{l}{r}\right)^2}, \qquad f = \frac{11{,}000}{1 + \frac{87^2}{10{,}000}} = 6250 \text{ lbs.}$$

The total load on a 3.5 in. $\times$ 3.5 in. $\times$ $\frac{5}{16}$-in. angle is $6250 \times 2.09 = 13{,}000$ lbs. This satisfies the requirements.

The diagonals will carry their maximum stresses when the trolley fully loaded is at one side of the bridge. The maximum end shear on the horizontal girder is

$$R_1 = \frac{2600 \times 57}{60} + 375 = 2845 \text{ lbs.}$$

The force in the diagonals then is $2845 \times 1.41 = 4010$ lbs. The length of the diagonal is $60 \times 1.41 = 84.6$ ins. If $\frac{l}{r} \leqq 120$ then $r \geqq \frac{84.6}{120} \sim 0.70$. This will require $3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$-in. angles. The permissible fiber stress, according to Ritter's formula, is

$$f = \frac{11,000}{1 + \frac{1}{10,000} \times \left(\frac{l}{r}\right)^2} = \frac{11,000}{1 + \frac{120^2}{10,000}} = 4500 \text{ lbs. per sq. in.}$$

The load that can be carried by one $3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{5}{16}$-in. angle is $4500 \times 2.09 = 9400$ lbs.

No calculation will be made for the vertical latticed girder as it merely holds the horizontal girder in place; the smallest permissible angles, $2\frac{1}{2}$ ins. $\times$ $2\frac{1}{2}$ ins. $\times$ $\frac{1}{4}$ in., will be used. The lower horizontal girder will be made the same as the upper, excepting that the angles at right angles to the main girder will be omitted.

Riveting. — To determine the rivet spacing in the main girder it will be necessary to estimate the maximum shears along the girder. The maximum reactions on one girder are, live-load reaction, $26,000 \times \frac{57}{60} = 24,700$ lbs. The dead-load reaction is $\frac{15,000}{2} = 7500$ lbs. The combined live-load and dead-load reactions are $24,700 + 7500 = 32,200$ lbs. See Fig. 217, which is the diagram of maximum shears for this girder. The maximum wheel load will be assumed as transferred to the web in 24 ins. of flange length, or the shear per inch of span due to load concentration is $\frac{13,000}{24} = 540$ lbs. $= s_1$.

The horizontal shear at the ends per inch of span is the end shear divided by the vertical distance center to center of flange rivets $= \frac{32,200}{[60 - (2 \times 2.5)]} = \frac{32,200}{55} = 585$ lbs. $= s_2$.

$$s_3 = \sqrt{s_1^2 + s_2^2} = \sqrt{540^2 + 585^2} = 796 \text{ lbs.}$$

The rivet value of a $\frac{3}{4}$-in. rivet in double shear, bearing in a $\frac{1}{4}$-in. plate, is 2813 lbs. for a unit shearing stress of 7500 lbs. per sq. in. and 15,000 lbs. per sq. in. in bearing. The rivet spacing therefore will be $\frac{2813}{796} \sim 3\frac{1}{2}$ ins.

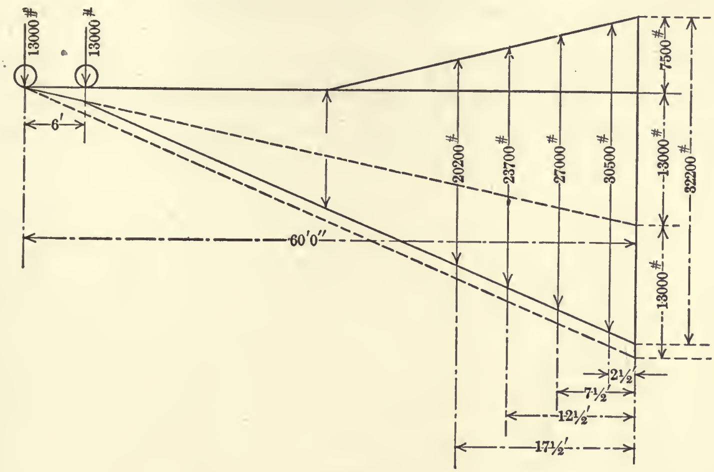


FIG. 217.

The rivet spacing will be determined for quarter points along the girder (see Fig. 218). At 15 ft. from the end the maximum

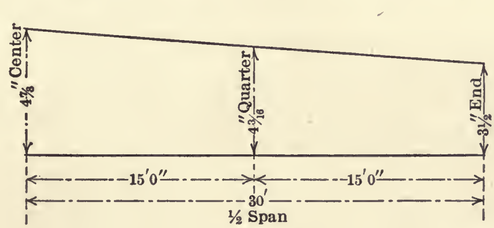


FIG. 218.

shear is 22,000 lbs. and the horizontal shear per inch of span is

$$\frac{22,000}{55} = 400 \text{ lbs.}$$

$$s_3 = \sqrt{400^2 + 540^2} = 672 \text{ lbs.}$$

The rivet spacing is $\frac{2813}{672} = 4.18$ ins.

At the middle the shear is 11,700 lbs. and the horizontal shear per inch of span is $\frac{11,700}{55} = 213$ lbs., hence

$$s_3 = \sqrt{213^2 + 540^2} = 580 \text{ lbs.}$$

The rivet spacing is $\frac{2813}{580} = 4.85$ ins.

From the diagram, Fig. 218, the rivet spacing for any point along the girder is readily scaled.

CHAPTER XIV

REINFORCED CONCRETE

Reinforced concrete is designed by an extension of the principles applied to the discussion of structural steel. The modification is necessary since reinforced concrete is a composite of several materials and therefore, unlike steel, is nonhomogeneous.

In ordinary practice the concrete varies from 1, 2, 4 (1 cement, 2 sand and 4 stone, slag or gravel) to 1, 3, 6. The best mixture depends upon the character of the materials, the purpose being to have the cement fill the voids in the sand, while the mortar thus produced shall fill the voids in the stone, slag or gravel.

Mr. N. C. Johnson, in the *Engineering Record* of Jan. 23, Feb. 6, 13 and 27, March 6 and 13, 1915, discusses the nature of concrete as revealed by a microscopic study. He draws attention to the fact that all concretes are very much weaker than the weakest of their constituents and explains this by showing how far the commercial mixing of the cement, sand and stone varies from that theoretically desirable.

The presence of air and water voids in the concrete prevents its attaining its maximum density and consequent strength. The concrete also contains unhydrated cement in considerable quantities which is revealed by the microscope; this also prevents its attaining its maximum strength.

His researches showed that concrete exposed to water, either fresh or salt, deteriorated due to the capillary action of cracks in the concrete drawing water into it; this water produced internal stresses by mixing with any unhydrated cement and possibly also by depositing salt crystals within the concrete.

In a concluding statement, Mr. Johnson estimates that less than 20 per cent of the cement added to concrete is used effec-

tively. It is to be hoped that the final results of such investigations may lead to a commercial method whereby a concrete of maximum density may be readily obtained.

Cement. — Portland cement only should be used. This is practically the only cement available, as very little natural cement is now manufactured.

The cement should meet the usual standard specifications.

Sand. — The sand should be a clean, coarse, sharp sand free of clay and dirt. Fine, rounded or dirty sand greatly reduces the strength of the concrete.

Stone and Gravel. — Stone is usually broken so that the maximum size does not exceed from $\frac{3}{4}$ in. to $1\frac{1}{4}$ ins. Stone and gravel should be freed from dust, sand and dirt, but uniformity of size is not desirable as it increases the percentage of voids.

Mixing. — Practice formerly demanded a fairly dry mixture, tamped until the water came to the top, but present practice uses a very wet mixture, which greatly reduces the tamping required and makes a more solid concrete. The wet concrete is weaker than the other when new, but its strength rapidly approaches that of the other with age.

Physical Properties. — When one month old and made from ordinary materials the compressive strength of 1, 2, 4 stone concrete can be assumed at from 1800 lbs. to 2200 lbs. per sq. in., while a 1, 3, 6 mixture will have a strength of about 20 per cent under these figures.

The modulus of elasticity increases with age and decreases with the unit stress, so that only approximate values can be assumed. A commonly accepted modulus of elasticity for making reinforced-concrete calculations is 2,000,000 lbs. per sq. in., being one-fifteenth that of steel.

Elastic Limit. — There is no true elastic limit for concrete as it shows permanent deformation under the lightest loads. According to Bach and others it seems fair to assume that what is generally accepted as the elastic limit in concrete occurs at a

unit stress of from one-half to two-thirds of the ultimate compressive strength.

PHYSICAL PROPERTIES OF STONE OR GRAVEL CONCRETE

	Age, days.	1, 2, 4 Mixture.	1, 3, 6 Mixture.
Compressive strength....	30	1,800–2,200	1,450–1,750
Tensile strength.........	30	150–200	100–125
Shearing strength........	30	1,000–1,400	900–1,100
Modulus of elasticity.....	90	2,500,000–3,500,000	2,500,000–3,500,000
Coefficient of expansion, 1° F..................	....	0.0000060	0.0000060
Weight per cubic foot including steel...........	....	150	150

NOTE. — All stresses in pounds per square inch. Slag or cinder concrete will only develop about one-third the strength of a good stone concrete. Its modulus of elasticity will range from 1,000,000 for lean mixtures up to 2,500,000 for 1, 2, 4 and richer mixtures. It weighs about 120 lbs. per cu. ft.

USUAL UNIT WORKING STRESSES. FLEXURE

	Lbs. per sq. in.
Extreme fiber stress on concrete, compression.............	500–650
Extreme fiber stress on concrete, tension.................	0
Steel, soft, tension......................................	12,000
Steel, mild and hard, tension.............................	16,000
Bonding stresses, straight bars...........................	60–80
Bonding stresses, straight bars, over supports.............	90–120
Bonding stresses, bent or twisted bars....................	150
Shearing, plain web, no steel stirrups....................	30
Shearing, when reinforced with stirrups, or bars...........	100
Tension, diagonal, on concrete............................	30
Modulus of elasticity of concrete.........................	2,000,000
Modulus of elasticity of steel............................	30,000,000

USUAL UNIT WORKING STRESSES. COLUMNS

	Lbs. per sq. in.
Columns not reinforced...................................	450
Columns, reinforced longitudinally only....................	450
Columns, reinforced with hoops or bands...................	550
Columns, reinforced with hoops or bands, and with from 1 to 4 per cent of longitudinal steel......................	650
Columns, structural steel thoroughly inclosing concrete.......	650

In general the following nomenclature and formulæ conform to the report of the Joint Committee on Reinforced Concrete of the American Society of Civil Engineers, American Society of

Testing Materials, American Railway Engineering and Maintenance of Way Association and the American Portland Cement Manufacturers.

f_s = tensile unit stress in steel, pounds per square inch.
f_c = compressive unit stress in concrete, pounds per square inch.
E_s = modulus of elasticity of steel, pounds per square inch.
E_c = modulus of elasticity of concrete, pounds per square inch.
$\frac{E_s}{E_c} = n.$
M = moment of resistance or a bending moment, inch pounds.
A = area of steel, square inches.
b = width of beam, inches.
d = depth of beam to center of reinforcement, inches.
k = ratio of depth of neutral axis to effective depth, d.
x = depth of resultant compression from top.
j = ratio of lever arm of resisting couple to depth, d.
$j \cdot d = d - x$ = arm of resisting couple.
p = ratio of steel to concrete. For rectangular beams,

$$p = \frac{A}{b \cdot d}.$$

B = width of flange of T beams, inches. Here $p = \frac{A}{B \cdot d}$.
b^1 = width of stem of T beams, inches.
t = thickness of flange of T beams, inches.
$z = \frac{f_s \times E_c}{f_c \times E_s}.$

Shear and Bond Stresses

V = total shear, pounds.
v = shearing unit stress, pounds per square inch.
u = bond stress, pounds per square inch of bond area.
o = circumference or perimeter of bar, inches.
Σ_0 = sum of the perimeters of all bars, inches.
T = allowable tensile load in bar or stirrup, pounds.

BEAMS WITH DOUBLE REINFORCEMENT

A^1 = area of compression steel.

p^1 = steel ratio of compression steel, not percentage.

$$p^1 = \frac{A^1}{B \times d}.$$

f_s^1 = unit compression in steel.

C = total compression in concrete.

C^1 = total compression in steel.

d^1 = depth to center of compressive steel.

e = depth to resultant of C and C^1.

$$\phi = \frac{e}{d}.$$

COLUMNS

A = total net area.

A_s = area of longitudinal steel.

A_c = area of concrete.

P = total safe load.

THEORETICAL DISCUSSION OF REINFORCED-CONCRETE BEAMS

Rectangular Beams. — The usual formulæ for the design of reinforced-concrete beams are developed upon the assumptions, (*a*) that a cross-section plane before bending continues a plane during bending, and (*b*) that stress and strain are proportional.

From these assumptions the strain in compression a unit distance above the neutral axis must equal that in tension a unit distance below that axis, or, from Fig. 219,

$$\frac{f_c}{kd \times E_c} = \frac{f_s}{(d - kd)\,E_s} = \frac{f_s}{(jd - \frac{2}{3}\,kd)\,E_s} = \frac{f_s}{d\,(j - \frac{2}{3}\,k)\,E_s},$$

$$\frac{f_s \times E_c}{f_c \times E_s} = \frac{jd - \frac{2}{3}\,kd}{kd} = \frac{d - kd}{kd} = z,$$

from which
$$k = \frac{1}{z + 1}, \tag{1}$$

and
$$j = \frac{3\,z + 2}{3\,(z + 1)}. \tag{2}$$

According to the principles of mechanics the sum of the horizontal forces acting on a vertical section must be zero, hence the resultant force in the compression flange must equal the resultant force in the tension flange, or

$$A \times f_s = \frac{f_c}{2} \times k \cdot d \times b,$$

and the moments are also equal, $M_s = M_c$.

$$M_s = A \times f_s \times j \cdot d. \qquad (3)$$

$$M_c = \frac{f_c}{2} \times k \cdot d \times b \times j \cdot d = \frac{f_c}{2} \times k \cdot j \times b \cdot d^2. \qquad (4)$$

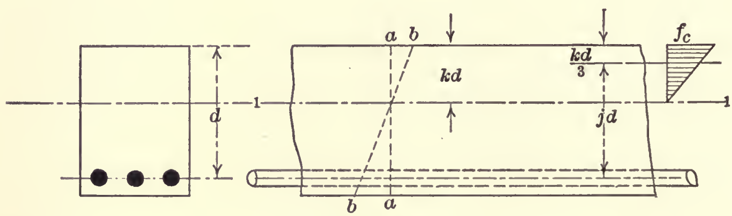


FIG. 219.

These equations apply when the maximum fiber stresses f_c and f_s are known; the reinforcement then is the critical ratio for these fiber stresses.

It is sometimes desirable to discuss the beam when a ratio of reinforcement is assumed which is not necessarily the critical ratio.

From the fundamental equations,

$$\frac{f_s}{f_c} = (1 - k)\frac{E_s}{k \cdot E_c} = \frac{n(1-k)}{k}.$$

From equations (3) and (4),

$$\frac{f_s}{f_c} = \frac{k}{2p}; \qquad (5)$$

hence $\dfrac{n(1-k)}{k} = \dfrac{k}{2p}$ and $p = \dfrac{k^2}{2 \cdot n(1-k)}$.

Solving for k we find

$$k = \sqrt{2 \cdot p \cdot n + (p \cdot n)^2} - p \cdot n. \tag{6}$$

$$j = 1 - \frac{k}{3}.$$

The fiber stresses may be found by substituting these values of k and j in equations (3) and (4).

Approximate Formulæ

The curves on page 202 give the calculated relations between z, k, j, and $n \cdot p$. It will be noted that for the usual values of p, j approximates $\frac{7}{8}$ and k approximates $\frac{3}{8}$. The following approximate formulæ, using these values of k and j, are sometimes used.

$$M_s = \tfrac{7}{8} \times A \times f_s \times d. \tag{7}$$

$$M_c = \tfrac{1}{6} \times f_c \times b \cdot d^2. \tag{8}$$

The curves, Fig. 220, give the relations between the several quantities.

These curves are plotted upon a base line giving the values of z; therefore when $z = \dfrac{f_s \times E_c}{f_c \times E_s}$ is known, the other values $k, j, j \cdot k$, $p \cdot n$ and p are read on the perpendicular to z. Thus for $z = 2.2$ the values are read on the dotted line a–a through 2.2. The values are $j = 0.895$; $k = 0.307$; $k \cdot j = 0.275$; $p \cdot n = 0.072$ and p for $n = 15$ is $p = 0.0048$.

Had the value of p for $n = 15$ been given as 0.0048 instead of the value of $z = 2.2$ the horizontal line b–b would have been drawn through $p = 0.0048$ until it cut the curve of p for $n = 15$, and through this point of intersection the line a–a would be drawn perpendicular to the z axis and upon this line a–a the other values would be read as before.

The following example will illustrate the use of the formulæ and curves.

Example. — A rectangular beam in which $d = 22$ ins. is to resist a bending moment of 384,000 in. lbs. Assume $n = 15$,

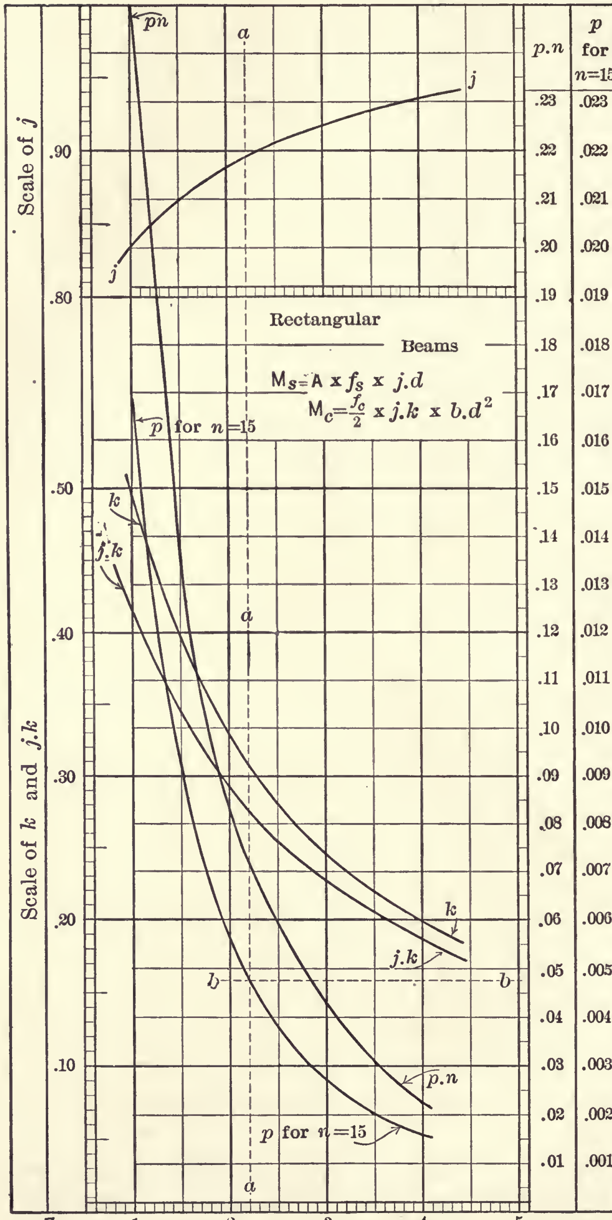

Scale of j
Scale of k and $j.k$
.90
.80
.50
.40
.30
.20
.10
pn
a
j
$p.n$
p for n=15
.23 .023
.22 .022
.21 .021
.20 .020
.19 .019
.18 .018
.17 .017
.16 .016
.15 .015
.14 .014
.13 .013
.12 .012
.11 .011
.10 .010
.09 .009
.08 .008
.07 .007
.06 .006
.05 .005
.04 .004
.03 .003
.02 .002
.01 .001
Rectangular Beams
$M_s = A \times f_s \times j.d$
$M_c = \frac{f_c}{2} \times j.k \times b.d^2$
p for n=15
k
$j.k$
b
Z = 1 2 3 4 5

FIG. 220.

$f_s = 16{,}000$ and $f_c = 650$; find the width of the beam and the area of the steel.

$$z = \frac{f_s \times E_c}{f_c \times E_s} = \frac{16{,}000}{(650 \times 15)} = 1.64.$$

Interpolating from the curves, $k = 0.379$, $j = 0.874$, $k \cdot j = 0.331$ and $p = 0.0077$. Substituting these values in equation (4) gives

$$b = \frac{2\,M_c}{f_c \times k \cdot j \times d^2} = \frac{2 \times 384{,}000}{650 \times 0.331 \times 22^2} = 7.37 \text{ ins.}$$

$$A = b \times d \times p = 7.37 \times 22 \times 0.0077 = 1.25 \text{ sq. ins.}$$

The solution of this problem by the approximate method gives almost identical results.

$$M = \tfrac{7}{8} \times A \cdot f_s \cdot d,$$

$$\text{therefore } A = \frac{8 \cdot M}{7 \cdot f_s \cdot d} = \frac{8 \times 384{,}000}{7 \times 16{,}000 \times 22} = 1.25 \text{ sq. ins.}$$

$$M = \frac{1}{6}(f_c \cdot b \cdot d^2) \quad \text{or} \quad b = \frac{6 \cdot M}{f_c \cdot d^2},$$

$$b = \frac{6 \times 384{,}000}{650 \times 22^2} = 7.32 \text{ ins.}$$

Another example will illustrate the procedure when it is required to check an existing beam. A reinforced-concrete beam has the following dimensions: $b = 9$ ins., $d = 20$ ins., $A = 1.50$ sq. ins., $n = 15$, and it is desired that f_s should not exceed 16,000 lbs. per sq. in. nor f_c exceed 650 lbs. per sq. in.

$$p = \frac{A}{b \cdot d} = \frac{1.50}{(9 \times 20)} = 0.00833.$$

From equation (5),

$$\frac{f_s}{f_c} = \frac{k}{2\,p} = \frac{0.390}{(2 \times 0.00833)} = 23.4$$

and $\qquad f_s = 23.4 \times 650 = 15{,}200$ lbs. per sq. in.

This fiber stress being under the 16,000 lbs. per sq. in. is satis-

factory. The bending moment corresponding to these fiber stresses can be found from either equation (3) or (4).

$M_s = A \times f_s \times j \cdot d = 1.5 \times 15{,}200 \times 0.870 \times 20 = 397{,}000$ in. lbs.

Checking this by the other formula gives

$$M_c = \tfrac{1}{2}\,(f_c \times k \cdot j \times b \cdot d^2) = 396{,}000 \text{ in. lbs.}$$

This is about as close agreement as can be expected considering the curve readings, the difference being under 1 per cent.

T Beams

In reinforced-concrete design the section most frequently met is the T beam, shown in Fig. 221. The floor is usually a slab of

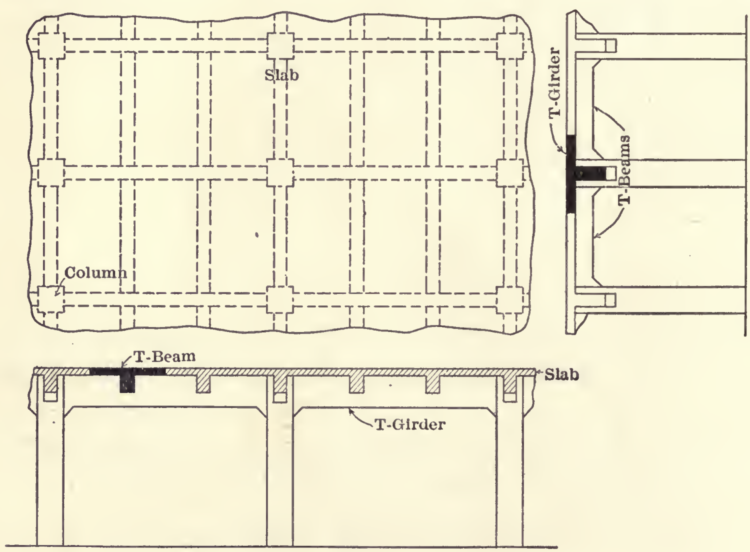


FIG. 221.

reinforced concrete laid on floor beams, corresponding in the ordinary timber construction to joists. In the case of concrete construction the floor beam and the slab over it become one piece, the web stresses at the junction of the two being resisted by the concrete and usually by reinforcing stirrups or rods.

When the neutral axis 1–1, in Fig. 222, falls in the flange or slab the formulæ and table previously derived for rectangular

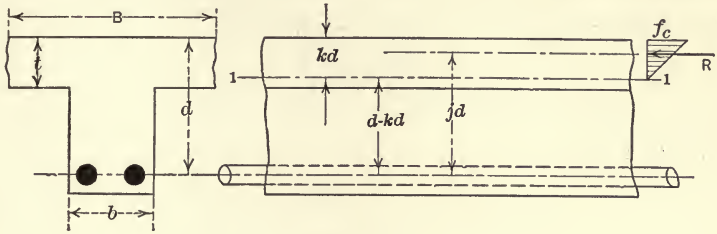


FIG. 222.

beams are sufficient. When, however, the neutral axis falls in the stem, as is usually the case, another set of formulæ is required.

General Formulæ for T Beams Reinforced for Tension Only

Neglecting the compression in the stem, and on the same conditions *a* and *b* that applied to rectangular beams we have as before:

$$\frac{f_s \times E_c}{f_c \times E_s} = z; \quad \frac{f_s}{f_c} = \frac{n\,(1 - k)}{k}; \quad \text{and} \quad k = \frac{1}{z + 1}.$$

Also from Fig. 223 $\quad \dfrac{f_c}{f_1} = \dfrac{k \cdot d}{k \cdot d - t}.$

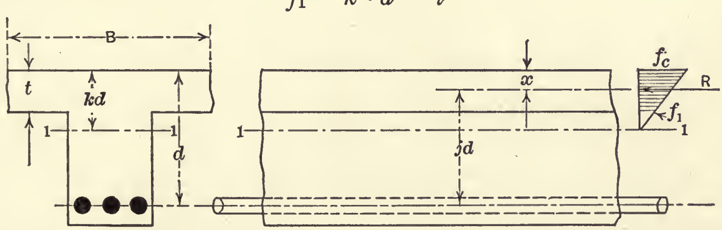


FIG. 223.

Assuming that the fiber stress in the concrete varies as its distance from the neutral axis, the distance x from the top of the beam to the resultant compression R is

$$x = \frac{f_c + 2f_1}{f_c + f_1} \times \frac{t}{3} = \frac{3 \cdot k \cdot d - 2 \cdot t}{2 \cdot k \cdot d - t} \times \frac{t}{3};$$

this gives $$j \cdot d = d - x = d - \left(\frac{3 \cdot k \cdot d - 2\,t}{2 \cdot k \cdot d - t} \times \frac{t}{3}\right). \tag{9}$$

The average fiber stress in the concrete is

$$f_a = \frac{f_c + f_1}{2} = f_c\left(1 - \frac{t}{2\,kd}\right).$$

It follows that

$$M_c = B \times t \times f_c\left(1 - \frac{t}{2\,kd}\right) j \cdot d, \tag{10}$$

and $$M_s = A \times f_s \times j \cdot d. \tag{11}$$

Equation (11) can be approximated as

$$M_s = A \times f_s \left(d - \frac{t}{2}\right).$$

When the ratio of the reinforcement is known, not the fiber stresses, equation (1) cannot be used to find k as z is not known. Taking equations (10) and (11), we find

$$\frac{f_c}{f_s} = \frac{A}{B \cdot t} \times \frac{1}{1 - \frac{t}{2\,kd}} = \frac{pd}{\left(1 - \frac{t}{2\,kd}\right)t} = \frac{k}{n\,(1 - k)}.$$

It follows that $$k = \frac{np\,d + \frac{t^2}{2\,d}}{t + np\,d}.$$

This may also be written

$$k \cdot d = \frac{2\,n\,dA + Bt^2}{2\,nA + 2\,Bt}.$$

To facilitate the solution of problems the following curves, Fig. 224, give the values of j and k for varying values of p and $\frac{t}{d}$ and also the ratios of $\frac{f_s}{f_c}$ for various values of k. These values have been calculated with $n = 15$, this being a very commonly assumed value for rock concrete. For other values of n similar curves may be drawn or the formulæ used.

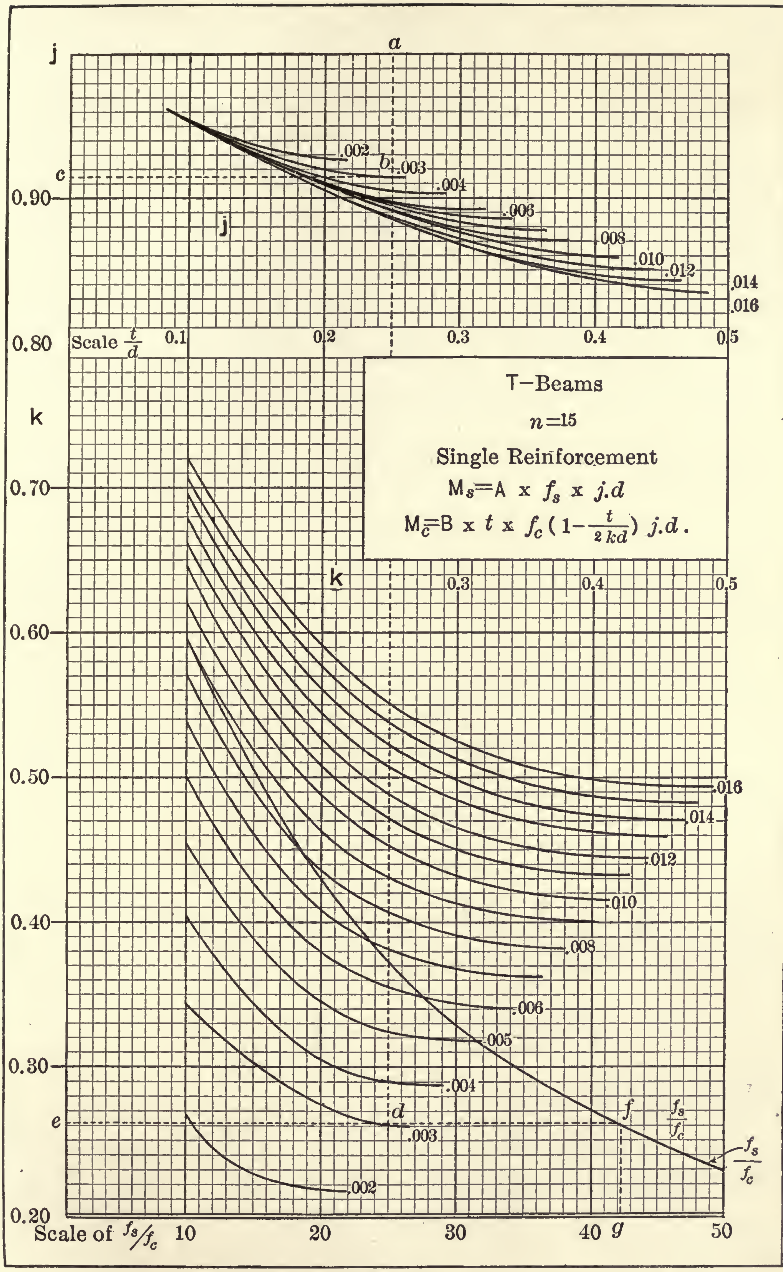


FIG. 224.

NOTE. — The values 0.002 to 0.016 placed at the curves are the values of $p = \frac{A}{B \cdot d}$. Moving the decimal point two places to the right expresses the reinforcement as a percentage.

To find j if $\frac{t}{d} = 0.25$; from 0.25, at a on the upper scale, drop a perpendicular until it cuts the curve corresponding to the steel ratio of, say, 0.003 in b; project the point b horizontally until it cuts the vertical scale in c. Here the value of j is given as 0.914. To find k continue the vertical a–b until it cuts the curve of k corresponding to a steel ratio of 0.003 in d. This point d projected horizontally cuts the vertical scale in e or $k = 0.262$.

To find the ratio of $\frac{f_s}{f_c}$ produce the value of k horizontally until it cuts the curve of $\frac{f_s}{f_c}$ in f. The vertical projection of f cuts the scale of ratios of $\frac{f_s}{f_c}$ in g or $\frac{f_s}{f_c} = 42.3$.

An example will further illustrate the uses of the formulæ and curves.

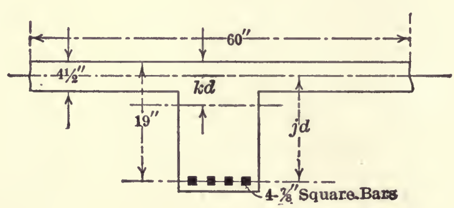


FIG. 225.

Given $n = 15, f_c \equiv 600, f_s = 15{,}000$, and the following section, Fig. 225. Area of bars $= 4 \times 0.766 = 3.06$ sq. ins.

$$p = \frac{3.06}{60 \times 19} = 0.0027. \qquad \frac{t}{d} = \frac{4.5}{19} = 0.237.$$

Consulting the curves,

$$j = 0.918, \quad k = 0.252, \quad \frac{f_s}{f_c} = 44.5.$$

The resisting moment then is

$$M_s = A \times f_s \times j \cdot d$$

or

$$M_s = 3.06 \times 15{,}000 \times 0.918 \times 19 = 801{,}000 \text{ in. lbs.}$$

The fiber stress in the concrete is

$$\frac{f_s}{f_c} = 44.5 = \frac{15{,}000}{f_c} \quad \text{or} \quad f_c = 330 \text{ lbs. per sq. in.}$$

Slabs

In using the formulæ and table for slabs it is customary to assume a width of slab of 12 ins.; thus assuming a slab whose span is 6 ft. and whose total dead and live load is 200 lbs. per sq. ft., taking the bending moment at $M = \frac{Wl}{10}$, $n = 15$, $f_c = 600$ lbs. per sq. in., $f_s = 12{,}000$ lbs. per sq. in., the thickness of the slab and area of reinforcing steel can be found by the following calculations:

$$M = \frac{Wl}{10} = \frac{(200 \times 6) \times 6 \times 12}{10} = 8640 \text{ in. lbs.}$$

$$z = \frac{E_c \times f_s}{E_s \times f_c} = \frac{12{,}000}{15 \times 600} = 1.33.$$

From the curves, page 202, the values corresponding to $z = 1.33$ are $k = 0.433$, $j = 0.855$, $k \cdot j = 0.370$ and $p = 0.011$; substituting these values in equation (4) gives

$$d = \sqrt{\frac{2\,M_c}{b \times f_c \times kj}} = \sqrt{\frac{2 \times 8640}{12 \times 600 \times 0.37}} = 2.55 \text{ ins.}$$

The area of the reinforcement then is given by equation (3) or by taking $b \cdot d \cdot p = 12 \times 2.55 \times 0.011 = 0.337$ sq. in.

If $\frac{3}{4}$ in. is allowed below the center of the reinforcing steel the slab thickness will be $2.55 + 0.75 \sim 3.25$ ins.

Bending Moments

Owing to the monolithic construction of reinforced concrete the bending moments on beams are frequently estimated as lower than those occurring on similarly loaded but merely supported beams. Thus for continuous slabs the moments are commonly taken as $M = \frac{Wl}{10}$, at the middle of the span and at supports.

The rods inserted for slab reinforcement should be bent up near the $\frac{1}{4}$ points of the span and carried over the supports in the upper portion of the slab. Although some designers bend all the rods, it would seem sufficient and better practice to run from one-quarter to one-half of the rods through straight, only bending the remaining bars. Although the angle of inclination of the bars should not be too slight it should preferably not exceed 30 degrees with the horizontal. See Specifications, § 212.

Square slabs reinforced in both directions and supported on four sides have the moments at the center of the span in one direction taken as $M = \frac{Wl}{20}$. Beams and girders although sometimes figured as fixed or continuous beams with $M = \frac{Wl}{10}$ are more commonly taken as supported beams with $M = \frac{Wl}{8}$. In the above, W = the total load on the span in pounds and l = the span in inches.

Beams with Double Reinforcement

In the preceding discussions reinforcing steel has been assumed in the tension side of the beam. The case will now be considered

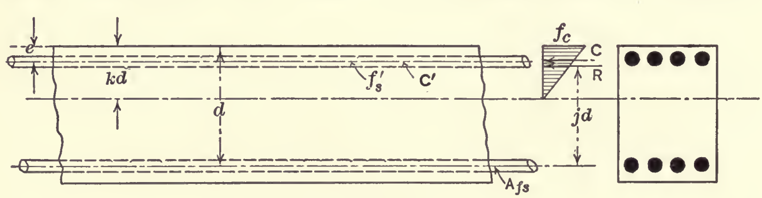


Fig. 226.

where the reinforcement is in both flanges. This condition occurs regularly in ordinary construction where some of the reinforcing bars in the tension flange are bent up and carried over the support in the upper flange. In this case the beam resists a negative moment over the supports which creates tension in the upper flange and compression in the lower flange. In the case

of T beams the lower flange width being b the width of the stem is much less than the width B of the upper flange, and unless this narrow flange is reinforced for compression the fiber stress f_c might easily be excessive.

To meet the usual specifications (see Specifications, paragraph 212), which demand that continuous beams be able to develop the same resisting moment at the supports and at the middle of the span, requires a tensile reinforcement at the supports equal to that at the center of the span if the beam is of uniform depth.

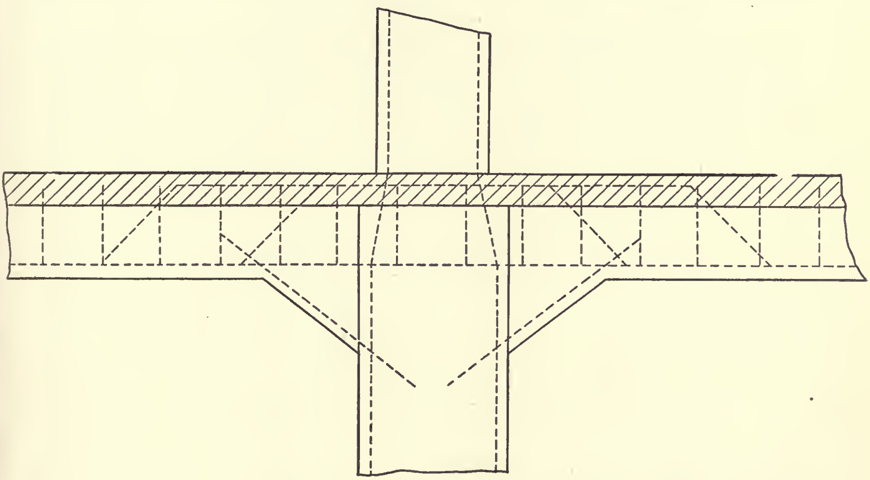


FIG. 226a.

As the bending moments reduce very rapidly near the supports towards the center of the span, the forces acting in both tension and compression flanges are readily reduced by haunching the column where the beam or girder runs into it. This amounts to increasing the depth of the beam adjacent to the column, as shown in Fig. 226a.

The following theory applies to beams with double reinforcement.

The nomenclature is that previously given, page 199.

Assuming the compression in the concrete as varying propor-

tionally to the distance from the neutral axis and that the concrete carries no tension, we have, in Fig. 226,

$$\frac{f_s}{nf_c} = \frac{d - kd}{kd} \quad \text{and} \quad \frac{f_s'}{nf_c} = \frac{kd - e}{kd}.$$

Also $\qquad A \times f_s = C' + C = A'f_s' + \frac{1}{2} f_c \cdot kd \cdot b,$

from which it follows that

$$kd = \sqrt{\frac{2\,n\,(A + \phi A')\,d}{b} + \left[\frac{n\,(A' + A)}{b}\right]^2} - \frac{n\,(A' + A)}{b}$$

and

$$k = \sqrt{2\,n\,(p + \phi p') + n^2\,(p + p')^2} - n\,(p + p').$$

To find the lever arm of the couple acting in the beam

$$A \times f_s \times jd = \frac{1}{2} f_c \cdot kd \cdot b \left(d - \frac{kd}{3}\right) + A'f_s'\,(d - e),$$

$$jd = \frac{f_c \cdot kd \cdot b \left(d - \frac{kd}{3}\right)}{2\,Af_s} + \frac{A'f_s'\,(d - e)}{A \cdot f_s}.$$

Expressing f_c, f_s, f_s' and d in terms of k, ϕ, p and p' gives

$$j = \frac{k^2\left(\frac{1}{2} - \frac{k}{6}\right) + np'\,(1 - \phi)\,(k - \phi)}{np\,(1 - k)}.$$

The relations between the fiber stresses are

$$f_s = \frac{M}{A \times j \times d}; \qquad f_c = \frac{k}{n\,(1 - k)} f_s,$$

and $\qquad f_s' = \dfrac{n\,(k - \phi)}{k} \times f_c = \dfrac{(k - \phi)}{(1 - k)}\, f_s.$

In the *Engineering News* of August 22, 1912, Mr. Arthur G. Hayden gives the following approximate formulæ for j of reinforced-concrete beams with double reinforcements.

$$j = 0.93 + 2\,p' + 100\,pp' - 4\,p - 0.1\,\phi - 20\,p'\phi.$$

Mr. Hayden states that the maximum error does not exceed $1\frac{1}{2}$ per cent for all proportions of reinforcement between $\frac{1}{2}$ per

cent and 2 per cent, and that for ordinary cases the error is inappreciable. This formula assumes that $n = 15$.

The following example will illustrate the use of the formulæ.

A T beam in which $d = 21$ ins. and $b = 12$ ins. is reinforced over the supports by two $\frac{7}{8}$-in. round bars in each flange. If $f_s = 16{,}000$ lbs. per sq. in., $\phi = \frac{1}{10}$ and $n = 15$, what moment can the beam resist at the supports and what are the fiber stresses f_c and f_s'?

$$kd = \sqrt{\frac{2n(A + \phi A')d}{b} + \left[\frac{n(A + A')}{b}\right]^2} - \frac{n(A + A')}{b},$$

$$kd = \sqrt{\frac{30\left(1.2 + \frac{1.2}{10}\right) \times 21}{12} + \left[\frac{15 \times 2.4}{12}\right]^2} - \frac{15 \times 2.4}{12},$$

$$kd = 5.85 \text{ ins.}$$

$$j = \frac{k^2\left(\frac{1}{2} - \frac{k}{6}\right) + np'(1 - \phi)(k - \phi)}{np(1 - k)},$$

$$j = \frac{0.28^2\left(0.50 - \frac{0.28}{6}\right) + 15 \times 0.0048(1.0 - 0.1)(0.28 - 0.1)}{15 \times 0.0048(1 - 0.28)},$$

$$j = 0.91 \quad \text{and} \quad jd = 21 \times 0.91 = 19.11 \text{ ins.}$$

By the approximate method $jd = 18.97$ ins.

The bending moment, allowing $f_s = 16{,}000$ lbs., is

$$M = A \times f_s \times jd = 1.2 \times 16{,}000 \times 19.11 = 366{,}912 \text{ in. lbs.}$$

$$f_c = \frac{kd}{n(d - kd)} f_s = \frac{5.85}{15(21 - 5.85)} \times 16{,}000 = 410 \text{ lbs. per sq. in.}$$

$$f_s' = \frac{k - \phi}{1 - k} f_s = \frac{0.28 - 0.10}{1 - 0.28} \times 16{,}000 = 4000 \text{ lbs. per sq. in.}$$

The following problems are intended to be solved by means of the theory just given and it is recommended that they be first

done by using the formulæ and then checked by the assistance of the curves.

Problem. — The combined live and dead loads upon a floor slab are 140 lbs. per sq. ft.; assuming $f_c = 650$ lbs., $f_s = 16{,}000$ lbs. per sq. in., $\frac{E_s}{E_c} = 15$, and that $M = \frac{Wl}{10}$, determine the thickness of the slab from the top to the center of the reinforcement and the spacing of $\frac{3}{8}$-in. round bars if the span of the slab is 10 ft. 8 ins.

Problem. — Assume a continuous girder whose span is 25 ft. 0 ins., carries 50,000 lbs., $d = 23$ ins., $B \sim 100$ ins., slab thickness $4\frac{1}{2}$ ins., $\frac{E_s}{E_c} = 15$, and $M = \frac{Wl}{10}$. Find the fiber stress in the concrete and steel if the steel reinforcement consists of two $1\frac{1}{4}$-in. and two $1\frac{1}{8}$-in. round bars.

Parabolic Variation of Stress in Concrete

In the preceding discussion of reinforced-concrete beams it has been assumed that the fiber stress in the concrete above the

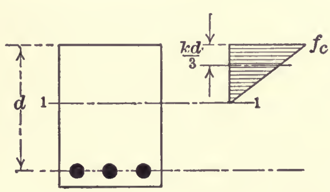


Fig. 227.

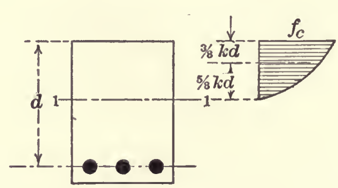


Fig. 228.

neutral axis varies as the intercepts in the triangle; this is represented by Fig. 227.

Some designers prefer to assume that this stress varies as the intercepts in the parabola as shown in Fig. 228. The resultant flange force acts at a distance $\frac{3}{8}\, k \cdot d$ from the top of the beam and the average fiber stress is $\frac{2}{3} f_c$. This treatment is commonly limited to rectangular beams as it becomes too complicated when extended to T beams. The following formulæ are derived

similarly to those previously determined and apply to rectangular beams only. The symbols are those previously used.

$$j = \frac{8z + 5}{8(z + 1)} \quad \text{and} \quad k = \frac{8}{3}(1 - j).$$

$$M_s = A \times f_s \times jd,$$

$$M_c = \tfrac{2}{3} f_c \times b \times kj \times d^2.$$

Where the ratio of reinforcement is known the formulæ are:

$$k = \sqrt{\tfrac{3}{2}\, p \cdot n + (\tfrac{3}{4}\, p \cdot n)^2} - \tfrac{3}{4}\, p \cdot n \text{ and } j = 1 - \tfrac{3}{8}\, k,$$

$$f_c = \frac{3\, M_c}{2\, b \cdot kj\, d^2},$$

$$f_s = \frac{M_s}{p \cdot b \cdot jd^2}.$$

The difference between these formulæ and those previously given will be evident upon working several of the problems by both methods. Considering the character of the materials used in reinforced concrete it is a question whether methods introducing any greater refinements than those used in the derivation of the first formulæ are warranted.

Most elaborate tests of reinforced-concrete beams have been made at the Experiment Station of the University of Illinois under the direction of Prof. Arthur N. Talbot, beginning in 1904 and reported in the Bulletins of that station since then. Professor Talbot has deduced formulæ and offers suggestions for reinforced-concrete design as the results of the tests; this data can be readily obtained by reference to the above bulletins.

Web Stresses

It is shown in the section of problems (see problems 65 to 68) that ordinarily in steel beams of usual sections the web stresses are of minor importance, while more careful consideration must be taken of them in timber beams, particularly of horizontal shear in timber beams of short spans. In reinforced concrete beams the consideration of the web stresses is of still greater

importance, and generally the webs of beams and girders must be carefully reinforced with steel to resist the web stresses.

Taking section 2–2 in Fig. 229 the horizontal shearing force will increase from zero at the upper fibers to a maximum at the neutral axis 1–1; this force F_2 is then transferred to the horizontal reinforcing steel. Assuming that the bending at 3–3 exceeds that at 2–2 the horizontal shearing force F_3 at 3–3 will

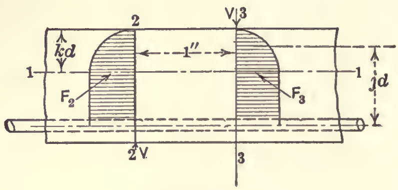


FIG. 229.

exceed F_2, and if b is the width of the beam at the neutral axis the unit shearing force on the section $b \times 1$ is $v = \frac{F_3 - F_2}{b \times 1}$. Equating the moments $V \times 1 = (F_3 - F_2)\, j \cdot d$ or $F_3 - F_2 = \frac{V}{j \cdot d}$ and $v = \frac{V}{b \cdot j \cdot d}$.

It has been previously shown that $j \cdot d \sim \frac{7}{8}\, d$, hence

$$v = \frac{V}{0.875\, b \cdot d}.$$

It should be noted that the reasoning is identical with that used in determining the flange riveting in plate girders.

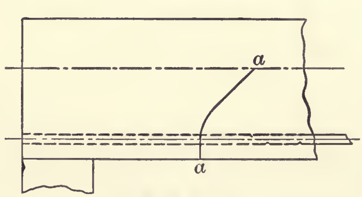


FIG. 230.

Tests of reinforced-concrete beams exhibit failures along lines a–a as shown in Fig. 230. This illustrates failure by diagonal tension. According to Prof. Arthur N. Talbot web failures of reinforced concrete beams occur at calculated shearing values much below the true shearing strength of concrete. Tests indicate these shearing strengths as ranging from 50 to 75 per cent of the compressive

strength. The usual method of web failure is by the tension resulting from the combination of the shearing and tensile web stresses. See Specifications, paragraph 218d.

The beam is strengthened to resist this tension, first, by bending the reinforcing bars when no longer required in the flanges (see Fig. 239); secondly, by vertical stirrups (Figs. 239 and 240) which carry the vertical component of the diagonal stresses, or thirdly, by a combination of vertical stirrups and bent rods, as shown in Fig. 239.

If we consider any point in the body of the beam, it will be acted on by a unit horizontal tension or compression, a unit horizontal shear and a unit vertical shear. The resulting maximum tensile or compressive stress is

$$f_{tm} = \tfrac{1}{2} f_t + \sqrt{\tfrac{1}{4} f_t^2 + v^2},$$

where f_{tm} = unit maximum diagonal tension,
f_t = unit horizontal tension,
v = unit shear, vertical or horizontal.

The direction of the maximum tension is given by

$$\text{tang } 2\,\theta = -\frac{2\,v}{f_t},$$

where $f_t = 0$, $\theta = 45$ degrees. It therefore follows that along the neutral axis and on the tension side of this axis $\theta = 45$ degrees.

Where $\theta = 45$ degrees, $f_t = 0$ and $f_{tm} = v$.

It is seen then that the maximum diagonal tension acts at an angle of 45 degrees with the neutral axis and that the maximum unit diagonal tension equals the unit shear at that point.

$$f_{tm} = v \sim \frac{V}{b \cdot jd}.$$

v = unit vertical shear.
b = width of beam if rectangular. Use b', the width of the stem if a T beam.
V = total vertical shear on section.

This formula applies for the worst condition, namely, that no tension is carried by the concrete. When it is desired to allow for the concrete the numerator of the fraction becomes $V - v_c \cdot b \cdot jd$ for rectangular beams and $V - v_c \cdot b' \cdot jd$ for T beams. Here $v_c =$ the allowable unit shear on the concrete.

Let $T =$ the total allowable tension in a diagonal bar, in pounds. If the bar makes an angle of 45 degrees with the horizontal, the spacing s along the horizontal in the first case is $s = \frac{1.41\ T \cdot jd}{V}$, while, when the allowance is made for the portion of the tension carried by the concrete,

$$s = \frac{1.41 \times T \times jd}{V - v_c \cdot b \cdot jd}.$$

The factor 1.41 is introduced since the diagonal tension is measured along a 45-degree line, while the spacing is laid off on a horizontal line. Where the angle of inclination of the bent rod with the horizontal is α degrees instead of 45 degrees the factor 1.41 should be replaced by $\frac{1}{\sin \alpha}$.

To be effective for web reinforcement the spacing of diagonal bars should not exceed $s = d$.

Vertical Stirrups

When vertical stirrups are used they carry the vertical component of the diagonal tension. This is assumed as measured by the vertical shear. The spacing for vertical stirrups when no allowance is made for the shearing strength of the concrete is $s = \frac{T \cdot jd}{V}$. Where a unit shearing stress v_c is allowed in the concrete the spacing is

$$s = \frac{T \cdot jd}{V - v_c \cdot b \cdot jd}.$$

In these formulæ j may be ordinarily assumed at $\frac{7}{8}$ without serious error. In the case of T beams b is the width of the stem. T is the total allowable tension in the stirrups, in pounds, s is

the stirrup spacing in inches, measured horizontally along the beam. This spacing should not exceed the effective depth of the beam, jd, and is commonly limited to from $\frac{1}{2}$ to $\frac{3}{4}$ the depth of the beam.

In important beams and girders where the web tension is carried by vertical stirrups the necessary stirrup spacing may

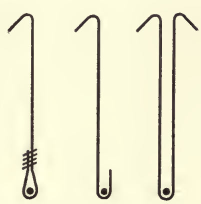


FIG. 231.

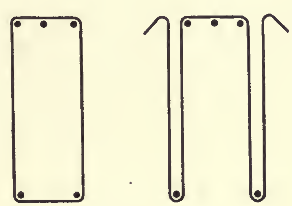


FIG. 232.

be determined similarly to the method employed for flange rivets in girders, on page 193, and as given by Taylor and Thompson in "Concrete Plain and Reinforced." Having determined by the formula the stirrup spacing at the end, the middle and several intermediate points on the span, lay off these spacings

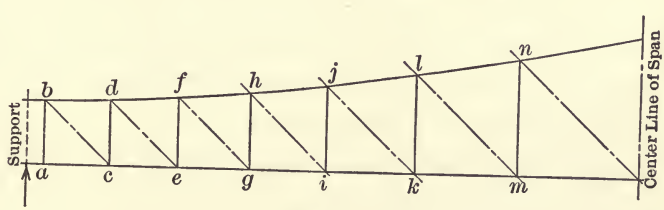


FIG. 233.

vertically upon a horizontal base line representing one-half the span, and at their proper positions, then draw a curve through these points. The required spacing can be scaled off for any point.

If the same scale is used for laying off the half span and the vertical ordinates representing the spacing, then the succeeding

positions for stirrups may be found by laying off $ac = ab$, $ce = cd$, $eg = ef$, etc. This is quite readily done by drawing bc, de, fg, hi, etc., making angles of 45 degrees with the horizontal and alternately drawing the vertical and inclined lines.

Bond Stress

The diagram, Fig. 234, shows how the flange force varies from the supports to the middle of the span for a uniformly distributed load upon a beam of constant depth. Evidently this change of force can only occur by the flange transferring force to the web, as was done in the case of the plate girder through the flange riveting. AB represents the maximum flange force. Passing from AB to EF the flange force has been reduced by the amount GF which has been transferred to the web. In the case of reinforcing steel the connection between the steel and the concrete must be strong enough to carry the force GF. This may be accomplished either by the adhesion between the concrete and smooth or plain bars, or corrugated or twisted bars may be used to increase the strength of the bonding.

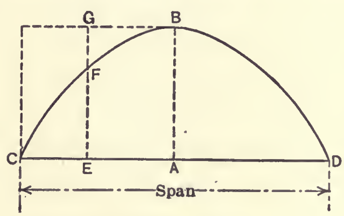


Fig. 234.

Considering plain round bars and letting

u = unit bonding stress, pounds per square inch,
δ = diameter of the bar, inches,
l = length of bar, inches,

then $$\pi \cdot \delta \cdot u \times \frac{l}{2} = \frac{\pi\delta^2}{4} \times f_s, \text{ from which } \frac{l}{\delta} = \frac{f_s}{2u}.$$

This means that to develop the full tensile strength of a round bar its total length must be $\frac{f_s}{2 \cdot u}$ times the diameter of the bar. This relation applies also to square bars. Illustrating this with

the $\frac{7}{8}$-in. square bars used in a former example, and allowing 60 lbs. per sq. in. for the working bond stress, we have

$$\frac{l}{\delta} = \frac{f_s}{2 \cdot u} \text{ or } l = \frac{0.875 \times 16{,}000}{2 \times 60} = 116.5 \text{ ins.}$$

This means that under the given conditions to develop the full strength of the bar, it must have a total length of at least 116.5 ins. and must be placed to have a length of not less than $58\frac{1}{4}$ ins. on each side of the center of the span. A bar's length beyond the point at which its allowed tensile stress is developed must be

$$l = \frac{f_s \times \delta}{4u}.$$

The bond stress varies along the bars; in fact, the increment of change is that of the flange force, and, similarly to the change in horizontal shear, we have

$$V \times 1 = (F_3 - F_2) j \cdot d, \quad \therefore \quad F_3 - F_2 = \frac{V}{j \cdot d}.$$

$$\text{The unit bonding stress} = \frac{F_3 - F_2}{\text{surface of bars for unit length}}.$$

$$u = \frac{V}{j \cdot d \, (\Sigma \text{ circumference of bars})}.$$

The bonding strength of bars may be reinforced by the use of corrugated or twisted bars and by hooks made at the end of the bars. These hooks when semi-circular and bent around a diameter of from 3 to 5 times the diameter of the bar, if properly imbedded in the concrete, will resist a pull beyond the elastic limit of the bar before the hook loses its grip. Right angled bends, when imbedded with sufficient concrete behind them to prevent kicking back, are also very effective.

Lengths of Reinforcing Rods

The lengths of the reinforcing bars besides being limited by the bond stress, as just explained, will also vary in length for the same reason the flange plates of girders do. That is, as the flange force decreases from the center towards the supports the

reinforcing area can be reduced. In Fig. 235, *CD* is the span and *AB* represents the total reinforcing area, *A*, at the center of the span. If 4 bars are used, the line *AB* is divided into 4 equal parts, each division representing the area of one bar; the lengths of the several rectangles, determined by the intersections of the horizontal lines with the parabola *CBD*,* give the respective lengths of the bars. In practice the bars instead of being cut off are frequently bent up, run to the top and then to the end of the beam, as in Fig. 236, thus reinforcing the upper flange of the beam over the support, where, owing to the monolithic character of a concrete beam, there is restraint and consequent

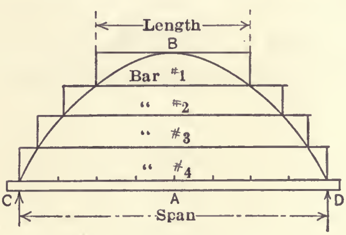


FIG. 235.

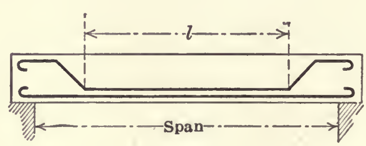


FIG. 236.

tension. The bars are usually bent up in pairs. The area run over the supports generally exceeds 25 per cent of the gross area *A*; the remainder of the bars are continued through the lower flange and over the supports. If it is preferred to calculate the length of the bars the following formula may be used,

$$l = L\sqrt{\frac{a}{A}},$$

where l = length of bar in lower flange between bends, inches.
a = area in square inches of bars, including all shorter bars and the bar whose length is desired. In Fig. 235, if the length of bar No. 3 is desired, a = area of bars Nos. 1, 2 and 3.

* NOTE. — Using a parabola assumes a uniform load on a supported beam. It may be replaced by the curve of maximum moments for other than uniform loads.

A = area in square inches of the total reinforcing at the center of the span.

L = span of beam, inches.

When the moment is g per cent of $\frac{WL}{8}$ the correction may be made by multiplying the value of l found above by $\sqrt{g}$.

The reinforcing bars must be spaced to have sufficient concrete around them to carry the web forces. This spacing must also be ample to permit of the thorough compacting of the concrete between and around the surfaces of the bars. The clearance between the surfaces of adjacent bars should generally be not under 1 inch nor $1\frac{1}{2}$ diameters of the bar.

Where the fireproofing required by the Specifications, paragraph 207, does not have to be enforced, the following clearances between the nearest surface of the rods and the surface of the beam may be followed.

Depth of slab or beam, inches.	Clearance between rods and nearest beam surface, inches.
Under $4\frac{1}{2}$	$\frac{3}{4}$
5 to $8\frac{1}{2}$	1
9 to 12	$1\frac{1}{4}$
13 to 18	$1\frac{1}{2}$
18 to 20	$1\frac{3}{4}$
Above 20	2

Design of a T Beam

A T beam spans 20 ft. 0 ins.; the beams are spaced 10 ft. 6 ins. and carry a uniform dead load of 30,000 lbs., while the uniform live load is 75,000 lbs.

The slab thickness is 7.5 ins., d = 29 ins., and the width of the stem b' is 20 ins. Assume the width of slab acting as flange at 80 ins.* The reinforcing bars are four $1\frac{1}{8}$-in. round bars and five $1\frac{1}{4}$-in. round bars.

The total area of the steel is

Four $1\frac{1}{8}$-in. round bars = 4×0.99 =	3.96	sq. ins.
Five $1\frac{1}{4}$-in. round bars = 5×1.23 =	6.15	sq. ins.
Total	10.11	sq. ins.

* Specification § 213 would have limited this to 60 ins. if enforced.

These bars to be spaced not less than $2\frac{1}{2}$ times their diameters.

From the dimensions given we have

$$\frac{t}{d} = \frac{7.5}{29} = 0.258,$$

$$p = \frac{10.11}{80 \times 29} = 0.0044.$$

From the curves we find that $k = 0.305$ and that $j = 0.90$, also $\frac{f_s}{f_c} = 34$.

The bending moment on the beam being assumed as $M = \frac{Wl}{8}$,

$$M = \frac{105{,}000 \times 20 \times 12}{8} = 3{,}150{,}000 \text{ in. lbs.}$$

$$M = A \cdot f_s \cdot j \cdot d;\ f_s = \frac{3{,}150{,}000}{10.11 \times 0.90 \times 29} = 12{,}000 \text{ lbs. per sq. in.}$$

Bent Bars. — Assuming that four $1\frac{1}{8}$-in. round bars are placed above the five $1\frac{1}{4}$-in. round bars and that these four bars are bent alike, the length of the four bars between the bends will be

$$l = L\sqrt{\frac{a}{A}} = 240\sqrt{\frac{3.96}{10.11}} = 150 \text{ ins.}$$

Had it been desired to bend up bars 1 and 2 before bars 3 and 4 were bent their lengths l would have been

$$l_1 = L\sqrt{\frac{a}{A}} = 240\sqrt{\frac{1.98}{10.11}} = 106 \text{ ins.}$$

The minimum length of the bars to develop the proper bonding strength is given by the formula

$$l = \frac{\delta \times f_s}{2 \times u} = \frac{1.125 \times 12{,}000}{2 \times 75} = 90 \text{ ins.}$$

It is seen that the 106 ins. required for flange strength provides ample bonding strength.

Web Reinforcement

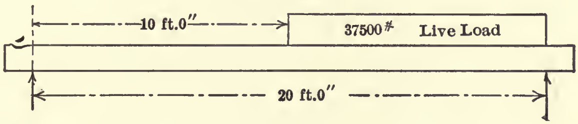
The maximum shear at the center of the span will occur when the beam is covered with the live load from its center to one of the supports.

The load on one-half of the span then is 37,500 lbs. The live-load reaction (Fig. 237) is

$$R_1 = \frac{37{,}500 \times 5}{20} = 9375 \text{ lbs.}$$

The maximum shear diagram is then given by Fig. 238.

Fig. 237.



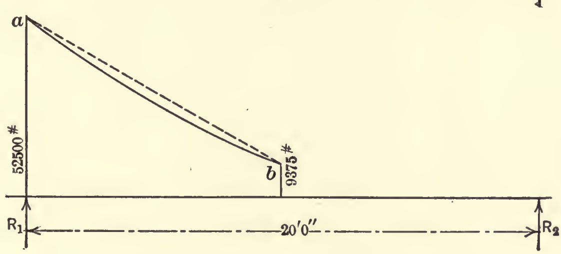


Fig. 238.

The straight-line approximation, *ab*, is commonly used.

Figure 239 represents a reinforced-concrete T beam from the center of the span to the right support.

From the point *i*, the intersection of the center line of the span and the neutral axis of the beam section, draw *ij*, making an angle of 45 degrees with the horizontal axis of the beam. The unit shear at any point below the axis equals the unit diagonal tension; at the center of the span this is

$$\frac{9375}{b' \times jd} = \frac{9375}{20 \times 29 \times 0.9} = 18 \text{ lbs. per sq. in.}$$

A line ih is laid off from i at right angles to ij and to a length representing 18 lbs. Through the point k, at the junction of the neutral axis and the line of reaction at the right support, draw jk at right angles to ij. The unit end shear is

$$\frac{52{,}500}{b' \times jd} = \frac{52{,}500}{20 \times 29 \times 0.9} = 100 \text{ lbs.}$$

This value is laid off to scale in jb. hb may be joined by a straight line or the true curve may be used as explained in Fig. 238. The former is done here, it being as accurate as the problem warrants. If it is assumed that the concrete can carry

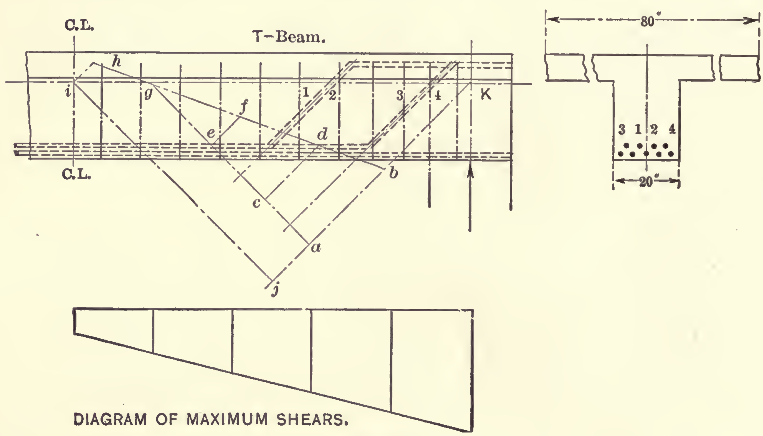


FIG. 239.

30 lbs. per sq. in. lay off ja to scale representing 30 lbs. and draw the line ag parallel to ij. The bars are bent after their lengths in the lower flange exceed the distances previously calculated as necessary, one-half these lengths being used as the lengths are measured from the center line *C.L.* The area of the trapezoid *abdc* multiplied by b' gives the load carried by the rods 3 and 4. The mean ordinate of the trapezoid is

$$\frac{cd + ab}{2} = \frac{52 + 70}{2} = 61.$$

ac = 19 ins. The area *abdc* is $61 \times 19 = 1160$ lbs.

The total load on the bars 3 and 4 = $1160 \times b' = 1160 \times 20 = 23,200$ lbs. Hence the unit tension in the bars is

$$\frac{23,200}{2 \times 0.99} = 11,700 \text{ lbs. per sq. in.}$$

The tension in the bars 1 and 2 will be considerably less, but this is made necessary, as the spacing must be less than d, or 29 ins., and the length of these bars in the lower flange measured from the center line of the span must exceed 53 ins. The bent rods are ordinarily assumed as acting approximately through the center of gravity of the trapezoid representing the load they carry; thus bars 3 and 4 should pass approximately the center of gravity of the trapezoid *abdc*.

Although the discussion errs on the side of safety it should be remembered that, the diagram being one of maximum shears, the shears given are not in existence at the same time. There is still a diagonal tension *efg* not cared for; this will be provided for with vertical stirrups and the whole beam will be strengthened by their insertion across the span. The vertical stirrups are also useful in holding the horizontal bars in place.

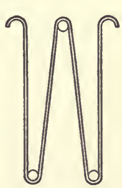


FIG. 240.

Had the web-stress been assumed as being resisted by the vertical stirrups and these stirrups taken as ½-in. round bars bent as shown in Fig. 240, making four sections of the rod that would have to be broken, and allowing 16,000 lbs. per sq. in. as the permissible fiber stress the stirrup value would be $T = 4 \times 0.196 \times 16,000 = 12,540$ lbs. The vertical shears are given by the diagram of maximum shear (Fig. 238), and have the following values at the several distances from the center of the span.

Distance from center of span, feet.	Max. shear, pounds.
8	43,875
6	35,250
4	27,000
2	18,000

The minimum spacing will be at the supports and will be

$$s = \frac{0.875\, d \times T}{V - 0.875 \times v_c \times b' \times d} = \frac{0.875 \times 29 \times 12{,}540}{52{,}500 - (0.875 \times 50 \times 20 \times 29)} \sim 8 \text{ ins.}$$

The spacing 8 ft. from the center of the span is approx. 11 ins. The spacing 6 ft. from the center of the span is approx. 16 ins. The spacing will be made, starting at the piers and running towards the center, 2 spaces at 8 ins., 2 spaces at 10 ins., and from there to the center the spacing will be 12 ins. This makes all the spacing less than $\frac{d}{2}$, or $\frac{29}{2} = 14.5$ ins., which would be the maximum for the best practice. The vertical stirrups might have been made much lighter considering what the bent rods do to strengthen the web, but in this case the vertical stirrups have been figured as resisting the entire web tension, the bent rods will strengthen the web and will strengthen the beam to resist bending by resisting a negative moment over the supports.

Economic Depth of Concrete Beams

The best practical depth to make a concrete beam, T beam or girder will depend upon the relative cost of the concrete and steel, upon the head-room or clearances, upon the necessary width to properly enclose the reinforcing bars and upon the required web area to care for the web stresses. Ordinarily where the beam is of sufficient importance the comparison of several beam sections will readily indicate the most satisfactory proportions. Turneaure and Maurer suggest the following formula for estimating the economic depth of T beams:

$$d - \frac{t}{2} = \sqrt{\frac{r \times M}{f_s \times b'}}.$$

Here r = ratio of the cost of equal volumes of steel and concrete.

The remaining symbols have the same significance as given on page 197.

Assume several different widths of stem and estimate the corresponding economic depths by the formula. The ratio r will

vary with different places and will ordinarily range from 50 to 80. Where the ratio is uncertain the mean of the above values will probably be a fair assumption.

Deflection of Concrete Beams

Owing to the varying moments of inertia along the span of a concrete beam together with the fact that the modulus of elasticity varies with the unit stress the calculation of the deflection of a concrete beam is at the best but an approximation. Mr. G. A. Maney in the *Transactions of the American Society for Testing Materials*, July, 1914, suggests the following simple formula whose accuracy, he states, has been checked by numerous tests on beams:

$$f = k\frac{l^2}{d}(e_c + e_s).$$

Here f = deflection.

k = a constant varying with the loading and nature of the support.

l = length of span.

d = depth of beam, to center of steel.

e_c = unit-deformation in extreme fibers of concrete, $\frac{f_c}{E_c}$.

e_s = unit-deformation in extreme fibers of steel, $\frac{f_s}{E_s}$.

The constant k for simply supported beams has the following values:

Uniform loading $k = 0.1041$; central loading $k = 0.0833$; third point loading $k = 0.1065$. In the case of fixed beams, for uniform loading $k = 0.0313$; central loading $k = 0.0416$ and third point loading $k = 0.0347$.

Columns

Concrete columns are reinforced in one or both of two ways: (1) by steel rods paralleling the axis of the column and (2) by spirally wound metal bands. In the first case the metal shares

the load with the concrete, while in the second case the bands strengthen the concrete by preventing lateral expansion which is assumed as occurring when the column is shortened along its axis by the load. As it is customary to tie the longitudinal reinforcements in place with steel wires or bands it virtually amounts to combining both forms of reinforcements.

The following is the usual discussion of the strength of columns having longitudinal reinforcements.

The nomenclature is

P = total load on column, pounds.
A_c = area of concrete inside reinforcements, not including steel, square inches.
A_s = area of steel, square inches.
$A = A_s + A_c$.
p = ratio, $\frac{A_s}{A}$.

If the column with longitudinal reinforcement is loaded, its length will be altered, the steel and concrete being shortened the same amount. The unit reduction is

$$\Delta = \frac{f_s}{E_s} = \frac{f_c}{E_c},$$

from which $$f_s = f_c \times \frac{E_s}{E_c} = n \times f_c.$$

The total load carried by the column then is

$$P = A_s \cdot f_s + A_c \cdot f_c = A_s \cdot nf_c + A_c \cdot f_c,$$

and $P = (A_s \cdot n + A_c) f_c$; also $P = A \cdot f_c [1 + p (n - 1)]$.

The working dimensions of the columns are generally assumed as those inclosed by the longitudinal or band reinforcements.

The proportions of concrete columns ordinarily work out so large that their ratio of length to smaller dimensions will usually be less than 15, thus making them short columns. The design may be safely made upon the basis just given, even where length divided by the smaller side reaches 25.

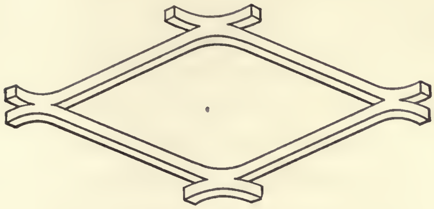


FIG. 241.

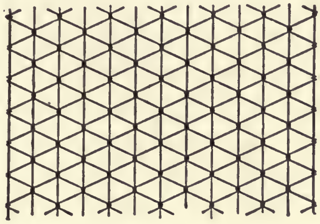


FIG. 242.

FIG. 243.

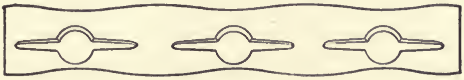


FIG. 244.

FIG. 245.

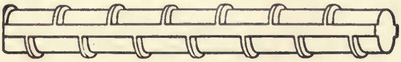


FIG. 246.

FIG. 247.

The usual coating of concrete is commonly put on the column outside the reinforcing steel to protect it and act as fireproofing.

A few of the numerous forms of metal reinforcements for concrete are shown in Figs. 241 to 247 inclusive. Fig. 241 shows a small section of expanded metal; this is made in varying sizes; the smaller, called metal lath, is intended to replace wooden laths. For concrete reinforcement the 3-in. mesh expanded metal is better. It is made in weights having a cross-sectional area of from 0.06 to 0.60 sq. in. per ft. of width. The triangular-mesh steel-wire reinforcement illustrated in Fig. 242 serves about the same purposes as the expanded metal. These reinforcements have the advantage of reinforcing in both directions, that is, across the span and at right angles to it.

Figure 243 is the Ransome twisted bar, Fig. 244 is the Thatcher bar and Figs. 245 and 246 are corrugated bars. These bars are designed to insure proper bonding between the steel and the concrete.

Figure 247 is the Kahn trussed bar; flanges along the side of the bar are bent up to resist the stresses in the web of the beam.

The Pittsburg Steel Products Company make a reinforcing frame consisting of steel bars through the lower part of the beam. To these bars on each side of the middle of the span are welded shear bars bent up and toward the adjacent support at an angle of 45 degrees.

To the upper end of these shear bars are welded horizontal bars which run back over the support. These frames are made in standard sizes and the reinforcement in the upper flange over the supports is said in all cases to be at least 25 per cent of the metal in the lower flange. These frames are built up for certain spans and are placed completed in the forms about which the concrete is then poured.

CHAPTER XV

FOUNDATIONS

THE designer recognizes two kinds of foundations, those for structures and others for machines. The function of the former is to distribute the pressure properly upon the soil and where settlement is inevitable to design the foundations that, as far as possible, such settlement shall be uniform throughout the structure. It may also maintain the structure upright against wind or other forces tending to upset it, as in the case of a chimney. In machine foundations besides these functions the mass of the foundation may play an important part in absorbing shock or may even prevent the motion of a machine when acting on other machines upon foundations external to its foundation. Properly balanced rotary converters and turbo-generators require only rigid support, while belt-driven machines, engines driving rolling mills, mill housings, etc., must be anchored.

With modern concrete floors, foundations for most machine tools may be dispensed with, the machines being placed where desired and rag bolts used to hold the machine to the floor.

The mass of a foundation to limit the vibration of an engine or other rapidly moving and imperfectly balanced machine within definite limits can only be determined when complete information of the design of the engine or machine is available. The determination of this mass therefore is properly the work of the engine or machine designer and not within the scope of this book. Numerous rules of thumb have been used in determining the minimum weights of foundations. One of these is to make the foundation weigh 1.5 times as much as the weight of the engine

or machine. Another rule credited to E. W. Roberts is that $F = 0.21\,E\sqrt{N}$, where

F = weight of the foundation, in pounds.
E = weight of the engine, in pounds.
N = R.P.M.

The wide variations in these two rules show how unsatisfactory such formulæ are. Vertical engines require heavier foundations than horizontal engines of the same capacity and speed.

The foundations of machines subjected to much shock should be kept free from other foundations and the building.

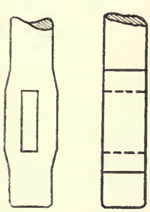
Although considering only pressure and settlement the best foundation bed is rock, yet vibrating machines placed directly upon the rock may result in the transfer of the vibrations through the rock for considerable distance. The usual precaution under such circumstances is to place 2 or 3 feet of sand held in a pocket in the rock; this may be either a natural pocket or it may be made of concrete. The foundation is then built upon this cushion of sand.

In setting an engine or important machine the foundation is brought to within $\frac{3}{4}$ in. to $1\frac{1}{4}$ ins. of its final top. The machine is then lined up by driving metal wedges between the foundation and machine bed. When the machine is properly in line a dam is built around the foundation top and a thin grout poured in, filling up the space between the bed plate and the foundation. After the grout hardens it firmly secures the bed and the foundation regardless of the finish of the engine bed or foundation top. When the grout is properly set the wedges may be removed and the spaces left by them filled up with cement mortar. The foundation bolts are preferably located and held in place during the construction of the foundation by templates of the bed plate. The bolts may either be set right in the concrete or may be placed in pipes whose inside diameters somewhat exceed the bolt diameters and the pipes are then set in the concrete. The bolts are carried by the templates; the space between the top of the

pipe and the bolt is filled with burlap, waste or paper, to prevent the concrete filling the clearance space between the pipe and the bolt.

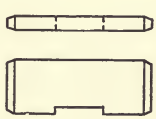
This clearance permits some adjustment of the bolts should the bed plate disagree with the template. The foundation bolts should be run well down into the foundation. Frequently the nuts or keys are placed in pockets made accessible from the outside of the foundation. This permits of the ready replacement of a bolt should it be found to be necessary.

In some cases what is known as a rust joint is made between the foundation and the bed plate. This is made by rusting cast-iron chips together with sal-ammoniac. Melted sulphur was



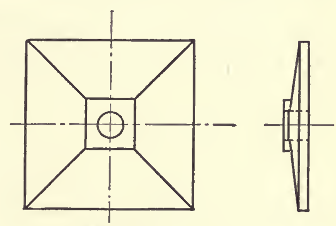
BOLT

FIG. 248.



KEY

FIG. 249.



WASHER

FIG. 250.

also much used before cement grout became so common. The foundation bolts generally have the end above the foundation threaded for a nut; the other end may either have a thread and nut or be slotted for a key. The washers in the foundation are of a great variety including scrap channels, angles or rails.

Cast-iron washers may also be used but of whatever kind they should be large to distribute the pressure well over the concrete. Fig. 248 illustrates a slotted bolt, while Fig. 249 shows the usual key and Fig. 250 the cast-iron washer. In other cases the adjustment of the bolt is provided for only at the top of the bolt for a distance of a couple of feet. This is done by using a short piece of pipe similar to the way mentioned before or by setting a wooden casing around the bolt which leaves a pocket in the finished foundation around the top of the bolt. The foundation

should be carried to proper soil and always below the frost line. The load upon the soil should be calculated to see that it does not exceed that permissible. The allowable pressure upon the soil is commonly given by the building codes where the foundation is within a city's limits.

It may frequently be had by obtaining data about neighboring foundations. The following table from "Baker's Masonry" is also available as a general guide.

Soil.	Bearing power, tons per sq. ft.	Soil.	Bearing power, tons per sq. ft.
Rock, thick layers.......	200 or over	Gravel and coarse sand..	8–10
Rock, equal to best brick.	15–20	Sand, compact..........	4–6
Clay, thick beds, dry....	4–6	Sand, clean.............	2–4
Clay, thick beds, moderately dry..............	2–4	Alluvial soils...........	$\frac{1}{2}$–1
Clay, soft...............	1–2		

Where the engine or machine is an important one and the soil of doubtful value, piles are frequently resorted to. The subject of piles is considered under building foundations.

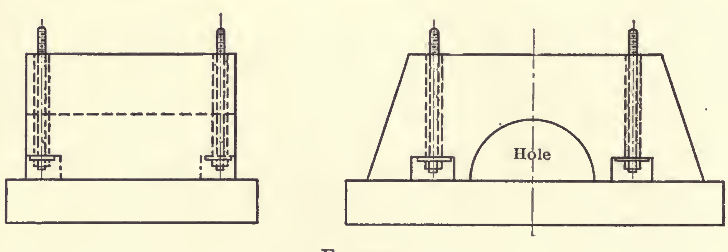


FIG. 251.

Figure 251 illustrates a foundation for a machine. The bolts have been set in pipes and run through to pockets accessible from the outside so that the bolts could be replaced if necessary. This foundation shows the base extended to reduce the pressure upon the soil and a semicircular hole has been cut through the foundation to reduce its weight.

In another class of foundations for machines the foundation requires weight to prevent the overturning of the machine. Fig. 252 illustrates such a foundation for a crane. The load of 33,000 lbs. is to be carried at a radius of 33 ft. The frame and

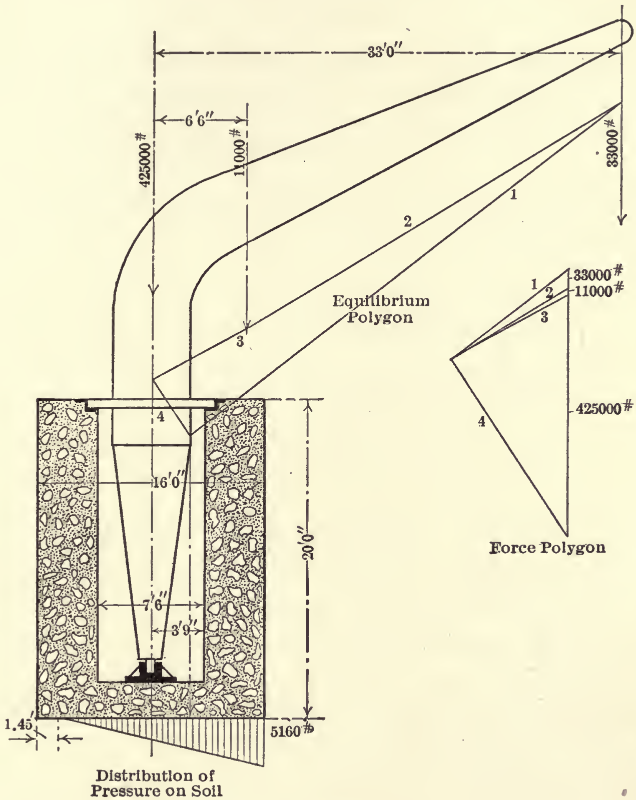


FIG. 252.

machinery is assumed as weighing 11,000 lbs. and as acting at a distance of $6\frac{1}{2}$ ft. from the post center. The foundation is circular and has been assumed as weighing 425,000 lbs. The total weight on the soil then will be the sum of these three amounts, $33{,}000 + 11{,}000 + 425{,}000 = 469{,}000$ lbs.

The distance of the line of action of the resultant of these forces can be found by taking moments about the vertical axis of the post; hence

$$x = \frac{(33 \times 33{,}000) + (6.5 \times 11{,}000)}{469{,}000} = 2.48 \text{ ft.}$$

The maximum pressure should now be determined as is done for a chimney (see page 254). The foundation has been assumed as circular. The kern radius of a circle is $\frac{D}{8}\cdot$ Here then $r = \frac{16}{8}$ $= 2.0$ ft. The resultant, therefore, falls outside the kern. Referring to Fig. 270, we first find

$$\beta = \frac{q}{D} = \frac{2.5}{16} = 0.156,$$

then $$\alpha = 0.355 \quad \text{and} \quad W = \alpha p_e D^2,$$

or $$p_e = \frac{W}{\alpha D^2} = \frac{469{,}000}{0.355 \times 16^2} = 5160 \text{ lbs.}$$

Whether or not this maximum pressure is permissible will depend upon the character of the soil or nature of the foundation under the section considered. The curves give ϕ as 41 per cent and the neutral axis therefore falls $16 \times 0.41 = 6.55$ ft. to the left of the vertical axis, or 1.45 ft. to the right of the left edge of the foundation when the full load is in the position shown in the figure. The distribution of this pressure is indicated below the assumed foundation bottom.

Building Foundations

Since settlement of foundations is bound to occur, unless foundations rest on bed rock, the foundations of a structure should be designed as far as possible to lead to uniform settlement. The effect of unequal settlement is frequently evident where chimney foundations have been carried into wall footings.

Here the settlement of the chimney may cause considerable cracking of the wall.

In designing foundations it is considered better to design the areas proportionally to the dead loads, which act continuously, and then see that they are sufficiently large to carry the combined dead and live loads without bringing excessive pressure upon the soil. The dead loads acting continuously will exert a greater effect upon the settlement of the foundation than live loads which may act but infrequently and then possibly be only a small part of the assumed live loads.

When the footings are proportioned for both dead and live loads only such portion of the live loads should be considered as may be assumed as acting continuously. It is also of prime importance that as far as possible the center of pressure on the soil shall coincide with the center of gravity of the footing. The building codes of the various cities give the requirements of footings and these are commonly required to be a certain enlargement of the wall upon them. Where these codes are not available the footings can be designed using the tables of allowable pressures previously given. The areas should be enlarged under pilasters carrying heavy concentrations.

Foundations for building columns, in addition to resisting vertical loads, may carry the horizontal reactions of wind loads, as in the case of the bent for the steel-mill building designed on page 121. Here there will be two cases, one (Fig. 253) where the column is assumed as hinged and then the horizontal reaction acts at the column base; the other (Fig. 254) where the column is considered as fixed and the point of application of the horizontal wind reaction is taken midway between the foot of the knee brace and the column base.

The base is commonly a rectangle. Were there no horizontal force the pressure upon the foundation would be uniformly distributed. The wind pressure tends to increase the pressure upon the leeward side of the foundation and decrease it upon the windward side.

As long as the resultant pressure R cuts the bottom of the foundation at a distance not exceeding $\frac{B}{6}$ from the center of the base there will be compression over the entire base. R is the resultant of W and H, where W = load carried by the column or wall plus the weight of the foundation and H = horizontal wind reaction on the column. Since neither masonry nor the surfaces of the foundation and soil in contact can be in tension,

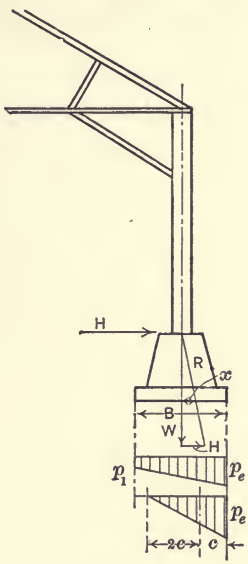


FIG. 253.

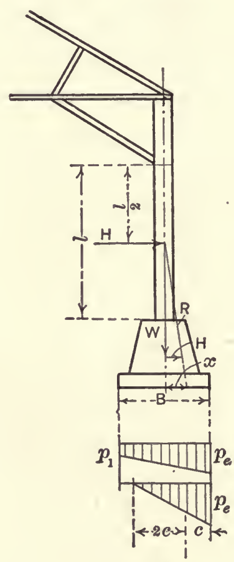


FIG. 254.

having R pass beyond the middle third will reduce the area of the foundation under pressure and will consequently increase the pressure on the leeward side, see Fig. 254.

When R falls in the middle third the extreme pressures on the edges of the foundation are given by

$$p_e = \frac{W}{b \cdot B^2}(B \pm 6x);$$

here b = width of the base at right angles to B.

When R falls beyond the middle third $p_e = \frac{2 \cdot W}{3 \cdot b \cdot c}$.

The distance x may be found graphically, as shown, or by moments, since

$$W \cdot x = H \cdot h \text{ or } x = \frac{H \cdot h}{W}.$$

h = distance from bottom of foundation to force H.

In the second case (Fig. 254) the attachment of the columns to the bases must be sufficiently strong to develop the bending moment $H \times \frac{l}{2}$ at this point.

The more general discussion of the kern of a section and the distribution of pressure upon the soil is given under chimney foundations, page 247. Where the base of a foundation extends beyond the main shaft, Fig. 255, the portion extended should be calculated as a cantilevered beam subjected to a uniform load p_e. This is commonly stated as tons or pounds per square foot. This extension e is given by $e = 4\,d\sqrt{\frac{3\,p}{p_e}}$. All dimensions in inches.

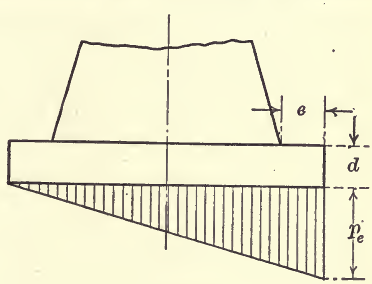


FIG. 255.

p = 30 to 50 lbs. per sq. in. in concrete.

p_e = maximum load on soil in pounds per square foot.

Figure 256 gives a foundation standard adopted by one company. Capstone — 1 Portland cement, 2 sand, 4 blast furnace slag. Foundation — 1 Portland cement, 2 sand, 6 slag, minimum bolt diameter — $1\frac{1}{4}$ in. — pressure on metal at base of column not over 10 tons per sq. ft. Pressure on capstone not over 5 tons per sq. ft. Bearing pressure per square foot on gravel 2.5 tons. Bearing pressure per square foot on other soil 2 tons.

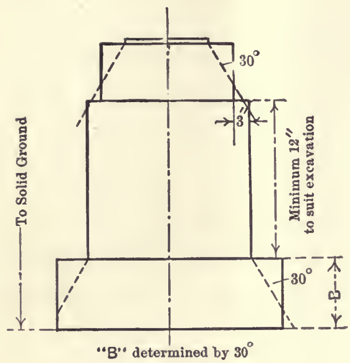


FIG. 256.

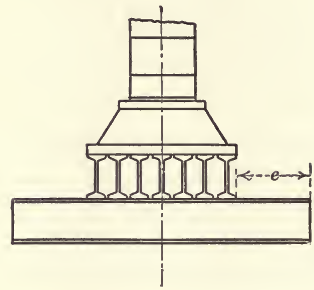


FIG. 257.

Where a shallow and light foundation is sufficient a grillage or reinforced-concrete footing can be used. The beam or rails

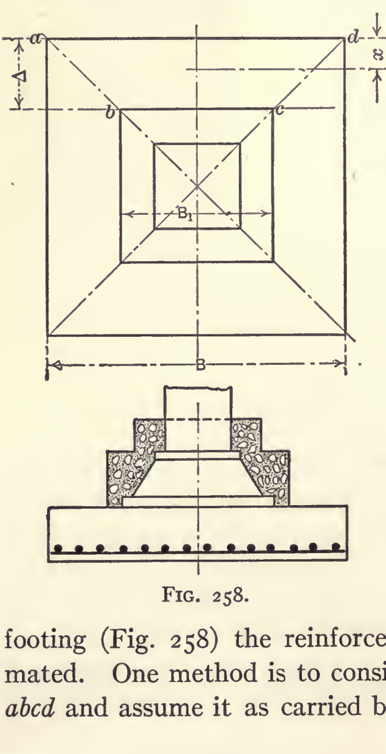


FIG. 258.

used in Fig. 257 may be calculated by assuming the bending moment on a tier of beams as $M = \frac{P \cdot e}{4}$, where P = the total load on the column, e = the extension of the beams beyond the superimposed beams, or column base. Owing to the uncertainty about the loading and deflection of the beams and the column base the load is assumed as uniformly distributed by the column base, or upper tier of beams on the beams below.

In the reinforced-concrete footing (Fig. 258) the reinforcement can be only roughly estimated. One method is to consider the load upon the trapezoid *abcd* and assume it as carried by the width *bc*. The load W on

this trapezoid is $p_e\left(\frac{B + B_1}{2}\right)\Delta$; the distance x to the center of gravity of the trapezoid is $x = \frac{B + 2\,B_1}{B + B_1} \times \frac{\Delta}{3}$.

The bending moment is $M = W\,(\Delta - x)$. In the formula for W, all dimensions are in feet, p_e = lbs. per sq. ft. and W is in pounds. In the formula for x all dimensions are in inches. In the equation for M all the terms are in pounds and inches. Having found the moment M the design is readily completed as explained under reinforced concrete and as illustrated by the design under a reinforced-concrete chimney.

Under the date of March 31, 1913, Bulletin No. 67 of the University of Illinois, entitled "Reinforced Concrete Footings and Column Footings," by Arthur N. Talbot, gives the results of tests and makes recommendations for the proper designing of footings based upon them. For a square footing and column base with two way reinforcement the suggestions are as follows. The moment on the section at *bc* is assumed as in the preceding case (Fig. 258). The resisting moment of the concrete footing is calculated considering the area of the steel in a distance across the footing equal to the width of the pier *bc* plus twice the thickness of the footing plus one-half the remaining distance on each side of the edge of the footing. This method may also be used when the spacing through the middle of the width of the footing is closer, or even when the bars are concentrated in the central portion.

With two way reinforcement evenly spaced over the footing, it seems that the tensile stress is approximately the same for bars lying within a space somewhat greater than the width of the pier and that there is also considerable stress in the bars lying near the edges of the footing.

Bond stresses are most important and the bulletin recommends that the bars shall not exceed $\frac{3}{4}$ inch.

The maximum bond stress will be that given on page 221, where V = load *abcd* and the number of rods is that stated above

and assumed as resisting the bending. Hooking the ends of the bars increases the bond resistance but is troublesome. Using long bars bent into horizontal loops covering the entire footing gave high bond resistance.

Diagonal Tension. — The vertical shear is taken as the measure of the diagonal tension. The unit shear is taken as that on the sides of a square concentric with that of the footing and the column base. The total shear, V, equals the load on the footing outside of the square being tested. If D is the depth of the footing and A a side of the pier, the square giving the maximum shear will have a side $(A + 2\,D)$ or its perimeter will be $4\,(A + 2\,D)$. The critical vertical shearing stress is

$$v = \frac{V}{4\,(A + 2\,D) \times jD}.$$

If B is the side of any other square, then

$$v_1 = \frac{V_1}{4\,B \cdot jD}.$$

The section under the pier will be subjected to punching shear; see Specifications, paragraph 218d.

Piles

Where the sustaining power of the soil is inadequate any of the preceding foundations may be placed upon piles. Piles were until a few years ago entirely of timber, preferably white oak, with a point diameter of at least 6 ins. but frequently specified as 8 to 10 ins. Their lengths will vary to suit the particular location. During the last few years extensive use has been made of concrete piles. Timber piles should be sharpened at the point, and are frequently shod with an iron band or point to protect them. The larger end is cut off square and is also sometimes protected with iron caps, hoops or bands. The brooming of the butt of a pile interferes materially with driving it, so that such piles should be trimmed during the driving to facilitate this

operation. The final or test blows should not be made on a broomed pile.

Piles may carry their load through friction between their sides and the surrounding soil, or they may be driven to rock when the load may be carried largely by the rock. A pile may therefore fail through crushing of the timber, or, where a pile is driven to rock or its equivalent through loose soil, it may fail as a column. Specifications limit the load on a pile to from 40,000 to 50,000 lbs., or 600 lbs. per sq. in. of its mean section. When a possible failure as a column is considered the load should be calculated for a column whose maximum stress does not exceed 600 lbs. per sq. in.

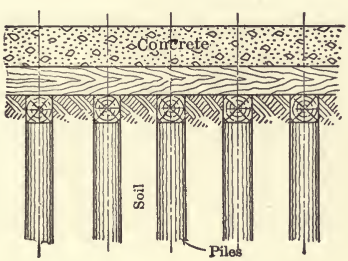


FIG. 259.

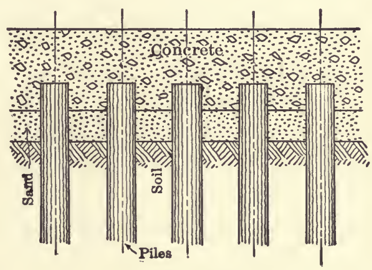


FIG. 260.

Timber piles when completely submerged in water and not exposed to the toredo (shipworm) are apparently preserved indefinitely. It is therefore necessary to cut timber piles below extreme low water. A bed is then built upon the tops of the piles to carry the upper part of the foundation (see Fig. 259). The usual spacing of the piles will vary from two to four feet and upon the tops of the piles which have been sawed off to a common level is placed a grillage of timbers secured to the piles with metal pins. The masonry is then placed upon this timber. All timber must be below water. Another and later practice, shown in Fig. 260, is to incase the tops of the piles in a bed of concrete. As in the first case the piles are cut off below the water level. The soil around the piles is excavated for two feet and a bed

of sand, say, 12 ins. thick, is put in, and upon this is laid the concrete whose thickness depends upon the load to be carried. This last method firmly incases the pile tops and distributes a part of the load upon the soil.

Concrete piles have an advantage over timber piles in not being attacked by the toredo and in not having to be continuously submerged.

There are several types of patented concrete piles. The Simplex is made in place by driving a hollow tube with a pointed end the entire depth required and then filling the hole with concrete as the tube is withdrawn. In the Raymond pile a thin tube is driven from which a collapsible case is then removed and replaced with concrete. Other forms of reinforced piles are made on the ground and when properly aged driven with a pile driver.

There is no very satisfactory method of determining the bearing power of a pile. The best known, where the pile is not driven to refusal, is the Engineering News Pile Formula. This formula is $P = \frac{2 \cdot W \cdot h}{s + 1}$ for drop-weight hammers and $P = \frac{2 \cdot W \cdot h}{s + \frac{1}{10}}$ for steam hammers.

P = safe load in tons, 2000 lbs.
W = weight of hammer in tons, 2000 lbs.
h = drop of hammer in feet.
s = penetration in inches due to the last blow.

CHAPTER XVI

CHIMNEYS

In the design of masonry chimneys it is usual to assume that the masonry cannot resist tensile stresses. If the weight of the column above the section 1–1, Fig. 261 (a), is W pounds this load will be uniformly distributed on the section when no other forces act on the column above this section. If then the area is A sq. ins. the uniform fiber stress is $\frac{W}{A}$ lbs. per sq. in. Assuming a force F acting on the right of the column, it will tend to reduce the fiber stress on the right and increase that on the left of the section. The original fiber stress f is compression. If the force F is increased sufficiently the fiber stress on the right would become zero while that on the left would be doubled, as is indicated in Fig. 261 (d). The unit stress would vary across the section as shown in Fig. 261 (d), being assumed as varying uniformly. The stresses would still all be compression. Under these conditions the resultant R of W and F must act through a point a distance q from the center of gravity of the section 1–1. Should the resultant R act at a greater distance from the center of gravity of the section than q, which would be due to a still further increase in F, then the fiber stress on the right would pass through zero and be-

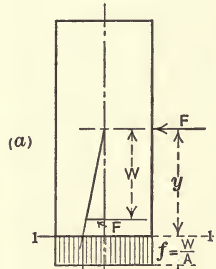


Fig. 261.

come tension, should the material resist tension, and the fiber stress on the left would be increased but would continue to be compression. Should the material not be able to resist tension the pressure will adjust itself to the left until the resultant stress R coincides with the point of intersection of the resultant of W and F with the section 1–1. When the extreme fiber stress on the left exceeds the strength of the material, the section 1–1 will fail. It is usual to assume that the resultant R must fall within the distance q from the center of gravity of the section. Had the force F been applied at all possible points around the column at the same distance y above the section 1–1 the points of intersection of the resultants R of W and F would have described the figure shown black in Fig. 261 (b); this is called the kern of the section. If the section 1–1 at no point is to be subjected to tension the resultant R must fall within the kern for all positions of the force F.

The moment on the section for a given loading will be the load W times the distance from the center of gravity of the section to the point of intersection of the resultant R with the plane of the section 1–1.

The allowable distance q from the center of gravity of the section to the resultant R that produces zero stress at the extreme fibers of the section for that position of F is given by $q = \dfrac{I}{A \times e}$,

where I = moment of inertia of the section, inches^4.

A = area of the section, square inches.

e = distance from center of gravity of section to extreme fibers whose stress is to be limited to zero.

The kerns for the commonest sections used for chimneys and foundations are given in Figs. 262 to 267.

The minimum radii for the kerns of these several sections are,

Square, Fig. 262, $r = 0.118\, h$.

Rectangle, Fig. 263, $r = \dfrac{b \cdot h}{6\sqrt{b^2 + h^2}}$.

Triangle, Fig. 267, $r_2 = \frac{h}{12}$, and $r_1 = \frac{h}{6}$.

Octagon, Fig. 264, $r_{\min} = 0.2256\,R$ (R = radius of corners).

Circle, Fig. 265, $r = \frac{d}{8}$ (constant).

Hollow square, outer side $= H$, inner side $= h$. Similar to Fig. 262

$$r_{\min} = 0.118\,H\left[1 + \left(\frac{h}{H}\right)^2\right].$$

Circular ring, Fig. 266, outer diameter $= D$, inner diameter $= d$.

$$r_{\min} = \frac{D}{8}\left[1 + \left(\frac{d}{D}\right)^2\right].$$

In the case of retaining walls it is necessary to consider the force F as acting in one direction; the base is then rectangular and the resultant R must not fall farther from the center of the base than one-sixth the base width, or if b = width of the base, $q \leqq \frac{b}{6}$. This is commonly expressed by saying that the resultant pressure must pass within the middle third.

In chimney design in this country the wind pressure is generally assumed at 30 to 50 lbs. per sq. ft. upon flat surfaces, and at 20 to 30 lbs. per sq. ft. on the projected area of cylindrical surfaces. The greater of these pressures considerably exceeds any pressure likely to occur in ordinary localities.

Ordinary masonry chimneys are commonly either square, octagonal or round. Square sections are suitable for short and unimportant chimneys while by far the greater number of chimneys are round.

Chimney sections are obtained by the use of specially shaped brick for the corners of octagonal chimneys and radial brick for round chimneys.

In small chimneys the upper 25 ft. should have a thickness of 8 to 9 ins.; this thickness should be increased about 4 to $4\frac{1}{2}$ ins.

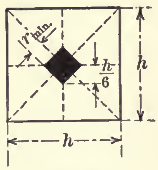


FIG. 262.

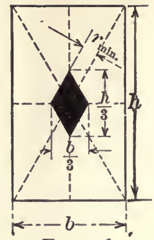
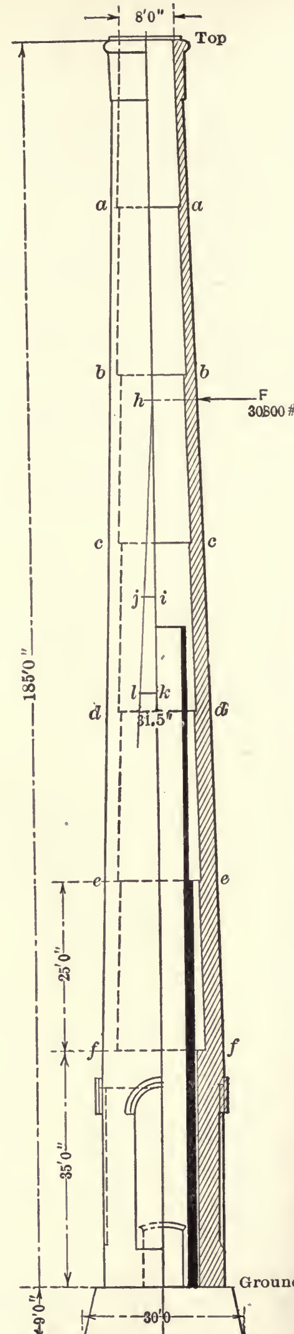


FIG. 263.



BRICK CHIMNEY
FIG. 268.

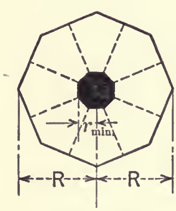


FIG. 264.

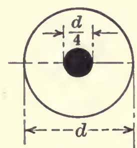


FIG. 265.

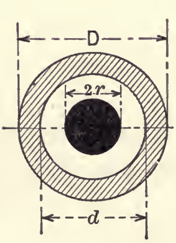


FIG. 266.

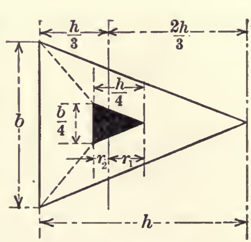


FIG. 267.

in each succeeding lower 25 ft. In large chimneys whose diameters exceed 60 ins., the thickness of the upper 25 ft. should be 12 to 13 ins.

Ordinary brick dimensions are, width 4 to $4\frac{1}{2}$ ins.; length 8 to 9 ins.; thickness 2 to $2\frac{1}{4}$ ins.

Chimneys are commonly provided with a fireproof lining in the lower portion extending up to from one-third to one-half the chimney height. This lining, being protected from the wind pressure by the main walls of the chimney, can be made quite thin, frequently not exceeding from 4 to 8 ins.

Designing a Chimney

The method of designing a chimney will be illustrated by making the calculations for the section *d–d* of the chimney in Fig. 268. The weight of the material above the section has been estimated as 545,000 lbs., being taken at 110 lbs. per cu. ft.

The wind pressure has been considered as 25 lbs. per sq. ft. of projected area for a round chimney and has been estimated as 30,800 lbs. This wind force has been assumed as acting at the center of gravity of the trapezoid which is the projected area of the chimney above the section.

The distance from the section *d–d* to the center of gravity is

$$x = \frac{b + 2\,b_1}{b + b_1} \times \frac{d}{3} = \frac{15.33 + 20}{15.33 + 10} \times \frac{100 \times 12}{3} = 558 \text{ ins.}$$

b = base of trapezoid.
b_1 = top of trapezoid.
d = altitude of trapezoid.

From the intersection of the chimney axis and wind force lay off *hi* representing the weight to scale, 545,000 lbs., and at its lower extremity lay off *ij* to the same scale representing F = 30,800 lbs. Draw the hypothenuse of this right-angled triangle and it will cut the section *d–d* at *l*. To insure the masonry at this section not being subjected to tension the point *l* must fall within the kern of this section.

For accuracy it is better to calculate the length *lk* than to measure it from a drawing on so small a scale. The calculation can be made by the use of the similar triangles *hlk* and *hij*, hence

$$lk = q = \frac{hk \times ij}{hi} = \frac{558 \times 30{,}800}{545{,}000} = 31.5 \text{ ins.}$$

hk = distance from the section to the resultant wind force.

The kern radius is given by

$$r = \frac{D}{8}\left[1 + \left(\frac{d}{D}\right)^2\right] = \frac{184}{8}\left[1 + \left(\frac{136}{184}\right)^2\right] = 35.6 \text{ ins.}$$

Since 31.5 ins. is less than 35.6 ins. it is evident that the resultant falls within the kern, and the section is subjected to compression only.

The weights have been calculated, using the volume of a frustum of a cone as

$$V = \frac{h}{3}[A + a + \sqrt{A \times a}].$$

h = height of the frustum.
A = area of the base.
a = area of the top.

The following table gives the properties of the several sections.

Section above.	Distance from top, ft.	Diameter O.D. I.D.	Mean diameter.	Mean area, sq. ft.	Weight above, lbs.	F at 25 lbs.	r''	q''
At top	0	10′–0″ 8′–0″						
a–a	25′ 0″	11′ 4″ 9′ 4″	10′ 8″ 8′ 8″	28	77,000	6,700	28.6	12.7
b–b	50′ 0″	12′ 8″ 10′ 0″	12′ 0″ 9′ 8″	40	187,000	13,750	30.8	21.3
c–c	75′ 0″	14′ 0″ 10′ 8″	13′ 4″ 10′ 4″	56	341,000	21,800	33.2	27.3
d–d	100′ 0″	15′ 4″ 11′ 4″	14′ 8″ 11′ 0″	74	545,000	30,800	35.6	31.5
e–e	125′ 0″	16′ 8″ 12′ 0″	16′ 0″ 11′ 8″	94	803,000	40,700	38.0	34.8
f–f	150′ 0″	18′ 0″ 12′ 8″	17′ 4″ 12′ 4″	117	1,127,000	52,500	40.1	37.9
Top of foundation	185′ 0″	18′ 0″ 11′ 0″	......	229	2,008,650	67,500	Max. 46.8, Min. 33.4	33.4
Base foundation. Concrete at 140 lbs. per cu. ft.		Top 22′ 0″, Bottom 30′ 0″	......	...	2,867,130	67,500	42.5 Min.	26.2

It is seen in the above table that since the resultant pressure always falls within the kern the maximum pressure never exceeds twice the fiber stress due to the weight above the section assumed as uniformly distributed over that section. Considering the section at *f–f*, the load above that section being 1,127,000 lbs., and the area of the section 117 sq. ft., the load per square foot is

$$\frac{1,127,000}{117} = 9630 \text{ lbs.}$$

Twice this is well below the allowable, or $144 \times 150 = 21,600$ lbs. per sq. ft.

Some designers prefer to use the inertia of the section in making their calculations. Making the calculations for the section 150 ft. from the top we have

The section modulus of the ring

$$= \frac{I}{e} = \frac{\pi (D^4 - d^4)}{32 D} = \frac{\pi (D^2 - d^2)(D^2 + d^2)}{4 \times 8 D}.$$

Since $\frac{\pi}{4}(D^2 - d^2)$ is the area of the ring section which we can call A we have

$$\frac{I}{e} = \frac{A}{8}\left(D + \frac{d^2}{D}\right).$$

The dimensions for the section 150 ft. from the top are $A = 117$ sq. ft., $D = 18$ ft. 0 ins., $d = 12.67$ ft., hence

$$\frac{I}{e} = \frac{117}{8}\left(18.0 + \frac{12.67^2}{18.0}\right) = 393.$$

The projected area of the chimney is $\frac{10 + 18}{2} \times 150 = 2100$ sq. ft. The wind pressure on this area is $2100 \times 25 = 52,500$ lbs. The distance x from the section to the center of gravity is

$$\frac{38}{28} \times \frac{150}{3} = 68 \text{ ft.}$$

The moment of the wind about the section is $52{,}500 \times 68 = 3{,}570{,}000$ ft. lbs., since

$$f = \frac{Me}{I} = \frac{3{,}570{,}000}{393} = 9100 \text{ lbs. per sq. ft.}$$

The direct pressure upon this section is $\frac{1{,}127{,}000}{117} = 9630$ lbs. per sq. ft. The maximum pressure upon the leeward side is $9630 + 9100 = 18{,}730$ lbs., while that on the windward side is $9630 - 9100 = 530$ lbs. per sq. ft., both being in compression.

Some designers assume an allowable tension on the windward side of one-tenth the maximum compression on the leeward side. It is then understood that the material does not exert this tensile fiber stress but that the effect is to make the resultant compression pass outside of the kern and increase the maximum unit pressure. This can be understood by referring to Figs. 261 (a) to 261 (f).

The determination of the maximum pressure under these circumstances presents considerable difficulties. As long as the resultant pressure at any section fell within the kern the margin of security would be given by the ratio of the ultimate crushing strength of the material to the extreme fiber stress in compression at that section. Keeping the resultant within the kern affords a ready means of determining the maximum fiber stress, thus assuring safety.

The maximum pressure upon the foundation will now be determined. The foundation being 30 ft. 0 ins. square the minimum radius of the kern is

$$r = 0.118 \times 30 \times 12 = 42.5 \text{ ins.}$$

As the resultant passes 26.2 ins. from the center line of the foundation it falls in the kern and the maximum unit compression will not exceed twice the unit uniform load on the foundation when no wind is blowing.

The maximum unit pressure will be on a corner when the wind blows parallel to a diagonal. The moment on the foundation

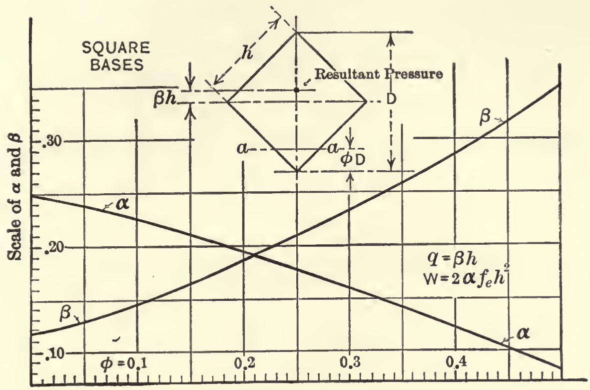


FIG. 269.

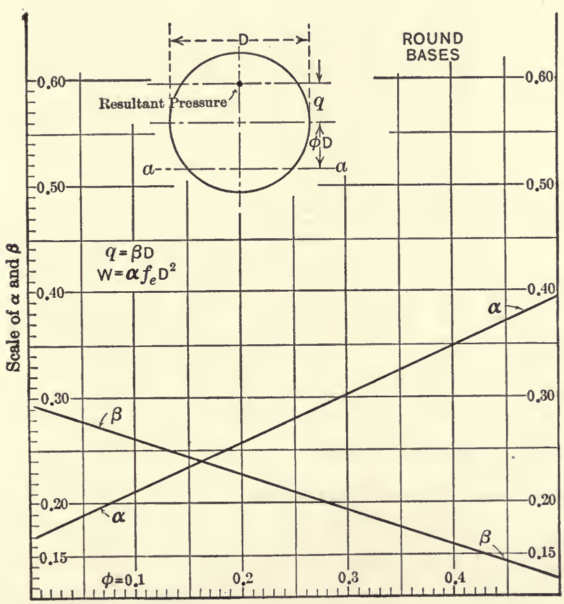


FIG. 270.

section is the product of the total weight on the soil multiplied by q or

$$M = \frac{2{,}867{,}000 \times 26.2}{12} = 6{,}260{,}000 \text{ ft. lbs.}$$

The resistance of a square referred to a diagonal as an axis is

$$\frac{I}{e} = 0.118\, h^3.$$

The base being 30 ft. square and keeping the resistance in feet^3 we have,

$$\frac{I}{e} = 0.118 \times 30^3 = 3186.$$

Since $f = \frac{M \cdot e}{I} = \frac{6{,}260{,}000}{3186} = 1965$ lbs. per sq. ft., the direct pressure is $\frac{2{,}867{,}000}{900} = 3190$ lbs. per sq. ft. The maximum compression then is $1965 + 3190 = 5155$ lbs. per sq. ft. The compression on the opposite corner is $3190 - 1965 = 1225$ lbs. per sq. ft.

These loads will be satisfactory if the soil can carry $2\frac{1}{2}$ tons per sq. ft.

When the resultant of the wind pressure and the chimney weight passes outside of the kern of the bottom of a chimney the maximum pressure on the corner of a square or the circumference of a circular section may be found by using the curves given in Figs. 269 and 270.

Fig. 269 represents a square base, Fig. 270 a circular base.

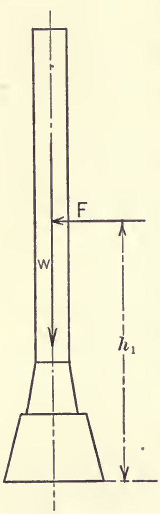


FIG. 271.

D = diagonal of square and diameter of circle.

h = side of square.

q = distance from center of section to resultant of wind and weight.

a–a = neutral axis, axis of zero pressure.

F = total wind force on chimney, acting at the center of gravity of the projected area of the exposed portion of the chimney.

W = total load on the soil, pounds.

h_1 = distance from force F to bottom of the foundation, inches. See Fig. 271. q (inches) $= h_1 \times \dfrac{F}{W}$.

Self-sustaining Steel Chimneys

A self-sustaining steel chimney, Fig. 272, is held upright by the resistance to overturning offered by the chimney, its lining and foundation.

The wind pressure is commonly assumed as 50 lbs. per sq. ft., acting upon the side of a square chimney, or 25 to 30 lbs. per sq. ft. of vertical projection upon round chimneys.

Usually the thickness of the upper sheets is determined for durability rather than for strength, being made not less than $\frac{3}{16}$ in. and frequently $\frac{1}{4}$ in. thick. This thickness is then increased by $\frac{1}{16}$ in. every 30 or 40 ft. Some designers vary by $\frac{1}{32}$ in. instead of by $\frac{1}{16}$ in. At the lower part of each 30 or 40 ft. section the fiber stress induced in it by the bending moment due to the wind pressure can be found and the section altered in thickness or length if deemed advisable.

The rivet spacing is made small to assure tightness, being not less than 2.5 times the rivet diameter nor more than 16 times the thickness of the plate. For most ring sections this gives excessive rivet strength.

An assumed ring section may be checked as follows:

M = bending moment in inch pounds on the section due to the assumed wind pressure above that section.

D = outside diameter of the chimney in inches.

h = distance from a ring section to the chimney top, inches.

t = thickness of shell, inches.

F = total wind pressure acting upon any portion of the chimney, pounds.

FIG. 272.

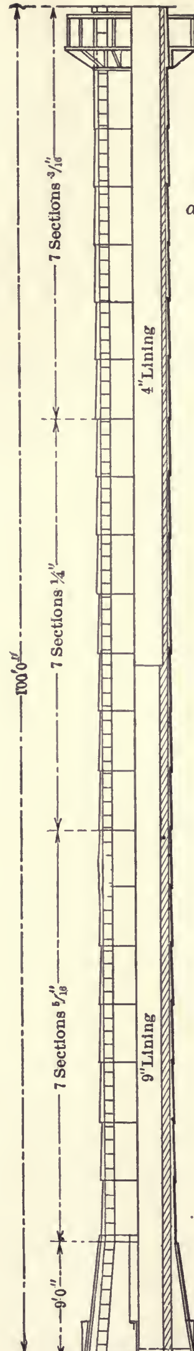
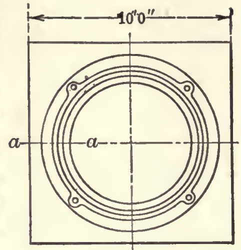
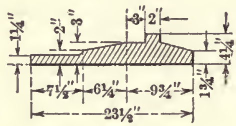


FIG. 273.



BASE PLATE



SECTION AT *a-a*

FIG. 274.

TOP PLATE

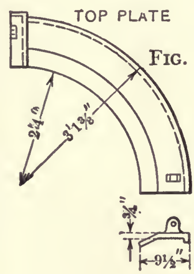


FIG. 275.

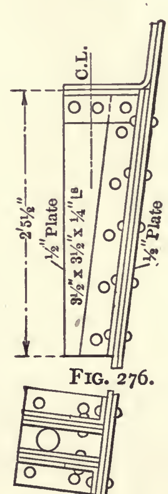


FIG. 276.

BRACKET

STEEL CHIMNEY

The chimney above the given ring section acts as a cantilever beam, and the bending moment equals the total wind pressure upon the vertical projection of the chimney above this ring section multiplied by one-half the distance from the ring section to the chimney top. The chimney will be assumed 66 ins. I.D. and 100 ft. high. The bell section is 9 ft. high and the section to be examined is 91 ft. from the chimney top. The total wind pressure on the vertical projection of this part of the chimney is $F = 5.5 \times 91 \times 25 = 12{,}510$ lbs. and

$$M = 12{,}510 \times \frac{91 \times 12}{2} = 6{,}830{,}460 \text{ in. lbs.}$$

The value of $\frac{I}{e}$ for a ring section is

$$\frac{I}{e} = \frac{\pi (D^4 - d^4)}{32 \times D},$$

where D = outside diameter.

d = inside diameter.

Where t, the thickness of the shell, is small compared with D, this value may be approximated as

$$\frac{I}{e} = \frac{4}{5} Dt (D - 3t).*$$

$\frac{I}{e}$ for the case in hand is $\frac{I}{e} = \frac{4}{5} \times 66.625 \times \frac{5}{16} (66.625 - 0.9375)$

$$= 1094.$$

$$f = \frac{Me}{I} = \frac{6{,}830{,}460}{1094} = 6250 \text{ lbs.}$$

This fiber stress is on the gross section. If the net section at the rivet circle is x per cent of the total section then the maximum fiber stress between rivets is

$$f_m = \frac{6250}{x}, \text{ or, if } \quad x = 80 \text{ per cent,} \quad f_m = \frac{6250}{.80}$$

$$= 7800 \text{ lbs. per sq. in.}$$

As these fiber stresses are satisfactory the $\frac{5}{16}$-in. shell will be used.

* NOTE.—$\frac{I}{e}$ is also frequently taken as the area inside the shell multiplied by the thickness of the shell, all measured in inches.

Riveting Ring Seams

s = pitch of rivets, inches. The force acting on the rivets per inch of circumference is $1 \times t \times f$, and to this must be added the weight of the shell 1 in. wide from the given section to the top. The calculation of this weight can be facilitated by remembering that a $\frac{1}{4}$-in. plate 12 ins. $\times$ 12 ins. weighs 10.2 lbs., then estimating the weight of a vertical strip 12 ins. wide and finally dividing the weight found by 12.

For the section 91 ft. from the top, $1 \times t \times f = 1 \times \frac{5}{16} \times 6250 = 1950$ lbs.

1. 30 ft. of $\frac{3}{16}$-in. plate, $30 \times 1 \times (\frac{3}{4} \times 10.2) =$	230
2. 30 ft. of $\frac{1}{4}$-in. plate, $30 \times 1 \times (1 \times 10.2) =$	306
3. 30 ft. of $\frac{5}{16}$-in. plate, $30 \times 1 \times (\frac{5}{4} \times 10.2) =$	383
Total	919
Adding 10 per cent for rivets, laps, etc.	91
	1010

or $\frac{1010}{12} = 84$ lbs. per in. The total force per inch of circumference acting on the rivets is $1950 + 80 = 2030$ lbs. Allowing 10,000 lbs. per sq. in. in single shear and 20,000 lbs. per sq. in. in bearing the value of a $\frac{3}{4}$-in. rivet in a $\frac{5}{16}$-in. plate is 4420 lbs., and the rivet spacing required is $\frac{4420}{2030} = 2.18$ ins. or, say, not exceeding $2\frac{1}{4}$ ins.

A double row of staggered rivets will be used at this seam spaced about, but not exceeding, $2\frac{1}{4}$ ins. Ordinarily the rivet spacing need be determined for only the lowest row of rivets in sheets of the same thickness.

Riveting Vertical Seams

The riveting in the vertical seams must provide for the shear along the neutral axis of the chimney, and as the riveted edge may be in compression the rivets must be spaced to prevent

the buckling of the plate between them; this demands that the rivet spacing shall not exceed 16 times the thickness of the plate.

The unit shearing stress at any point along a cylindrical beam

$$f_s = \frac{V \cdot r}{\frac{I}{e}},$$

where f_s = unit shear in pounds per square inch at the section.

V = shear at right angles to the neutral axis, pounds.

r = radius at the section, inches.

$\frac{I}{e}$ = resistance of the section, inches^3.

In this chimney at the section under consideration the unit shearing stress is

$$f_s = \frac{V \cdot r}{\frac{I}{e}} = \frac{12{,}510 \times 33}{1094} = 380 \text{ lbs.}$$

This per inch of chimney height is $380 \times \frac{5}{16} = 120$ lbs.

The rivet spacing for shear then is $\frac{4420}{120} \sim 37$ ins.

This spacing of course is not permissible, 16 times the plate thickness limiting the spacing to 5 ins. It generally will be the buckling of the plate edge that will determine this riveting.

The ring sheets are preferably of one piece, but in large chimneys this becomes impractical both for manufacture and shipment.

The base of the chimney is usually a bell, and is preferably a frustum of a cone, although sometimes flared to improve the appearance. The former shape is a much simpler and cheaper design. The height of the bell and the diameter at its base vary from $1\frac{1}{2}$ to 2 times the chimney diameter. The bell section is usually made of a number of pieces, butt jointed with single outside straps as shown in the drawing. The bell sheets are usually $\frac{1}{16}$-in. thicker than the ring sheets immediately above them.

Lining. — The method of lining varies. Where the temperatures do not exceed 650° F. (and they ordinarily do not) common red brick are satisfactory; otherwise a No. 2 fire brick is required. At the top the lining should have a thickness $4\frac{1}{2}$ ins., which can be increased by $4\frac{1}{2}$ ins. every 30 to 40 ft. In some cases the chimney is only partially lined, say one-half the way up, and in other cases the thinnest lining is made $2\frac{1}{4}$ ins.; the former, however, is the better practice.

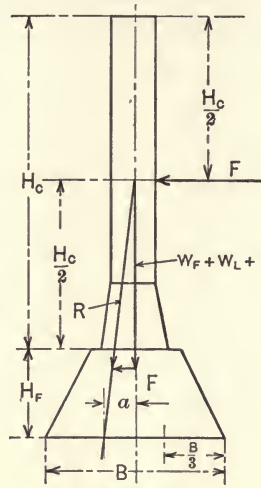


Fig. 277(a)

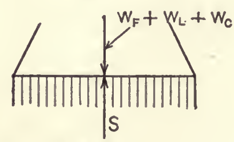


Fig. 277(b)

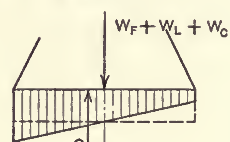


Fig. 277(c)

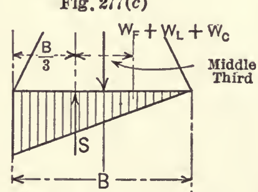


Fig. 277(d)

Where the breeching from the boilers enters the chimney above the base care should be taken not to weaken the chimney by cutting away too much, and the chimney should be sufficiently reinforced at this point. The practice of having the gases enter the chimney through flues below the steel base makes the chimney stronger and of better appearance.

Foundations

The shell is maintained upright against the moment of the wind pressure by the weight of the foundation and chimney with lining assumed as acting about the lower edge of the foundation.

The foundation is concrete and the chimney is secured to it by heavy foundation bolts. The concrete can be assumed as weighing from 125 to 150 lbs. per cu. ft. When

there is no wind acting upon the chimney the combined weights of the foundation, W_f, the steel chimney, W_c, and the lining, W_l, are distributed uniformly over the soil through the base of the foundation, as is shown in Fig. 277 (b). Now as a wind pressure F begins to act the uniform pressure previously on the soil will be altered, the pressure being reduced on the side upon which the wind acts and increased on the opposite side. In Fig. 277 (b) S and $W_f + W_c + W_l$, which are always equal, lie in the same line. In Fig. 277 (c) the force acting upon the base from the soil, S, has moved over, acting through the center of gravity of the trapezoid which here represents the distribution of the soil pressure.

When the wind pressure becomes great enough the distribution of pressure on the soil will become that shown in Fig. 277 (d), being zero at the right, while the original pressure at the left is doubled. The force S will pass through the center of gravity of the triangle, or $\frac{1}{3}$ B from the edge of the foundation. It is commonly assumed in masonry construction that the resultant pressure upon any section must pass within the kern of that section. Were the resultant S to pass to the left it would pass out of the kern. See page 247. In Fig. 277 (a) the moment due to the wind force must equal the moment of the forces W_f, W_c and W_l, with the arm a; hence

$$F\left(\frac{H_c}{2} + H_f\right) = (W_f + W_c + W_l)\, a;^*$$

the limit being $a = 0.118\,B$ we have

$$F\left(\frac{H_c}{2} + H_f\right) = (W_f + W_c + W_l) \times 0.118\,B.$$

The volume of a frustum of a cone or pyramid is

$$V = (A_t + A_b + \sqrt{A_t \times A_b})\frac{h}{3},$$

where A_t = area of the top of the frustum.

A_b = area of the bottom of the frustum.

h = altitude of the frustum.

* NOTE. — Some designers neglect the weight of the lining, as it may be omitted or removed during the life of the chimney.

Maximum Pressure on the Soil.— When the resultant pressure passes through the kern the maximum pressure will be less than twice the mean pressure. The maximum pressure per square foot then is

$$P \leqq 2\frac{(W_f + W_c + W_l)}{A_b}.$$

P = pounds per square foot.

A_b = area of the base of foundation, square feet.

Where the foundation is not reinforced the angle of the sides with the vertical should not exceed 30 degrees.

The maximum pressure should not exceed that permitted on the soil (see page 236).

In the chimney under discussion assume the base 18 ft. square and that the concrete weighs 140 lbs. per cu. ft. Estimating the weight of the steel of the chimney, the diameter being 5 ft. 6 ins., the circumference is 17.3 ft.

Sheets.	Dimensions, feet.	Weights, lbs.
$\frac{3}{16}$	$17.3 \times 30 \times 10.2 \times \frac{3}{4}$	3,980
$\frac{1}{4}$	$17.3 \times 30 \times 10.2 \times 1$	5,300
$\frac{5}{16}$	$17.3 \times 30 \times 10.2 \times \frac{5}{4}$	6,620
$\frac{3}{8}$	$17.3 \times 10 \times 10.2 \times \frac{6}{4}$	2,650
		18,550 lbs.
Allowing 10 per cent for bolts, rivets, laps, etc.,		1,850
	Total	20,400 lbs.

The weight of the lining assuming the chimney to be lined to a height of 62 ft.,

Section.	Lining, ins.	Area of lining, sq. ft.	Volume, cu. ft.
Base to 40 ft.	9	11.2	448
40 to 62 ft.	$4\frac{1}{2}$	6.05	135
		Total	583

Weight of lining, $580 \times 125 = 72{,}500$ lbs.

The height of the foundation will be assumed at 10 ft.

If the resultant of wind pressure and total weight is to fall within the kern its distance from the center line of the chimney must not exceed $0.118 \times B = 0.118 \times 18 \times 12 = 25.5$ ins.; this is the distance along the diagonal. At right angles to the side of the square it would be $\frac{h}{6} = 18 \times \frac{12}{6} = 36$ ins.

Trying the following foundation, top 10 ft. square, bottom 18 ft. square and height 10 ft., the volume of this foundation is

$$V = \frac{h}{3}(A_t + A_b + \sqrt{A_t \times A_b} = \frac{10}{3}(10^2 + 18^2 + \sqrt{10^2 \times 18^2})$$
$$= 2013 \text{ cu. ft.}$$
$$\text{Weight} = 2013 \times 140 = 281{,}800 \text{ lbs.}$$

The total weight on the soil is $281{,}800 + 20{,}400 + 72{,}509 = 374{,}700$ lbs. The uniform pressure per square foot is $\frac{374{,}700}{324} = 1150$ lbs. The resultant of wind pressure and total weight cuts the bottom of the foundation a distance q from the center line of the chimney. The total wind force is $F = 5.5 \times 100 \times 30 = 16{,}500$ lbs. This force acts at a distance of 50 ft. from the base of the chimney or 60 ft. from the bottom of the foundation. Hence $q = \frac{60 \times 12 \times 16{,}500}{374{,}700} = 31.7$ ins.

The resultant evidently falls outside the kern on the diagonal but passes within it at right angles to the side.

The maximum pressure on the corner can be found by the assistance of the curves, Fig. 269.

$$\beta = \frac{q}{h} = \frac{31.7}{18 \times 12} = 0.147.$$

From the curves for a square base when $\beta = 0.147$, $\alpha = 0.22$, and since

$$f_e = \frac{W}{2\,\alpha h^2} = \frac{374{,}700}{2 \times 0.22 \times 18^2} = 2625 \text{ lbs. per sq. ft.,}$$

the foundation will be satisfactory if this load of 2625 lbs. per sq. ft. is permissible upon the soil under the chimney.

Foundation Bolts

The chimney is assumed as tending to overturn about the axis a–a tangent to the bolt circle and differing only slightly from the lower edge of the bell. The maximum fiber stress will be f_m and will occur in bolts No. 3 and No. 4 (see Fig. 278), these being farthest from a–a. The fiber stress in any other bolt will be f_m multiplied by the ratio of its distance from a–a to the distance of the bolts farthest from a–a; thus the fiber stress in the bolts 2 and 5 is $f_m \times \frac{\rho}{\beta}$. The resisting moment due to any bolt is the product of its area, the fiber stress acting in it and its distance from the axis a–a. The resisting moment of the entire group of bolts will be the sum of the moments of the individual bolts. Taking the case of the six bolts shown, and letting a be the area of one bolt, at the root of the thread, and R the radius of the lower bell circle, we have,

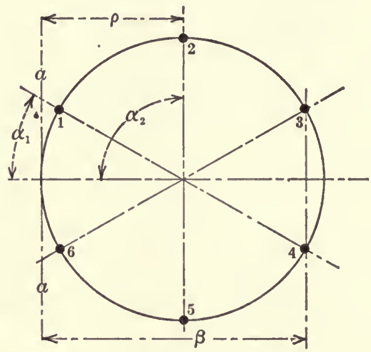


Fig. 278.

No. bolts.	ρ	Fiber stress.	Moment.
1 and 6	$R(1 - \cos 30°) = 0.134R$	$f_m \times \frac{0.134R}{1.866R}$	$2 \cdot a \cdot f_m \cdot R \frac{0.134^2}{1.866}$
2 and 5	$R(1 - \cos 90°) = R$	$f_m \times \frac{R}{1.866R}$	$2 \cdot a \cdot f_m \cdot R \frac{1}{1.866}$
3 and 4	$R(1 - \cos 150°) = 1.866R$	$f_m \times 1$	$2 \cdot a \cdot f_m \cdot R \cdot 1.866$

In a similar manner take moments around one of the bolts and use the lower of the two values.

This gives a total moment of $M = 4.5 \times a \cdot f_m \cdot R$.

In this way it may be shown that in the expression $M = C \cdot a \cdot f_m \cdot R$, C equals three-quarters of the number of bolts.

The moment on the chimney about its base due to the wind is 8,250,000 in. lbs. The moment due to the weight of the chimney which tends to hold it upright is $M_1 = 20{,}400 \times 45 = 918{,}000$ in. lbs. The net moment to be resisted by the bolts is $8{,}250{,}000 - 918{,}000 = 7{,}332{,}000$ in. lbs.

Trying six bolts, $M = 4.5\ a \cdot f_m R$, or $a = \dfrac{7{,}332{,}000}{4.5 \times 10{,}000 \times 47} =$ 3.46 sq. ins. This corresponds to $2\frac{1}{2}$-in. diameter bolts. This method being approximate, low fiber stresses should be used in bolts and brackets. Fig. 276 shows an effective bolt-bracket. The bolts should be kept close to the edge of the bell, and the bell sheet reinforced with straps. The lower edge of the bell should be reinforced with a circular band. This lower edge then rests upon a cast-iron base plate, Fig. 273, that distributes the pressure over the top of the foundation. The base plate may be made in one piece for small chimneys, as shown, but is usually made of a number of sections for convenience in casting and handling. These sections when placed in position are secured with a few $\frac{3}{4}$-in. diameter bolts. After erection the bell of the chimney and the foundation bolts will hold these sections in place.

The foundation bolts should extend almost to the bottom of the foundation and should be fitted with a lock nut and washer or key and washer.

The chimney should have a light ladder running to a platform at the chimney top. Dropping the platform a little below the top protects the handrail from the chimney gases.

A somewhat more conservative treatment might consider the pressure acting upon one-quarter of the area of the bell rim rather than being concentrated upon its leeward edge. The center of pressure might then be considered as acting on the leeward side, nine-tenths of R from the bell center. R is the radius of the bolt circle which has been considered as being about the center of the bell rim angle. Under these conditions C equals two-thirds of the number of bolts.

Reinforced-concrete Chimneys

In a reinforced-concrete chimney the weight of the concrete above the section under consideration will produce a compression on that section similar to that produced in a masonry chimney. Take any unit area having a ratio of reinforcement of p; under a given load the shortening of the two materials will be equal and the loads shared proportionally to their moduli of elasticity.

Let f_c = compressive unit stress in concrete, pounds per square inch.

$\frac{E_s}{E_c} = n$; then the fiber stress on the steel is $f_s = n \cdot f_c$ and if L is the load on 1 sq. in. of section $f_c = \frac{L}{1 + p\,(n - 1)}$. The load can be assumed as the weight of the concrete for the ordinary percentages of steel reinforcement.

With wind acting on one side of a chimney the pressure upon the leeward side is increased while that on the windward side is decreased, finally becoming zero while that upon the leeward side becomes double the original direct unit pressure. The concrete not being assumed as resisting tension may however be considered as acting like a beam until the compression at the windward side becomes zero. Under these circumstances the resistance of the section may be used as in the case of a beam resisting both tension and compression. In calculating the moment of inertia of the section proper allowance must be made for the modulus of elasticity of the steel differing from that of the concrete.

$$I = I_c + I_s = \frac{A}{16}(D^2 + d^2) + 0.394\, n \cdot D_1^2 t_s (D_1 - 3\, t_s);$$

here, I = total inertia of concrete and steel.

I_c = inertia of concrete ring.

I_s = inertia of steel.

A = area of concrete ring, square inches.

D = outside diameter of section, inches.

d = inside diameter of section, inches.
D_1 = diameter of steel cylinder having same area as total reinforcement at the section.
t_s = thickness of equivalent cylindrical steel reinforcement.

Generally t_s will be so small that $3 \cdot t_s$ may be neglected, making

$$I = \frac{A}{16}(D^2 + d^2) + 0.394\, nD_1^3 t_s.$$

Where the ratio of reinforcement is known $t_s = \frac{p}{2}(D - d)$.

When the point has been reached where the extreme fiber stress on the concrete on the windward side is zero any further increase of the wind pressure F will be resisted by increased compression in the concrete on the leeward side and by the reinforcing steel taking tension on the windward side. This causes the axis of zero stress to move across the section from the extreme fibers on the windward side towards the leeward side. Considering now only the additional moment applied after the extreme fiber stress on the windward side becomes zero there will be created additional compression on the leeward side shown shaded, while on the windward side the fiber stress will be tensile and must be carried by the steel. Considering now the fiber stress as indicated in Fig. 279, it will vary uniformly on each side of the neutral axis a–a, the stress in the steel being n times the stress in the concrete at the same distance from the axis a–a.

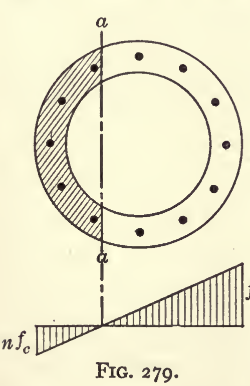


Fig. 279.

The resultant of the flange stresses in compression, shown in the shaded portion in Fig. 279, must equal the resultant of the tensile stresses in the steel in the remaining part of the section, the portion not shaded. The resisting moment induced in any section equals either flange force multiplied by the distance between

the lines of action of the resultant flange forces on each side of the axis a–a. This distance, where $R = R_s$, has been found to be constant and to equal 1.56 R_s, where R_s = radius of the steel circle.

The values of the flange forces and the moments have been calculated based upon the assumption that the stresses vary proportionally to the distances of the fibers from the neutral axis a–a, and that the fiber stress in the steel is n times that in the concrete for similar positions. The thickness of the wall $R_1 - r$ (Fig. 281) has been considered small compared with R; that is, the flanges have been considered as lines. The flange forces are

Tensile steel, $F_t = 2\, t_s f_t R_s \times C_1$.
Compression steel, $F_c = 2\, t_s f_c R_s \times C_3$.
Compression concrete, $F_{co} = 2\, f_{co} t_{co} R \times C_3$.

The moments due to these forces are

Tensile steel, $M_t = 2 f_t t_s R_s^2 \times C_2$.
Compression steel, $M_c = 2 f_c t_s R_s^2 \times C_4$.
Compression concrete, $M_{co} = 2 f_{co} t_{co} R^2 \times C_4$.

Here t_s = thickness of equivalent cylindrical steel reinforcement, inches.
f_t = extreme fiber tension in steel, pounds per square inch.
R_s = radius of steel, inches.
f_c = extreme fiber compression in steel, pounds per square inch.
t_{co} = thickness of concrete, inches.
f_{co} = maximum compression in concrete at mean radius R, pounds per square inch.
R = mean radius of concrete, inches.
f_a = fiber stress in the concrete at the radius R_s.

The values of these coefficients C_1 and C_3 have been calculated for the several values of k and are given in the curves, Fig. 280. The following relations are important. $F_t = F_c + F_{co}$. The total resisting moment of a section $M = M_t = M_c + M_{co}$. The distance between the lines of action of the resultants of the

flange forces has been previously stated as $1.56\, R_s$; hence the moment may be expressed as

$$M = F_t \times 1.56\, R = (F_c + F_{co})\, 1.56\, R.$$

These curves are based upon $n = 15$.

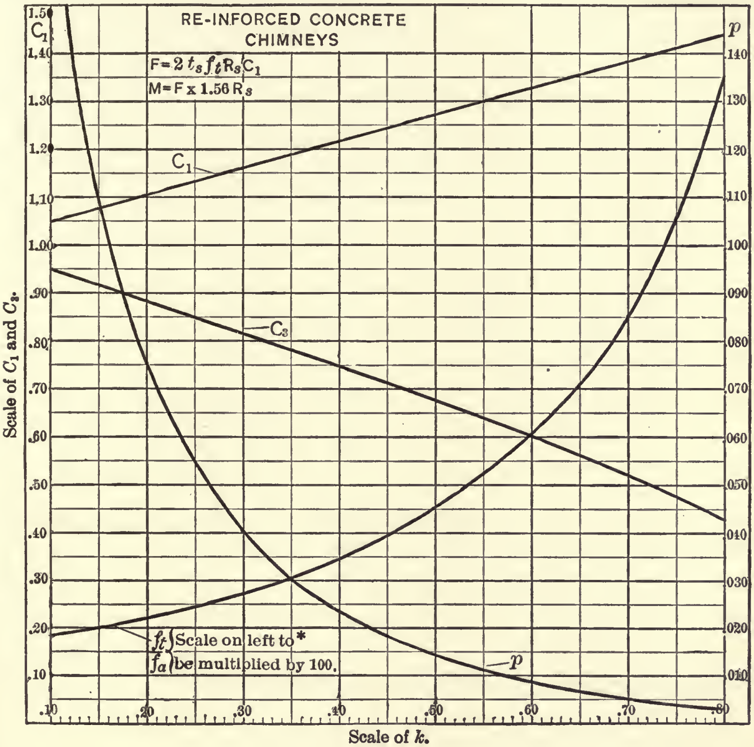


FIG. 280.

The curves also give the relations between the several fiber stresses and the critical percentages of reinforcements for the various values of k. $k = \dfrac{\Delta}{R}$. Here R is the radius of the flange being considered, that is, R_s for the steel and the mean radius for the concrete. The following problem will illustrate the calculations of a chimney as outlined.

* NOTE. — The values of $\dfrac{f_t}{f_a}$ and p have been estimated upon the basis that $R = R_s$. This materially simplifies the problem.

Problem. — A chimney is 150 ft. high. Its outside diameter is 10 ft. and the walls at its base are 9 ins. thick. Assume a wind pressure of 50 lbs. per sq. ft., giving an equivalent pressure of 30 lbs. per sq. ft. of projected area of the round chimney. The pressure on the concrete is not to exceed 600 lbs. per sq. in., while the unit stress on the steel must not be over 15,000 lbs. per sq. in. The chimney walls are 6 ins. thick for the upper 100 ft. and 9 ins. thick for the lower 50 ft. In Fig. 282 assume a reinforcement of 1.75 per cent and that a bar of concrete 1 sq.

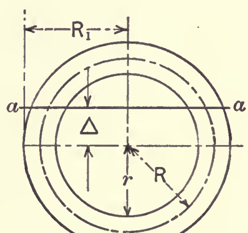


FIG. 281.

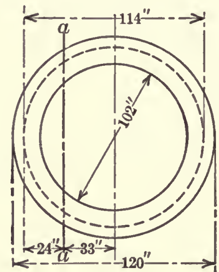


FIG. 282.

in. in section and 12 ins. high weighs 1 lb. The direct compression in the concrete at the base of the chimney is

$$f_c = \frac{(100 \times \frac{6}{9}) + 50}{1 + p\,(n - 1)} = \frac{117}{1.24} \sim 100 \text{ lbs. per sq. in.}$$

The inertia of the section with 1.75 per cent steel is

$$I = \frac{A}{16}(D^2 + d^2) + 0.394\, nD_1^3 t_s.$$

$$I = \frac{3140}{16}(120^2 + 102^2) + 0.394 \times 15 \times 114^3 \times 0.158$$

$$= 6{,}252{,}000 \text{ in.}^4.$$

$$M = \frac{fI}{e} = \frac{100 \times 6{,}252{,}000}{60} = 10{,}420{,}000 \text{ in. lbs.}$$

The total wind force is $W = 10 \times 150 \times 30 = 45{,}000$ lbs. The moment due to this wind pressure is $M = 45{,}000 \times 150 \times \frac{12}{2}$

= 40,500,000 in. lbs. The remaining moment to be resisted by the reinforced section after the stress in the extreme fibers on the windward side has become zero is 40,500,000 − 10,420,000 = 30,080,000 in. lbs. The compression on the leeward side at this time is 2 × 100 = 200 lbs. per sq. in. The permissible increase in the compression on the leeward side is 600 − 200 = 400 lbs. per sq. in.

The additional flange force is

$$F = \frac{30{,}080{,}000}{(1.56 \times R)} = \frac{30{,}080{,}000}{1.56 \times 57} = 346{,}000 \text{ lbs.}$$

$$F_t = 2 \times t_s \times f_t \times R_s \times C_1 = 2 \times 0.158 \times 15{,}000 \times 57 \times 1.25$$
$$= 338{,}000 \text{ lbs.}$$

This is sufficiently near the required 346,000 lbs. From the curves, Fig. 280, the ratio of stress in the steel to that in the concrete is 41 to 1; hence

$$f_c = \frac{15{,}000}{41} = 365 \text{ lbs. per sq. in.}$$

The approximate total stress in the concrete = 200 + 365 = 565 lbs. It should be noted that the 365 lbs. is the flexural stress at a distance R_s from the center of the chimney and that this can be easily corrected.

The value of k from the curves corresponded to 0.46, making

$$\Delta = k \times R_s = 0.46 \times 57 = 26.2 \text{ ins.}$$

The distance from the neutral axis to the steel 57 − 26.2 = 30.8 ins. The distance to the outside of the concrete is 60 − 26.2 = 33.8 ins. By similar triangles,

$$\frac{f_{\max}}{f_c} = \frac{33.8}{30.8}, \quad \text{or} \quad f_{\max} = 400 \text{ lbs.}$$

The extreme fiber stress then becomes 200 + 400 = 600 lbs. per sq. in.

Design of a Reinforced-concrete Chimney

In Fig. 283, assuming the chimney wall 5 ins. thick at the top, the area of this section is 1210 sq. ins. The radius of the kern is

$$r = \frac{D}{8}\left(1 + \frac{d^2}{D^2}\right) = \frac{82}{8}\left(1 + \frac{72^2}{82^2}\right) = 18.3 \text{ ins.}$$

This means that before any reinforcement would be required the resultant of wind force and weight could pass 18.3 ins. beyond the axis of the chimney. It follows that the weight above a section l feet from the top, divided by the wind force acting on the portion above that section equals $\frac{l}{2}$ ins., divided by 18.3, or

$$\frac{1210 \times l}{30 \times 6.8 \times l} = \frac{\frac{l}{2}}{18.3}, \quad \text{or} \quad l = \frac{1210 \times 18.3 \times 2}{30 \times 6.8 \times 12} = 18.1 \text{ ft.}$$

Hence the upper 18 ft. would require no reinforcement. It is usual to place a small reinforcement in this portion. Assuming a reinforcement used here of $\frac{1}{2}$ per cent (0.005) the distance that this will serve down the chimney can be found in a similar way. From the curves the ratio of the fiber stresses for $p = 0.005$ is 85 to 1; hence taking the fiber stress in the steel at 15,000 lbs. per sq. in. would make the value of $f_c = \frac{15{,}000}{85} = 177$ lbs. per sq. in.; as this is very low the tensile steel will determine the strength of the section. The value of C_1 is 1.38. The flange force in the tensile steel then is

$$F_t = 2\, t_s f_t \times R_s \times C_1 = 2 \times 0.025 \times 15{,}000 \times 38.5 \times 1.38$$
$$= 39{,}800 \text{ lbs.}$$

$$M_1 = F_t \times 1.56 \times R_s = 39{,}800 \times 1.56 \times 38.5 = 2{,}395{,}000 \text{ in. lbs.}$$

The inertia of the section is

$$I = \frac{A}{16}(D^2 + d^2) + 0.394\, nD_1^3 t_s.$$

$$I = \frac{1210}{16}(82^2 + 72^2) + 0.394 \times 15 \times 77^3 \times 0.025 = 968{,}406.$$

Fig. 283

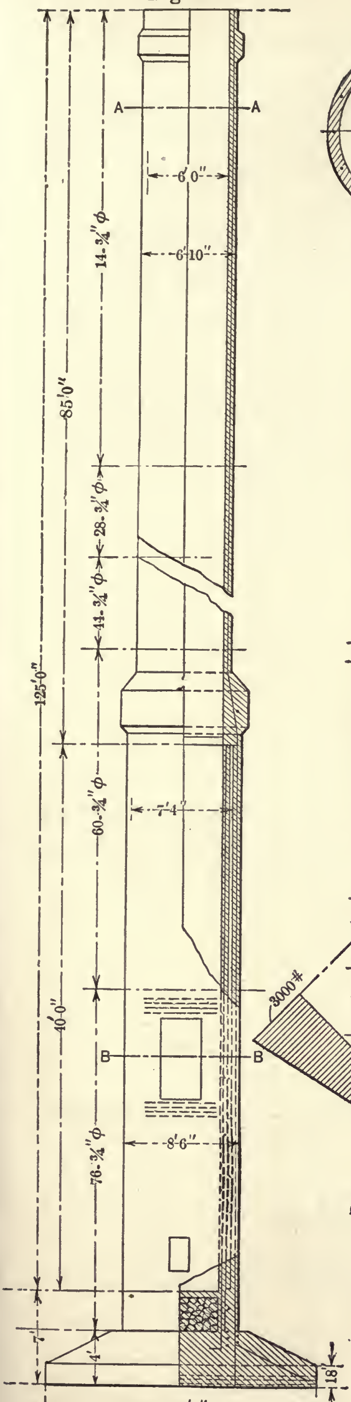
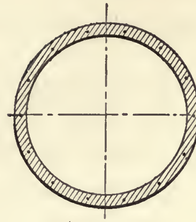
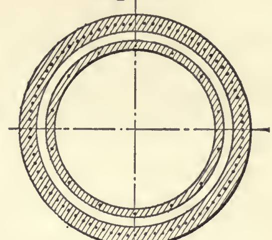


Fig. 284



SECTION A-A

Fig. 285



SECTION B-B

Fig. 286

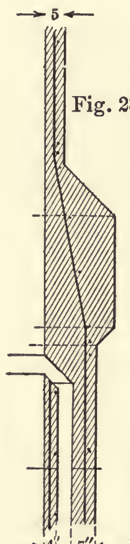
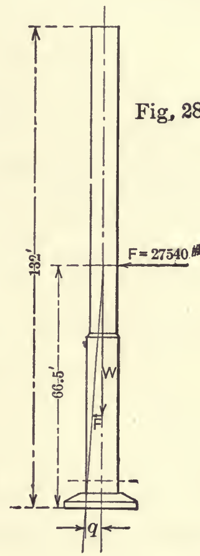


Fig. 287



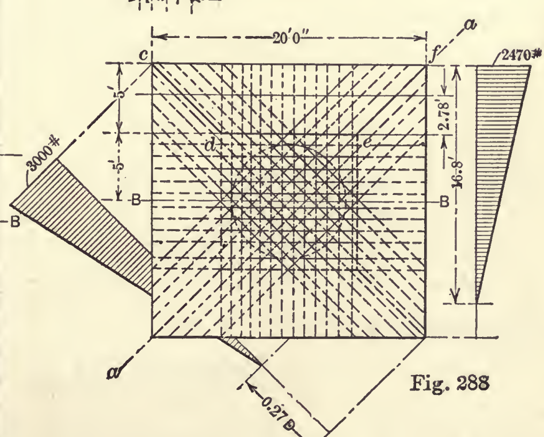


Fig. 288

REINFORCED CONCRETE CHIMNEY.

Now $M = \frac{fI}{e}$, and when the fiber stress on the windward side becomes zero that on the leeward side has become double the stress due to the weight above the section.

If the concrete weighs 144 lbs. per cu. ft., and the distance from the top to the section is l feet, then $f = l$, and we have

$$M_2 = \frac{lI}{e} = \frac{l \times 968{,}406}{41} = 23{,}600\, l.$$

The bending due to the wind acting on the portion above the section equals the sum of M_1 and M_2. From the wind pressure and the chimney dimensions the bending equals

$$M_3 = \frac{30 \times 6.8 \times l^2 \times 12}{2} = 1220\, l^2.$$

Equating these values of M we have

$$1220 \times l^2 = 23{,}600 \times l + 2{,}395{,}000,$$

from which

$$l^2 - (19.3 \times l) + \left(\frac{19.3}{2}\right)^2 = 2058, \quad \text{or} \quad l = 55.0 \text{ ft.}$$

This reinforcement will have an area of $1210 \times 0.005 = 6.05$ sq. ins., and will require 14 – $\frac{3}{4}$-in. round bars. Allowing a bond stress of 80 lbs. per sq. in. the length of the bars at laps is

$$l_1 = \frac{f_s \times d}{4 f_b},$$

where l_1 = length of the lap in inches.
f_s = fiber stress in steel, pounds per square inch.
f_b = bond stress, pounds per square inch.
d = diameter of round bar or side of square bar, inches.

In the problem

$$l_1 = \frac{15{,}000 \times d}{4 \times 80} = 46.9\, d,$$

d being $\frac{3}{4}$ in. the lap should be $46.9 \times 0.75 = 35.2$ ins.

The reinforcement will now be determined for the section at the base. The moment due to wind at this section of the chimney is

$$M = \left[(85 \times 6.8 \times 30)\left(\frac{85}{2} + 40\right)\right] + [(40 \times 8.5 \times 30 \times 20)]$$

$$= 1{,}634{,}550 \text{ ft. lbs.}$$

$$M = 19{,}614{,}600 \text{ in. lbs.}$$

$D = 102$ ins. $d = 88$ ins. The area of the section is 2085 sq. ins. The weight of the upper 85 ft. is $1210 \times 85 = 103{,}000$ lbs. The load per square inch on the lower section is $f_c = \frac{103{,}000}{2085} + 40$ $= 50 + 40 = 90$ lbs. per sq. in. Trying a reinforcement of 0.0175, from the curves $C_1 = 1.25$, $\frac{f_s}{f_a} = 41$, and $k = 0.46$.

Revising f_c for the steel percentage

$$f_c = \frac{90}{1 + p\,(n - 1)} = \frac{90}{1 + (0.0175 \times 14)} = 72.$$

The compression on the leeward side when the tension on the windward side is zero is $2 \times 72 = 144$ lbs. per sq. in. The tension in the steel corresponding to this is $(500 - 144)\, 41 =$ 14,596 lbs. per sq. in.

$$F = 2\, t_s f_t R_s C_1 = 2 \times 0.1225 \times 14{,}600 \times 48 \times 1.25$$

$$= 214{,}600 \text{ lbs.}$$

$$M = F \times 1.56 \times R_s = 214{,}600 \times 1.56 \times 48 = 16{,}069{,}250 \text{ in. lbs.}$$

The inertia of the section is

$$I = \frac{A}{16}(D^2 + d^2) + 0.394 \times n \times D_1^3 t_s$$

$$= \frac{2085}{16}(102^2 + 88^2) + 0.394 \times 15 \times 96^3 \times 0.1225,$$

$$I = 2{,}365{,}000 + 640{,}000 = 3{,}005{,}000,$$

$$M = \frac{fI}{e} = \frac{72 \times 3{,}005{,}000}{51} = 4{,}240{,}000.$$

The remaining moment to be resisted by the chimney is

$$19{,}614{,}600 - 4{,}240{,}000 = 15{,}374{,}600 \text{ in. lbs.}$$

$$F = M \div (1.56 \times R_s) = 15{,}374{,}600 \div (1.56 \times 48) = 205{,}000 \text{ lbs.}$$

$$f_t = F \div (2 \times t_s \times R_s \times C_1) = 205{,}000 \div (2 \times 0.1225 \times 48 \times 1.25)$$

$$= 13{,}900 \text{ in. lbs.}$$

$$f_c = \frac{13{,}900}{41} = 340 \text{ lbs.}$$

The extreme fiber stress will be somewhat in excess of this.

$$k = 0.46, \quad \Delta = k \times R_s = 0.46 \times 48 = 22 \text{ ins.}$$

By similar triangles the extreme fiber stress is $\frac{26 + 3}{26} \times 340 = 378$ lbs. per sq. in. The extreme fiber stress then becomes $(2 \times 72) + 378 = 522$ lbs. per sq. in. The reinforcing bars will require an area of $2085 \times 0.0175 = 36.5$ sq. ins. Using $\frac{3}{4}$-in. ∅ bars will demand $\frac{36.5}{0.44} = 83 - \frac{3}{4}$-in. ∅ bars.

Similar calculations will indicate the following bars in the several sections; beginning at the bottom, we have

Section.		Section.	
First,	22 ft., 83 $\frac{3}{4}$-in. round bars.	Fourth,	22 ft., 28 $\frac{3}{4}$-in. round bars.
Second,	25 ft., 60 $\frac{3}{4}$-in. round bars.	Fifth,	34 ft., 14 $\frac{3}{4}$-in. round bars.
Third,	22 ft., 44 $\frac{3}{4}$-in. round bars.		

Horizontal reinforcing rings must be used to resist the web stresses and also to prevent cracking due to expansion from the heat. Concrete being a poor conductor of heat the inside becomes much hotter than the outside. This causes considerable circumferential stress in the chimney which must be resisted by horizontal rings, preferably placed near the outer surface of the chimney.

The amount of this circumferential reinforcement can be only roughly approximated, as the difference in temperature between the inner and outer faces of the wall must be guessed. In several chimneys this reinforcement was from $\frac{1}{4}$ to $\frac{1}{3}$ of 1 per cent.

The greater the steel ratio the higher is the compression in the concrete.

These chimneys are generally lined at least one-third their height. The inner tubes, being protected from the wind, are merely called upon to resist temperature stresses. These are not usually calculated. The linings may be made of fire brick or reinforced concrete and will have horizontal ring reinforcement somewhat lighter than in the outer wall, say from $\frac{1}{6}$ to $\frac{1}{4}$ of 1 per cent, and vertical reinforcement to resist temperature stresses of from $\frac{1}{4}$ to $\frac{1}{2}$ of 1 per cent.

The horizontal reinforcing rings are commonly $\frac{1}{2}$-in. or $\frac{5}{8}$-in. rounds, and are spaced from 12 ins. to 18 ins. vertically. The spacing should be closer at the point on the chimney where the lining stops, Fig. 286, and additional vertical reinforcement should also be placed at this part.

An approximation of the amount of steel for web reinforcement may be made following the formulæ given on page 218. Assuming no tension carried by the concrete we find

$$p \sim \frac{w \times H}{18.7 \times t_{co} \times f_t}.$$

Here w is the effective horizontal wind pressure, in pounds per square foot.

H is the height of the chimney above the section being examined, in feet.

t_{co} is the thickness of the chimney wall at the section, in inches.

f_t is the unit tensile working stress on the steel, in pounds per square inch.

p is the ratio of steel reinforcement.

To this calculated percentage there should be added a small amount of steel to provide against vertical temperature cracks. Taylor and Thompson place this amount at $\frac{1}{4}$ of one per cent, and recommend the placing of these horizontal rings at intervals of from 6 to 10 inches, rather than at 12 inches or over.

Chimney Base, Fig. 288. — Estimate of total weight of chimney:

Section.	Area, sq. ft.	Volume, cu. ft.	Weight, lbs.
Upper	$\frac{1210}{144} \times 85$	$= 715 \times 144$	$=$ 103,000
Lower	$\frac{2085}{144} \times 40$	$= 580 \times 144$	$=$ 83,500
Lining	$\frac{955}{144} \times 40$	$= 265 \times 144$	$=$ 38,200
Base	56×3	$= 168 \times 144$	$=$ 24,190
Foundation			$=$ 167,000
		Total	415,890

The total wind moment previously found for the base of the chimney is 1,634,550 ft. lbs. The total force is $(85 \times 6.8 \times 30) + (40 \times 8.5 \times 30) = 27{,}540$ lbs. The resultant wind pressure then must act $\frac{1{,}634{,}550}{27{,}540} = 59.5$ ft. above the ground line. The resultant of weight and wind will act a distance q from the axis of the chimney and

$$q = \frac{(59.5 + 7)\,F}{W} = \frac{66.5 \times 27{,}540}{415{,}890} = 4.4 \text{ ft.} = 52.8 \text{ ins.}$$

The radius of the kern for a square is $r = 0.118 \times h = 0.118 \times 20 \times 12 = 28.4$ ins. From page 255,

$$\beta = \frac{q}{h} = \frac{4.4}{20} = 0.22, \text{ and } \alpha = 0.173,$$

since

$$W = 2\,\alpha f_e h^2, \quad \text{or} \quad f_e = \frac{W}{2\,\alpha h^2} = \frac{415{,}890}{2 \times 0.173 \times 20^2} = 3000 \text{ lbs.}$$

$\phi = 0.27$. The resultant pressure passes 4.4 ft. from the center, so that the pressure extends a distance $(5.6 \times 3) = 16.8$ ft. over the base. The area of the pressure on the base is $16.8 \times 20 = 336$ sq. ft. The total pressure equals the entire weight of the chimney, hence $336 \times \frac{p}{2} = 415{,}890$ and

$$p = (415{,}890 \times 2) \div 336 = 2470.$$

The reinforcement can be only approximated, the simplest way being to consider the part *cdef*, Fig. 288, as a cantilevered beam.

The distance from the center of gravity of the portion *cdef* to the line *cf* is

$$\begin{array}{llr} & \text{Area.} & \text{Statical moment.} \\ +\,20 \times \frac{10}{2} = & 100 \times \frac{10}{3} & = 333.33 \\ -\,10 \times \frac{5}{2} = & \underline{25} \times (5 + \frac{5}{3}) = & \underline{166.75} \\ & 75 & 166.58 \end{array}$$

$$X = \frac{166.58}{75} = 2.22 \text{ ft.}$$

The load on *cdef* approximates $75 \times 2400 = 180{,}000$ lbs. The approximate bending moment is $M = 180{,}000 \times (5 - 2.22) = 500{,}400$ ft. lbs. The bending moment per foot along the line *de* is $(500{,}400 \times 12) \div 10 = 600{,}480$ in. lbs. Assuming $n = 15$; $d = 42$ ins. and $p = 0.0025$; $j = 0.922$ and $kj = 0.218$.

Using the formula for reinforced concrete design, we have

$$f_s = \frac{M_s}{A \times jd} = \frac{600{,}480}{(42 \times 12 \times 0.0025) \times 0.922 \times 42}$$
$$= 12{,}300 \text{ lbs. per sq. in.}$$

and

$$f_c = \frac{2 \times M_c}{kj \times bd^2} = \frac{2 \times 600{,}480}{0.218 \times 12 \times 42^2} = 260 \text{ lbs. per sq. in.}$$

The depth of the beam being controlled by the necessity of the foundation reaching proper soil and extending below the frost line this fiber stress in the concrete is satisfactory. The spacing of $1\frac{1}{8}$-in. round bars will be

$$\text{spacing} = \frac{\text{area of bar} \times 12}{A} = \frac{0.994 \times 12}{42 \times 12 \times 0.0025} = 9\tfrac{1}{2} \text{ ins.}$$

The subject of stirrups for the web stresses may be considered as suggested on page 218.

CHAPTER XVII

RETAINING WALLS

Pressures on Retaining Walls

According to Coulomb's theory some wedge BAC, in Fig. 289, will produce a maximum pressure E against the wall. This force will make an angle δ with the face of the wall corresponding

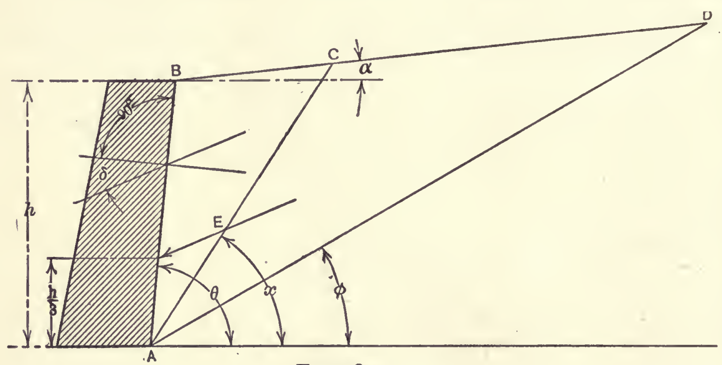


Fig. 289.

to the angle of friction between the face of the wall and the fill. The natural slope of the fill is the angle ϕ; the angle that the face of the wedge, producing the maximum pressure against the wall, makes with the horizontal is x.

h = height of wall in feet.

w = the weight of the fill in pounds per cubic foot.

According to this theory the general value of the maximum pressure in pounds per foot of length of wall is

$$E = \frac{1}{2} w \cdot h^2 \frac{\sin^2 (\theta - \phi)}{\sin^2 \theta \cdot \sin (\theta + \delta) \left(1 + \sqrt{\dfrac{\sin (\phi + \delta) \sin (\phi - \alpha)}{\sin (\theta + \delta) \sin (\theta - \alpha)}}\right)^2}.$$

This is simplified for the more commonly assumed conditions as follows:

For $\theta = 90$ degrees, and $\delta = \alpha$,

$$E = \frac{1}{2} wh^2 \frac{\cos^2 \phi}{\cos \alpha \left(1 + \sqrt{\frac{\sin(\phi + \alpha)\sin(\phi - \alpha)}{\cos^2 \alpha}}\right)^2}.$$

In Rankine's formula $\theta = 90°$, $\alpha = 0$, and $\delta = 0$; substituting these values in the general equation gives:

$$E = \frac{1}{2} wh^2 \frac{\cos^2 \phi}{(1 + \sin \phi)^2}.$$

$$E = \tfrac{1}{2} w \cdot h^2 \tan^2 (45° - \tfrac{1}{2} \phi).$$

$$E = \frac{1}{2} w \cdot h^2 \frac{1 - \sin \phi}{1 + \sin \phi}.$$

This theory, although usually developed for retaining walls, applies also to the sides of bins.

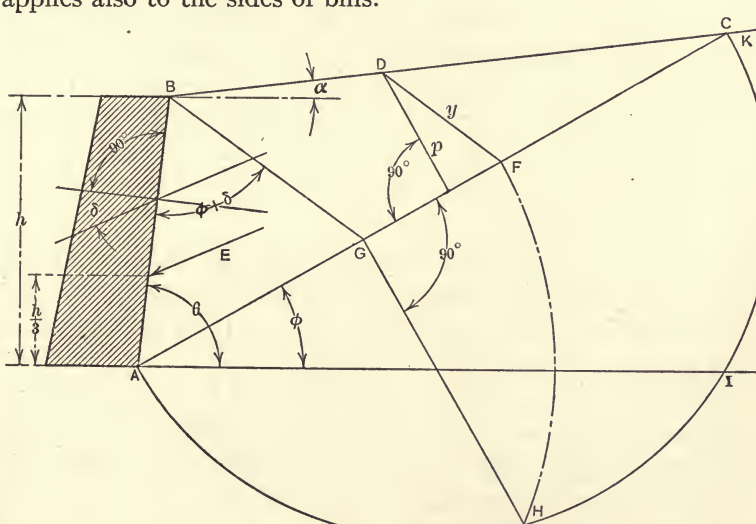


Fig. 290.

Graphical Solution. — The graphical method (Fig. 290) affords the neatest way of estimating the pressure E.

$$E = \tfrac{1}{2} w \cdot p \cdot y.$$

p and y = lengths of lines in the diagram measured to the same scale by which the wall or side is drawn. The other symbols are the same as in the preceding case.

Explanation of Diagram. — AB is the side of the wall or bin in contact with the fill. AI is a horizontal line through the base of the wall or side AB. AC is the line of natural slope of the fill making an angle ϕ with the horizontal. BC is the slope of the top of the fill making an angle α with the horizontal. Produce AC and BC until they intersect in C. Upon AC as a diameter draw the semicircle AHC. Draw BG making an angle $(\phi + \delta)$ with the wall AB.

At the point of intersection G of BG and AC draw GH perpendicular to AC. With AH as a radius draw the arc HF and through F draw DF parallel to BG. From D drop p perpendicular to AC.

The proofs and discussions of the preceding equations and diagrams may be found in books on the mechanics of retaining walls and earth pressures.

Figure 291 is the diagram when the angle of fill α equals the angle of natural slope ϕ. In this case since BK and AF are parallel p and y will be the same lengths wherever the point D is taken in the line BK.

Figure 292 shows the construction when the angle of fill falls below the horizontal and also below the line BC^1 making an angle $\phi + \delta$ with the back of the wall. In all the preceding cases $E = \frac{1}{2} w \cdot p \cdot y$.

If the fill, Fig. 293, carries a uniform load L lbs. per sq. ft. of horizontal projection, the height of the fill may be considered as increased sufficiently to bring such loading on the soil. The diagram is given in Fig. 294.

$$w' = w + \frac{2 \cdot L}{h} \quad \text{and} \quad E = \frac{1}{2}\left(w + \frac{2 \cdot L}{h}\right) p \cdot y.$$

Distribution of Pressure on the Wall or Side. — In the first cases, where the fill is not loaded, the pressure on the wall will

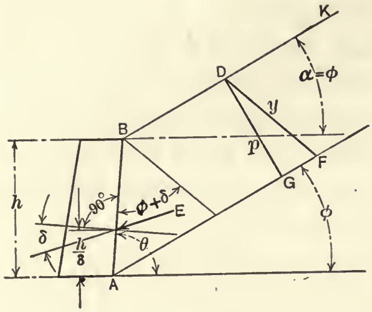


FIG. 291.

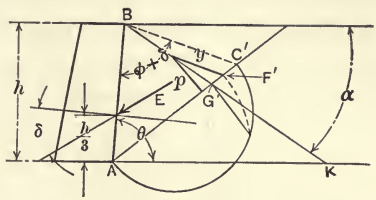


FIG. 292.

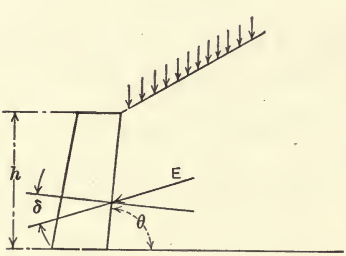


FIG. 293.

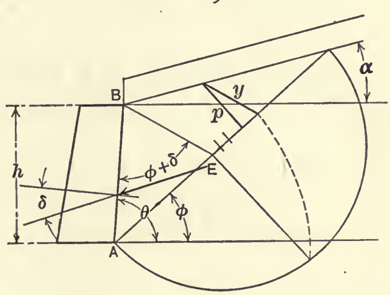


FIG. 294.

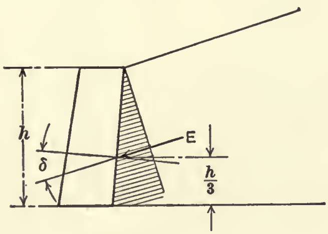


FIG. 295.

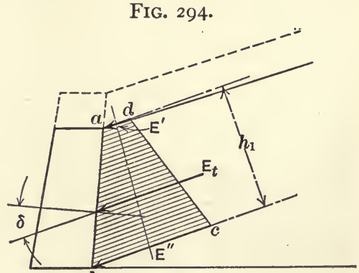


FIG. 296.

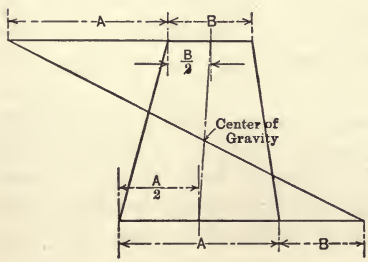


FIG. 297.

vary as a triangle and the center of pressure will correspond to the center of gravity of the triangle, as shown in Fig. 295.

Where the fill is loaded the usual assumptions regarding the distribution of the pressures behind the wall are shown in Fig. 296 and the magnitudes of these pressures are given by the following formulæ. The resultant pressure E_t acts through the center of gravity of the trapezoid of pressure *abcd*.

$$E_t = \frac{1}{2}\left(w + \frac{2 \cdot L}{h}\right) \cdot p \cdot y, \quad \text{and} \quad E_t = \left(\frac{E' + E''}{2}\right)h_1.$$

From which $$E'' = \frac{2\,E_t}{h_1} - E'. \quad E' = \frac{L \cdot p \cdot y}{h \times h_1}.$$

Center of Gravity of a Trapezoid. — Fig. 297 being a trapezoid its center of gravity can be readily found as shown.

WEIGHTS AND ANGLES OF REPOSE OF MATERIALS

Material.	Weight per cu. ft.	Angle of repose, ϕ. Degrees.
Clay, dry	90–110	30–40
Clay, damp	100–120	40–45
Clay, wet	120–135	15–25
Gravel, wet	100–120	25–40
Ashes	40–45	25–40
Coke	30	30–45
Earth, dry	80–90	30–40
Earth, moist	90–100	35–45
Earth, wet	105–120	17–30
Broken stone, wet	100	35–40
Coal, broken	56	45–50
Sand, dry	90–110	30–35
Sand, moist	100–110	30–45
Sand, wet	115–125	15–30
Water	62.5	0

The coefficient of friction between the usual masonry materials upon themselves or upon soil will range from 0.50 to 0.75.

Note. — To prevent the fill becoming saturated with water and greatly increasing the pressure on the wall the fill should be carefully drained. This may be done by suitably locating a drain of broken stone or gravel behind the wall and connecting at intervals by weepers draining outside the wall.

Retaining Wall

A retaining wall may fail by being rotated about the toe A or by sliding upon the base AB. There is usually little likelihood of failure by sliding as the coefficient of friction between the wall and the soil is high. The force E due to the earth pressure behind the wall creates an uneven pressure under AB as shown in Fig. 298.

A tilting of the wall may result from the side at A settling faster than that at B. To improve the condition it is not necessary to enlarge the entire wall but the base may be spread as shown in Fig. 299, so that the resultant R passes through the center of the base AB.

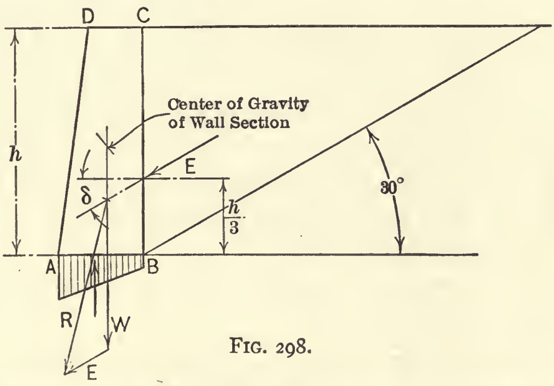


Fig. 298.

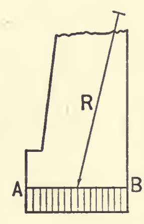


Fig. 299.

At best, the theories relating to retaining walls are unsatisfactory. This is due largely to the fact that the properties of the materials vary so widely that it is difficult to decide upon the conditions applicable to a particular case. The practical conditions differ greatly from the approximations necessary in the theory. The weight of the fill and its angle of repose will vary greatly as the fill is wet or dry, clean or dirty, loose or rammed, etc.

Other factors, such as shock, unexpected loading and frost, cannot be taken into account in the formulæ. According to Trautwine a practically vertical retaining wall sustaining a fill of sand, gravel or earth, without surcharge, with the fill loose, not rammed, the wall being of good common rubble or brick, should have a thickness at the top of the footing of four-tenths

the height of the wall. The Engineering News in its issue of Sept. 26, 1912, commenting upon the failure of a retaining wall, recommends the calculation of a retaining wall upon the assumption that a wall with no surcharge carries a load at least equivalent to a mass of water behind the wall of two-thirds its depth. The following problem will illustrate the method of designing a retaining wall.

Problem. — Design a section for a retaining wall, height 24 ft., weight of earth 110 lbs. per cu. ft., natural slope 30 degrees, maximum earth pressure not to exceed 5000 lbs. per sq. ft. Assume that the concrete of the wall weighs 140 lbs. per cu. ft.

The center of gravity of the wall must be located with reference to the back of the wall. Dividing the wall section (see Fig. 300) into rectangles and triangles, multiplying their several areas by the distances of their centers of gravity from the back of the wall and then dividing the sum of these moments by the total area of the wall section gives the center of gravity of the entire wall section as 46.6 ins. from the back of the wall. The area of the wall section is 143 sq. ft.

The weight of 1 ft. of length of the wall is $143 \times 140 = 20{,}020$ lbs. The resulting earth pressure is

$$E = \tfrac{1}{2} \cdot w \cdot p \cdot y = \tfrac{1}{2} \times 110 \times 12.2 \times 14 = 9394 \text{ lbs.}$$

The resultant R passes 9.3 ins. to the left of the center line a–a of the base. Using the formula deduced for footings with varying pressures on the soil, page 240, $f_e = \dfrac{W}{b \cdot B^2}(B \pm 6\,x)$, here $W = 25{,}000$ lbs. and is the vertical component of E and the weight of the wall, $b = 1$ foot, and the maximum pressure under the foundation is

$$f_e = \frac{25{,}000}{1 \times 10^2}\left(10 + \frac{6 \times 9.3}{12}\right) = 3670 \text{ lbs.}$$

Since $$\frac{f_1 + f_2}{2} \times B = 25{,}000,\ f_1 = 1330 \text{ lbs.}$$

An inspection of the wall will show that by increasing the width of the footing to 12 ft., as shown in Fig. 301, the resultant

would be brought practically through the center of the base thus making the pressure uniform across it.

To prevent sliding the coefficient of friction would have to be 8100 ÷ 25,000 = 0.324. As the coefficient of friction probably lies between 0.500 and 0.750 there is evidently ample margin of safety against failure in this way.

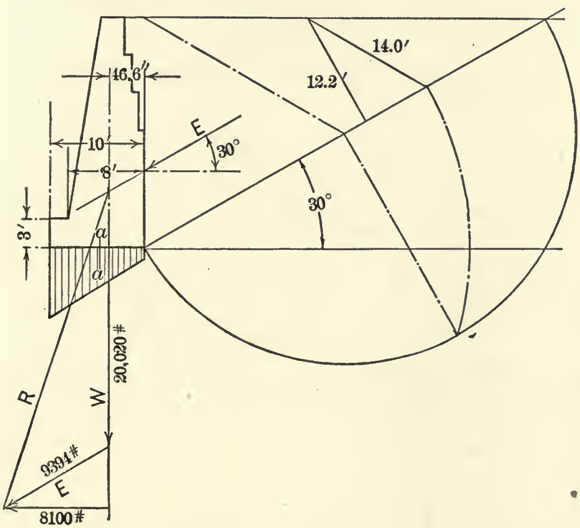


FIG. 300.

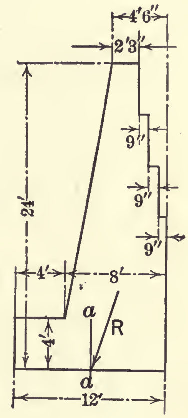


FIG. 301.

REINFORCED-CONCRETE RETAINING WALLS

The use of reinforced concrete in retaining walls has led to some modification of the ordinary design. This results in considerable economy in walls exceeding 25 ft. in height. The common section of such a wall is shown in Fig. 302. The vertical load upon the soil is the weight of the wall plus the weight of the soil carried directly by the wall, that is, the soil prism *abcd* and the vertical component of the force *E* acting upon the back of the wall. The general lines of the reinforcement are indicated by the steel shown in Fig. 302. The buttresses *B* are placed at intervals of from 8 to 10 ft. along the wall. Walls under 18 ft.

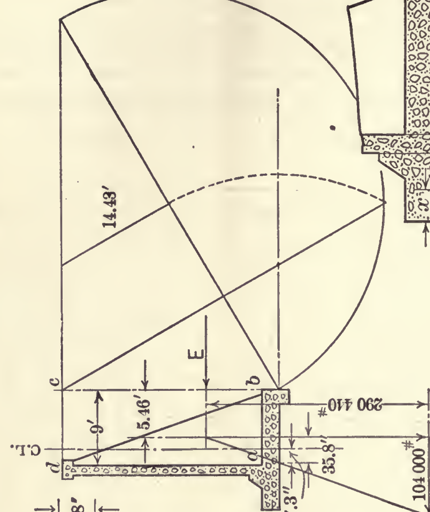


FIG. 303.

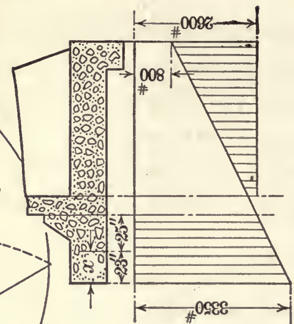


FIG. 305.

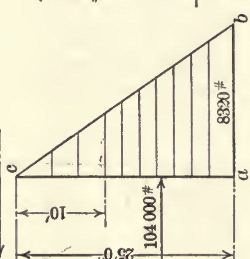


FIG. 304.

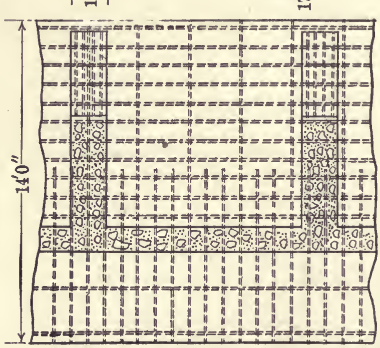


FIG. 306.

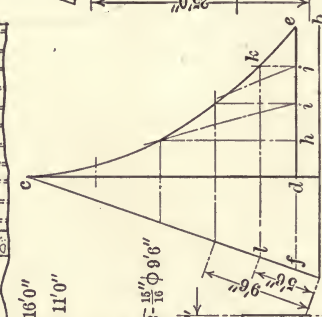


FIG. 307.

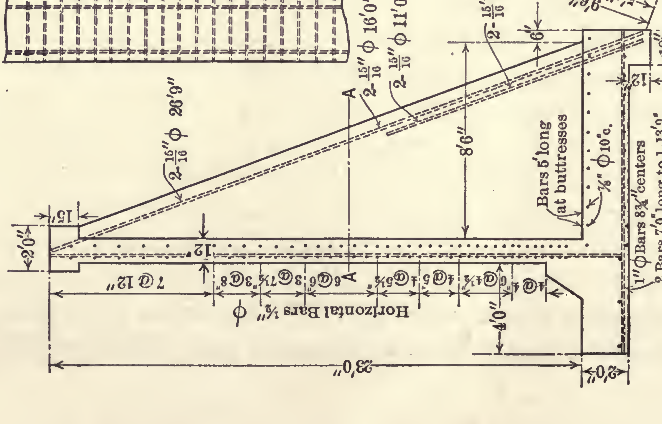


FIG. 302.

may have the buttresses omitted and the wall designed as a cantilever beam.

Problem. — Design a reinforced-concrete wall for a height of 25 ft.

Assume the weight of the fill as 100 lbs. per cu. ft. The angle of repose of the material behind the wall can be taken as 30 degrees. The maximum soil pressure is not to exceed 2 tons per sq. ft. The working unit stress in the steel is not to exceed 12,000 lbs. per sq. in., while that in the concrete is not to be over 500 lbs. per sq. in.

The fill *abcd*, Fig. 303, in the wall will act to prevent overturning, so that we will first want the weight and the center of gravity of the wall, buttresses and prism *abcd* of the fill.

Section.	Volume.	Weight.	Arm.	Moment.
Coping	25×140=	3,500	9.33	32,670
Face of wall	217.5×140=	30,450	9.50	289,275
Fillet	7.5×140=	1,050	10.25	10,760
Fillet	7.5×140=	1,050	10.83	11,372
Base	280×140=	39,200	7.00	274,400
Base	15×140=	2,100	0.75	1,575
Buttress	128.1×140=	17,934	5.55	99,534
Buttress	22.9×140=	3,206	8.67	27,785
Fill, prism over base	1759.5×100=	175,950	4.50	791,775
Fill, prism over buttress	160.0×100=	16,000	3.00	48,000
Total for 10 ft. length		290,440		1,587,146

$$\text{Distance to center of gravity of wall, } x = \frac{1,587,146}{290,440} = 5.46 \text{ ft.}$$

$$E = \tfrac{1}{2}(w \cdot p \cdot y) = \tfrac{1}{2}(100 \times 14.43 \times 14.43) = 10,400 \text{ lbs.}$$

For a length of 10 ft. this is 104,000 lbs.

The diagram made to an enlarged scale of Fig. 303 gives the line of action of the resultant of E and the total weight, 290,440 lbs., as passing through a point 35.8 ins. to the left of the vertical line through the center of gravity. The distribution of this

pressure upon the foundation is given by the formula

$$f_e = \frac{W}{b \cdot B^2}(B \pm 6\,x).$$

$$B = 14, \quad x = \frac{17.3}{12} = 1.44 \text{ ft.}$$

$$f_e = \frac{290{,}440}{10 \times 14^2}[14 \pm (6 \times 1.44)] = \begin{matrix} + \ 3350 \text{ lbs.} \\ + \ \ 795 \text{ lbs.} \end{matrix}$$

Sliding of the Wall. — The coefficient of friction between the wall and the soil will lie between 50 per cent and 75 per cent. The resistance to sliding, even assuming the coefficient at 50 per cent, would be $290{,}440 \times 0.50 = 145{,}220$ lbs. This, being considerably above the horizontal pressure on the wall, 104,000 lbs., indicates little possibility of failure by sliding.

Reinforcements

Face of the Wall. — The wall will be tied into the buttresses and the bending will be assumed as $M = \frac{W \cdot L}{12}$.* The reinforcements will be determined for sections down the wall and the bending moment will be calculated from the load taken from Fig. 304. As the pressure is assumed as varying proportionally to the depth, the distribution will be represented by a triangle. The area of this triangle equals 104,000 lbs., or

$$104{,}000 = 0.5 \times \overline{ac} \times \overline{ab} \text{ or } \overline{ab} = (104{,}000 \times 2) \div 25 = 8320 \text{ lbs.}$$

The load upon a strip 1 ft. wide and 10 ft. span is then given by an intercept in the triangle *abc* parallel to *ab*, the intercept being located the same distance from *c* that the center line of the section is from the top of the wall; thus the load on a strip 12 ins. wide whose center is 10 ft. from the top is 3320 lbs. when scaled from Fig. 304.

* Note. — If the negative moments are not to be carefully provided for M should be taken $\frac{WL}{10}$ or even $\frac{WL}{8}$.

The bending moment upon such a strip is $M = \frac{W \cdot L}{12} = 3320 \times 10 \times \frac{12}{12} = 33{,}200$ in. lbs. Using the approximate formula

$$M_s = \tfrac{7}{8} \times A \times f_s \times d.$$

Taking $d = 10.5$ ins. and $f_s = 14{,}000$

makes $A = \frac{33{,}200 \times 8}{7 \times 14{,}000 \times 10.5} = 0.258$ sq. in.

$\frac{1}{2}$-in. ∅ bars would require a spacing of

$$\frac{\text{Area of } \tfrac{1}{2}\text{-in. } \varnothing \times 12}{A} = \frac{0.196 \times 12}{0.258}, \text{ approximately 9 ins.}$$

If desired, this can be checked by the more accurate formulæ.

$$p = \frac{0.258}{10.5 \times 12} = 0.0020.$$

$p \cdot n = 0.0020 \times 15 = 0.032$. From the curves, page 202, $j = 0.924$; $k \cdot j = 0.231$.

Substituting these values in formulæ (3) and (4), page 200,

$$M_s = A \times f_s \times j \cdot d \quad \text{or} \quad f_s = \frac{M_s}{A \times j \cdot d}$$

$$f_s = \frac{33{,}200}{0.258 \times 0.924 \times 10.5} = 13{,}300 \text{ lbs.}$$

and $f_c = \frac{2\, M_c}{k \cdot j \times b \cdot d^2} = \frac{2 \times 33{,}200}{0.231 \times 12 \times 10.5^2} = 217$ lbs.

These values are satisfactory, it not being desired to make the wall thinner. The $\frac{1}{2}$-in. round bars will be spaced 8 ins. center to center in the section between 8 ft. and 10 ft. from the top of the wall.

The reinforcements may be determined for the other sections in a similar way. The results of such calculations are given in the table. The calculated spacing is given in the brackets, while that to be used is given outside.

Depth of section, feet.	Load, pounds.	$M = \frac{W \cdot L}{12}$, inch pounds.	Area of steel, square inches.	Size of bars and spacing, inches.
4	1400	14,000	0.109	$\frac{1}{2}''\varnothing$ spaced (21.6) 12″
7	2400	24,000	0.187	$\frac{1}{2}''\varnothing$ spaced (12.6) 12″
10	3320	33,200	0.258	$\frac{1}{2}''\varnothing$ spaced (9.1) 8″
12	4100	41,000	0.318	$\frac{1}{2}''\varnothing$ spaced (7.4) 7.5″
14	4800	48,000	0.373	$\frac{1}{2}''\varnothing$ spaced (6.3) 6″
16	5400	54,000	0.420	$\frac{1}{2}''\varnothing$ spaced (5.6) 5.5″
18	6100	61,000	0.474	$\frac{1}{2}''\varnothing$ spaced (4.96) 5.0″
20	6750	67,500	0.524	$\frac{1}{2}''\varnothing$ spaced (4.5) 4.5″
22	7400	74,000	0.575	$\frac{1}{2}''\varnothing$ spaced (4.1) 4.0″

The toe or base of wall to the left of the face will carry a load given in Fig. 305 and equals $\frac{3350 + 2600}{2} = 2975$ lbs. per sq. ft., making a load per foot of wall of $2975 \times 4 = 11{,}900$ lbs. The distance of the center of gravity of this loading from the face of the wall is

$$x = \frac{3350 + 2\,(2600)}{3350 + 2600} \times \frac{48}{3} = 23 \text{ ins.}$$

The moment on a strip 12 ins. wide then is $M = 11{,}900 \times 25 = 297{,}500$ in. lbs.

By the approximate formula $M_s = \frac{7}{8} \times A \times f_s \times d$.

$$A = \frac{297{,}500 \times 8}{7 \times 14{,}000 \times 22} = 1.10 \text{ sq. ins.}$$

The spacing for 1-in. $\varnothing$ bars is $\frac{0.785 \times 12}{1.1} = 8.6$ ins.

Rear Portion of Base. — The load upon this section (see Figs. 305 and 306) will be the difference between the pressure upon the soil and the weight of the fill vertically over this portion. This amounts to 23 ft. of soil and 2 ft. of concrete, and weighs $2300 + 280 = 2600$, approximately. The maximum loading is $2600 - 800 = 1800$ lbs. Estimating the reinforcement for maximum loading the total load on a strip 12 ins. wide and 10 ft. long is 18,000 lbs.

$$M = \frac{18{,}000 \times 10 \times 12}{12} = 180{,}000 \text{ in. lbs.}$$

Placing rods 3 ins. above the bottom of the slab and using the approximate formula gives,

$$A = \frac{8 \times M}{7 \times f_s \cdot d} = \frac{8 \times 180{,}000}{7 \times 14{,}000 \times 21} = 0.70 \text{ sq. in.}$$

The spacing of $\frac{7}{8}$-in. ⌀ bars will be $\frac{0.60 \times 12}{0.70} = 10$ ins., approximately.

In assuming the bending moment at $M = \frac{W \cdot L}{12}$ instead of $\frac{W \cdot L}{8}$ it becomes necessary to place reinforcement in the outer flange over the supports; this would have to equal the reinforcement at the middle. Here short lengths of rods the same as the full-length reinforcements (see Fig. 306) will be placed over the supports and spaced the same as the other rods.

Counterforts. — In this design (see Fig. 302) these will resist the moment due to the thrust on the wall. The thrust on the wall from the top to the upper face of the footing is $\frac{7650}{2} \times 23 = 87{,}975$ lbs.

The moment is $88{,}000 \times \frac{23 \times 12}{3} = 8{,}096{,}000$ in. lbs.

The design can be made as a T beam, the wall serving as the flange.

The distance from the face of the wall to the center of the reinforcement can be assumed as 108 ins. The ratio of flange thickness to depth of beam is $\frac{12}{108} = 0.11$. From the curves for T beams, $j = 0.95$, and substituting in the equation

$$M = A \times f_s \times j \cdot d,$$

we have

$$A = \frac{M}{f_s \times j \cdot d} = \frac{8{,}096{,}000}{14{,}000 \times 0.95 \times 108} = 5.65 \text{ sq. ins.}$$

This will require eight $\frac{15}{16}$-in. ⌀ rods.

The number of rods can be reduced towards the top of the wall as the bending moment becomes less.

In a beam similar to the counterfort the flange force at any distance x from the top can be approximately expressed as $F_x = \frac{(F \cdot x^2)}{h^2}$, where h = height of wall whose flange force is F. This is not accurate, as j will vary with the depth of the beam, but may be approximated at $\frac{9}{10}$. The flange forces will be proportional to the required areas. The varying flange forces can then be represented by laying off a base line equal to the height of the wall and plotting ordinates according to the formula just stated. The curve being a parabola can be laid off as in Fig. 307.

ca represents the side of the wall, here 25 ft., and *cd* represents the side extending to the top of the slab. *de* is the force in the steel 23 ft. from the top of the wall. *gc* represents the total length of diagonal steel reinforcements, the distance *df* measuring 8 ft. from the wall.

Eight rods taken in pairs will divide *de* into four equal parts and *dh*, *hi*, *ij* and *je* each represents the area of two of the rods. To find the lengths of the shortest rods produce *jk* perpendicularly until it cuts the curve, from *k* project a horizontal line until it cuts the diagonal line in *l*. The lengths of these two rods will then be given by the length *gl*. As this curve gives the rate of change of flange force it also gives the horizontal shear which will be the difference between two adjacent abscissas and the stirrups can be determined as shown for beams with uniform loads, page 218. The beam being so deep the concrete alone would probably supply ample strength for horizontal shear but in practice there are generally also metal stirrups inserted.

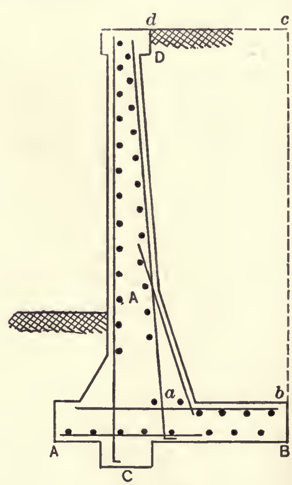


FIG. 308.

Another type of wall is the cantilevered wall. In it the

buttresses or counterforts are omitted. This type is suited to lower walls than the design just carried out. The principal reinforcements are shown in Fig. 308. The calculation of the thicknesses and reinforcing metal can be made on the same general lines as the preceding problem.

The vertical wall *CD* is treated as a cantilever and is assumed as carried by the base *AB*.

CHAPTER XVIII

BINS

Pressures on Bins

The methods used in obtaining the pressures upon retaining walls are also applicable to the pressures acting on the sides and bottoms of bins.

Bins may serve as retainers of any material but the discussions here will be confined to such bins as are ordinarily used for coal, sand, refractory materials at steel mills, and similar materials. Bins at grain elevators are generally high compared with their width; it therefore follows that the friction of the material against the sides of such bins may greatly assist in relieving the lower portion of the bin, both sides and bottom, of this accumulative pressure. The theory previously given for retaining walls does not apply in this case as the assumed plane of rupture to produce maximum pressure will not cut the surface of the grain. Several formulæ have been proposed for estimating these pressures; that of Janssen, taken from Hütte des Ingenieurs Taschenbuch, is

$$V = \frac{Rw}{fk}\left(1 - e^{-\frac{fk}{R} \times h}\right),$$

and $$L = V \cdot k.$$

L = lateral pressure of the grain, pounds per square foot.
V = vertical pressure of the grain, pounds per square foot.
f = coefficient of friction of grain on the bin wall.
w = weight of grain, pounds per cubic foot.
R = hydraulic radius of horizontal section of bin. For circular section $R = \frac{\text{diameter}}{4}$. For other sections $R = \frac{\text{area of section}}{\text{perimeter of section}}$. All dimensions in feet.

k = ratio of lateral to vertical pressure.
e = base of Naperian system of logarithms.
h = depth to bottom or point on side at which pressure is to be determined.

For wheat, w = 50 lbs. per cu. ft.; k = 0.60; f = 0.40.

Shallow Bins

The bins now taken up will be shallow compared with those just considered. In these the theory of retaining walls will apply. These bins will commonly be made of, (a) Timber, (b) Steel, unlined, or lined with timber or concrete, (c) Reinforced concrete.

The bin should be built or lining placed to avoid pockets that might hold materials indefinitely. This is particularly necessary in materials in which spontaneous combustion might occur. Timber lining should be tar coated on the sides and faces against the metal, when such treatment will not affect the material held.

The inclined sides of unlined steel bins frequently have wearing strips, flats about $3 \times \frac{1}{2}$ in., placed on about 12-in. centers and running at right angles to the direction of flow of the material, thus tending to retain a slight thickness of the contained material along the side or make the material slide upon itself rather than on the metal and so protect the metal from wear.

Stresses in Bins

Although the forces acting upon the bin sides may be estimated by formulæ, the graphical methods used on retaining walls will be largely used in the following discussions. For those who prefer formulæ those given by R. W. Dull in the Engineering News of July 21, 1904, are here given, with slight modification.

$$E = \left(\frac{\cos\phi}{n+1}\right)^2 \frac{wh^2}{2\cos\delta}$$

$$N = \left(\frac{\cos\phi}{n+1}\right)^2 \frac{wh^2}{2},$$

where $$n = \sqrt{\frac{\sin(\phi+\delta)\sin\phi}{\cos\delta}}.$$

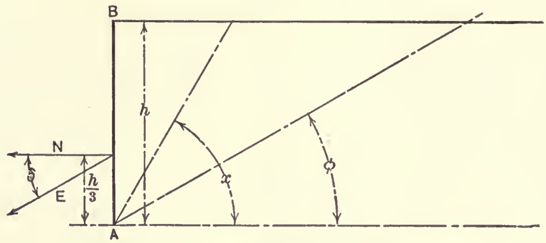


FIG. 309. Vertical wall, no surcharge.

$$E = \frac{wh^2}{2\cos\delta} \times \left(\frac{\cos\phi}{1+n}\right)^2,$$

$$N = \frac{wh^2}{2}\left(\frac{\cos\phi}{1+n}\right)^2,$$

where $$n = \sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\alpha)}{\cos\delta\cos\alpha}}.$$

If $\alpha = \phi$, then $n = 0$.

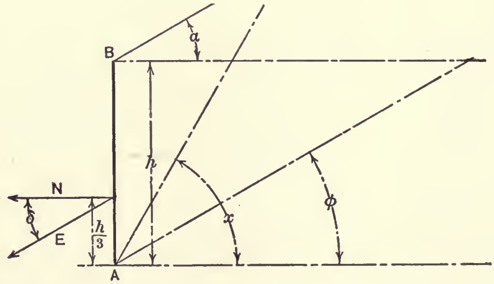


FIG. 310. Vertical wall, surcharged.

$$E = \frac{wh^2}{2\cos\delta} \times \left(\frac{\cos\phi}{1+n}\right)^2,$$

$$N = \frac{wh^2}{2} \times \left(\frac{\cos\phi}{1+n}\right)^2,$$

where
$$n = \sqrt{\frac{\sin(\phi+\delta)\sin(\phi+\alpha)}{\cos\delta\cos\alpha}}.$$

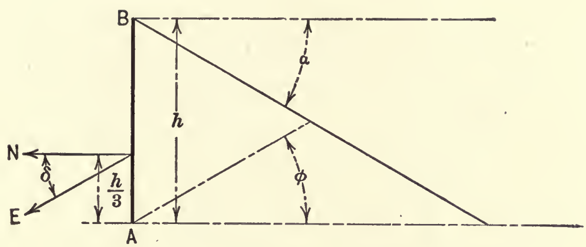


FIG. 311. Vertical wall, surcharge negative.

$$E = \frac{wh^2}{2\cos(\beta+\delta)} \times \left[\frac{\cos(\beta-\phi)}{(n+1)\cos\beta}\right]^2,$$

$$N = \frac{wh^2\cos\delta}{2\cos(\beta+\delta)} \times \left[\frac{\cos(\beta-\phi)}{(n+1)\cos\beta}\right]^2,$$

where
$$n = \sqrt{\frac{\sin(\phi+\delta)\sin\phi}{\cos(\delta+\beta)\cos\beta}}.$$

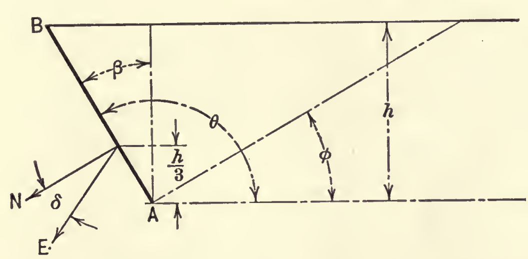


FIG. 312. Wall inclined outward. $\theta < 90° + \delta$. Surface horizontal.

$$E = \frac{wh^2}{2\cos(\delta + \beta)} \times \left[\frac{\cos(\phi - \beta)}{(n+1)\cos\beta}\right]^2,$$

$$N = \frac{wh^2 \cos\delta}{2\cos(\delta + \beta)} \times \left[\frac{\cos(\phi - \beta)}{(n+1)\cos\beta}\right]^2,$$

where
$$n = \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \alpha)}{\cos(\delta + \beta)\cos(\beta - \alpha)}}.$$

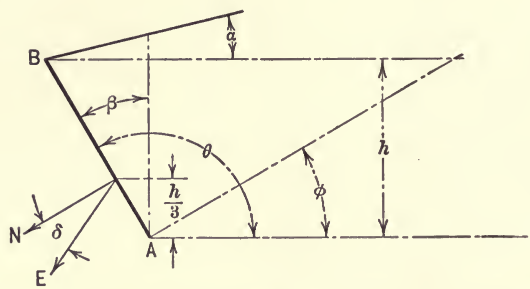


FIG. 313. Wall inclined outward. $\theta < 90° + \delta$. Surcharged.

$$N_1 = \tan^2\left(45° - \frac{\phi}{2}\right)\frac{wh^2}{2}.$$

W = weight of triangle $ABC = \dfrac{wh^2 \tan\beta}{2}$, $E = \sqrt{W^2 + N_1^2}$,

$$E = \frac{wh^2}{2}\sqrt{\tan^2\beta + \tan^4\left(45° - \frac{\phi}{2}\right)}. \quad \tan\mu = \frac{\tan\beta}{\tan^2\left(45° - \frac{\phi}{2}\right)}.$$

$Q = E\cos(\mu - \beta)$ and $T = E\sin(\mu - \beta)$.

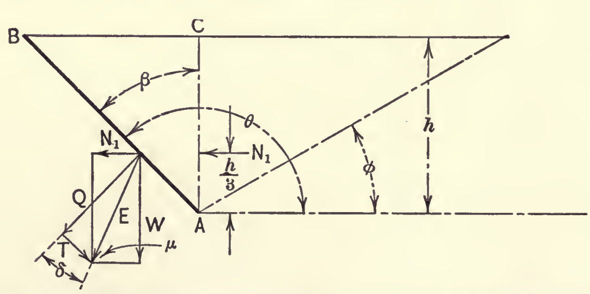


FIG. 314. Wall inclined outward. $\theta > 90° + \delta$. Surface level.

$$E_1 = \cos\alpha \times \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}} \times \frac{wh_1^2}{2}.$$

E_1 acts parallel to line of fill.

$$W = \frac{wh^2 \sin\beta \cos(\alpha - \beta)}{2 \cos^2\beta \cos\alpha},$$

$$E = \sqrt{E_1^2 + W^2 + 2\,E_1 W \sin\alpha}.$$

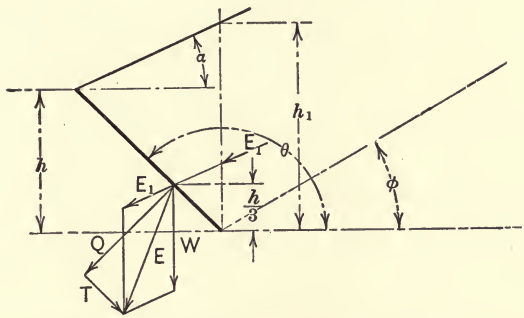


FIG. 315. Wall inclined outward. $\theta > 90° + \delta$. Surcharged.

$$N_1 = \tan^2\left(45° - \frac{\phi}{2}\right)\frac{w}{2}(H^2 - h^2),$$

$$W = \frac{w \tan\beta}{2}(H^2 - h^2) - E_2 \sin\delta_2,$$

where $E_2 = \left(\dfrac{\cos\phi}{n+1}\right)^2 \dfrac{wh^2}{2\cos\delta_2}$ and $n = \sqrt{\dfrac{\sin(\phi + \delta_2)\sin\phi}{\cos\delta_2}}$,

$$E = \sqrt{N_1^2 + W^2},$$

$$\tan\mu = \frac{\tan\beta - \dfrac{2\,E_2 \sin\delta_2}{w(H^2 - h^2)}}{\tan^2\left(45° - \dfrac{\phi}{2}\right)},$$

$$Q = E\cos(\mu - \beta);$$
$$T = E\sin(\mu - \beta).$$

NOTE. — The quantity $E_2 \sin\delta_2$ is the friction on the vertical side and reduces the weight acting upon the side AB.

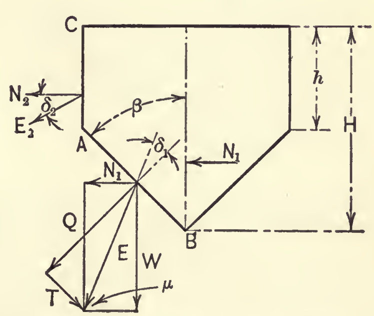


FIG. 316. Hopper bin, fill level.

$$E_1 = \cos\alpha \times \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}} \times \frac{w\,(h_1^2 - h^2)}{2}.$$

$$W = \frac{\sin\beta\cos(\alpha - \beta)}{\cos^2\beta\cos\alpha} \cdot \frac{w}{2}(H - h)^2 + wh\,(H - h)\tan\beta - E_2\sin\delta_2.$$

$$E_2 = \left(\frac{\cos\phi}{n + 1}\right)^2 \frac{wh^2}{2\cos\delta_2}, \quad \text{where} \quad n = \sqrt{\frac{\sin(\phi + \delta_2)\sin(\phi - \alpha)}{\cos\delta_2\cos\alpha}}.$$

$$E = \sqrt{E_1^2 + W^2 + 2\,E_1W\sin\alpha}.$$

E_1 is parallel to the angle of the fill.

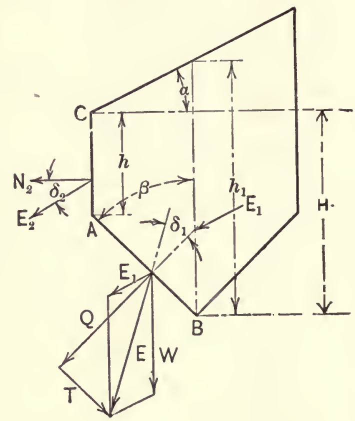


FIG. 317. Hopper bin, surcharged, unsymmetrical.

$$N_1 = \left(\frac{\cos\phi}{n + 1}\right)^2 \times \frac{w}{2}(h_1^2 - h^2),$$

where $n = \sqrt{\dfrac{\sin\phi\sin(\phi + \alpha)}{\cos\alpha}}.$

$$W = \frac{\sin\beta\cos(\alpha - \beta)}{\cos^2\beta\cos\alpha} \times \frac{w}{2}(H - h)^2 + wh\,(H - h)\tan\beta - E_2\sin\delta_2.$$

$$\tan\mu = \frac{W}{N_1}. \quad E = \sqrt{W^2 + N_1^2}.$$

$Q = E\cos(\mu - \beta)$ and

$T = E\sin(\mu - \beta)$.

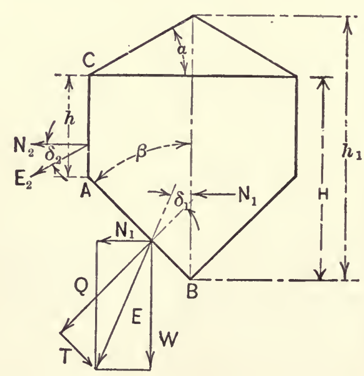


FIG. 318. Hopper bin, surcharged, symmetrical.

The following coefficients of friction are given by Mr. Dull in the article just referred to.

COEFFICIENTS OF FRICTION BETWEEN MATERIALS

Material.	ϕ Degrees.	ϕ^1 Degrees.	Weight per cu. ft., pounds.
Bituminous coal..................	35	18	50
Anthracite coal..................	27	16	52
Sand..............................	34	18	90
Ashes.............................	40	31	40

Where the material runs on concrete ϕ^1 may be assumed equal to ϕ.

ϕ = coefficient of friction of the material upon itself.

ϕ^1 = coefficient of friction of the material upon steel.

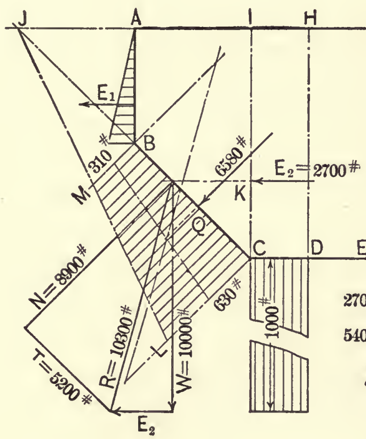


FIG. 319.

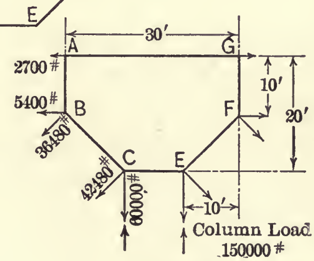


FIG. 320.

GRAPHICAL DETERMINATION OF FORCES ACTING ON BIN

Bin not Surcharged. — Assume Fig. 319 to be a bin for bituminous coal. The coal weighs 50 lbs. per cu. ft. $\phi = 35$ degrees. The natural sine of 35 degrees is 0.574.

The pressure against the side AB and the plane CI may be found as was done for retaining walls with Rankine's formula.

On AB,

$$E_1 = \frac{1}{2}wh^2\frac{1 - \sin\phi}{1 + \sin\phi} = \frac{50}{2} \times 10^2 \times \frac{1 - 0.574}{1 + 0.574} = 675 \text{ lbs.}$$

On IC,

$$E_2 = \frac{1}{2}wh^2\frac{1 - \sin\phi}{1 + \sin\phi} = \frac{50}{2} \times 20^2 \times \frac{0.426}{1.574} = 2700 \text{ lbs.}$$

The side BC is acted on by the weight of the superimposed coal and the horizontal pressure E_2. It is easier to consider the triangle of coal CJI, and then having found the pressure on CJ the pressure on the section CB may be scaled from the diagram. The weight of the triangle of coal CIJ is $W = \frac{1}{2} \times 20^2 \times 50 = 10{,}000$ lbs.

This resultant force acts at the center of gravity of this triangle or $\frac{1}{3}JI$ from CI. E_2 acts a distance $CK = \frac{CI}{3}$ from the bottom of the bin. Combining E_2 and W gives the resultant $R = 10{,}300$ lbs. This force resolved parallel and at right angles to the side BC gives $T = 5200$ lbs. and $N = 8900$ lbs. The normal pressure of 8900 lbs. is distributed along the line JC as the intercepts in the triangle LJC. Putting the area of the triangle $LJC = 8900$ lbs. and solving for the side LC gives $LC = 630$ lbs., which represents the pressure per square foot at the point C. The pressure at B is found, by scaling the intercept BM, to be 310 lbs.

The total pressure on the side BC is the area of the trapezoid $BCLM$ and equals 6580 lbs. The resultant of this pressure acts through the center of gravity of the trapezoid. The forces acting upon the points of the bin with supports every 12 ft. are given in Fig. 320, and are the force at $A = (\frac{675}{3}) \times 12 = 2700$ lbs. The horizontal force at $B = (\frac{2}{3}) \times 675 \times 12 = 5400$ lbs.

The force at B normal to the side BC must be estimated as

follows. The total normal pressure on BC is 6580 lbs. $CQ = 6.5$ ft. $CB = 14.1$ ft. The resultant at B is, therefore,

$$\frac{(6580 \times 6.5)}{14.1} = 3040 \text{ lbs.}$$

The total on 12 ft. of bin is $3040 \times 12 = 36{,}480$ lbs. The balance of the load represented by the trapezoid $MBCL$ acts at C and is $(6580 \times 12) - 36{,}480 = 42{,}480$ lbs.

The vertical pressure at C is $CD \times DH \times w \times 12 = 5 \times 20 \times 50 \times 12 = 60{,}000$ lbs.

The load on the columns due to the fill will be the total weight of the coal in the bin. The area of the bin section is 500 sq. ft., making the load in 12 ft. $500 \times 12 \times 50 = 300{,}000$ lbs.

The trussed bracing will be carried to the points A, B, C, E, F and G of the bin and taking these forces together with the total column load the several forces acting in the frame may be determined.

Surcharged Bins. — A bin similar to the last one but surcharged (Fig. 321) may have the forces acting upon it analyzed in much the same way. First find the pressure upon the side AB; this is done graphically in Fig. 322. The diagram gives $p = 8$ ft. and $y = 8.5$ ft., from which $E_1 = \frac{1}{2} \times w \times p \times y = \frac{1}{2} \times 50 \times 8 \times 8.5 = 1700$ lbs. The normal pressure is 1620 lbs.

Now finding the pressure upon IH, the axis of the bin, by using the graphical method and assuming $\phi^1 = 0$, we have $p = y = 16$ ft.

$$E_2 = 0.5 \times w \times p \times y = 0.5 \times 50 \times 16 \times 16 = 6400 \text{ lbs.}$$

The center of gravity of the triangle IJH is at M and the weight of a prism of coal 1 ft. high with the base IJH is

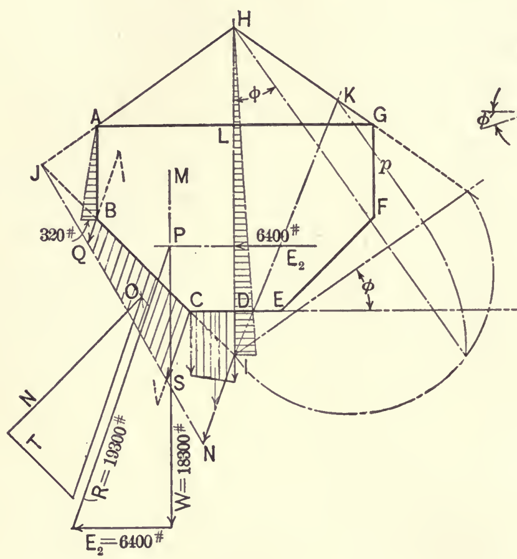
$$\frac{(35.4 \times 20.7 \times 50)}{2} = 18{,}300 \text{ lbs.}$$

The resultant of E_2 and W is $R = 19{,}300$ lbs. This is assumed as varying uniformly along the face JI. The pressure at I is

$$x = \frac{(19{,}300 \times 2)}{26.5} = 1450 \text{ lbs.} = IN.$$

The total pressure on the side BC is that represented by the trapezoid $QBCS$ whose altitude scales 12.8 ft.; the area then is

$$\frac{410 + 1100}{2} \times 12.8 = 9660 \text{ lbs.}$$



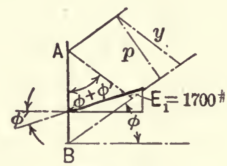


FIG. 322.

FIG. 321.

This acts through the center of gravity O of the trapezoid which has been located graphically.

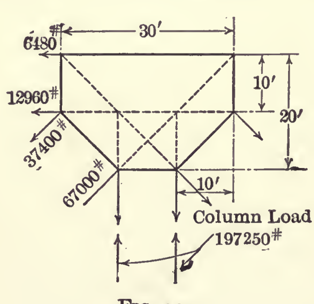


FIG. 323.

The normal pressure on the side BC may now be found by laying off 9660 lbs. through the point O and resolving it into its components parallel with and at right angles to BC. The load upon one-half of the base CD is 7210 lbs. The forces, due to the fill, acting at the points of the bin may be estimated as in the preceding problem and are given in Fig. 323. The distance between supports has been taken at 12 ft. The columns

will carry the total weight of the coal in the bin and the weight of the bin. The stresses in the internal framework of the bin may be calculated by treating these external loadings as is done for any other truss. The stress diagram may be drawn or the forces calculated by any of the methods previously given.

Suspension Bunkers

If the side plates of the ordinary hopper bunker are permitted to bulge but slightly they must be supported at frequent intervals by beams or other structural shapes. This added weight is avoided in the use of suspension bunkers. These bunkers are patented and are designed with the idea that the sides resist tension only. To fulfil this condition theoretically the sides would assume a shape peculiar to each possible loading. This could be illustrated by a model bin whose sides were a purely tension piece like muslin or duck. However, an actual bin differs greatly from the above illustration and any deflection due to bending in the sides when the bin is not loaded as theoretically assumed is ordinarily not sufficient to cause trouble. The lined bins will be stiffer than the unlined ones.

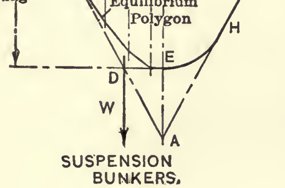


FIG. 324.

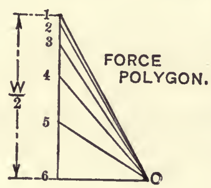


FIG. 325.

The theoretical curve of a bin for any assumed loading may be determined as follows. In Fig. 324, given the desired span and sag, the load in the left-hand side of the bin will not vary much from a triangle ABC. The weight of the material in the portion ABC of the bin will act through the center of gravity of this triangle ABC and lies in the vertical line DG or W, GF being $\frac{BF}{3}$.

Considering the side of the bin *BE* the force acting at *B* and that at *E* must hold *W* in equilibrium; hence they must pass through a common point *D*.

The line of action of *W* is known as is also the line of action of the force acting at *E* which must be horizontal if the bin is symmetrical and symmetrically loaded, both of which conditions have been assumed. If then a horizontal line is drawn through *E* it cuts *W* in *D* and the line of action of the force acting at *B* must lie in the line *BD* passing through the point *D*. The loading being assumed as varying as the intercepts of a triangle we may represent this loading by the triangle *FBC*, and having divided it into sections of equal widths the weight of each section will be proportional to its center ordinate; these have been shown dotted.

These lengths from 1–2 to 5–6 have been laid off on the load line of the force polygon, Fig. 325. The lines of action of the forces acting at *B* and *E* being known the pole *O* is readily found by drawing *O*1 parallel to *DB* and *O*6 parallel to *DE*. The other strings may now be drawn in the force polygon. If we complete the equilibrium polygon in Fig. 324 the broken-line side will approximate the theoretical side of the bunker. On the right-hand side the line of the bunker has been drawn as it is commonly made in practice. The portion *HI* is straight while the lower part *HE* is an arc of a circle. The sides of this bin approximate an equilateral triangle. The sag is about six-tenths of the span.

For a bin which is nearly an equilateral triangle the following figures will be approximately true.

Area of the bin section, fill level,	$= 0.40\ S^2$.
Area of the bin section, triangular surcharge,	$= 0.57\ S^2$.
Length of plate, no allowance for laps,	$= 19.8\ S$ (inches).
Force in plate per foot of length at *B* or *I*	$= 0.60\ W$ (pounds).
Force in plate per foot of length at *E*	$= 0.30\ W$ (pounds).

Here $S =$ the span in feet.

$W =$ the load in the bin for one foot of its length, pounds.

In the case of the suspension bunker with vertical sides above the points of suspension, B and I of the bin, Fig. 326, the cross section of the bunker will be increased by the area $BCIJKL$ equal to $S \times h$, and the load will be increased proportionally. In this case the weight of the material in one-half of the bin will act through the center of gravity of the section $KLBDEK$. This will be a greater distance from the center line EC than was the case in the preceding problem, and will make the bin somewhat wider below BI than at corresponding points in the former bin.

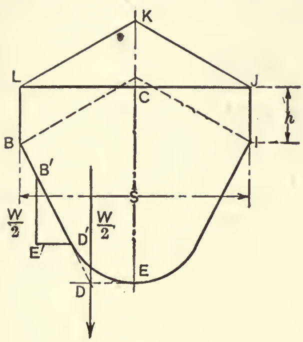


Fig. 326.

In either problem if $\frac{W}{2}$ is known the forces acting at B and E may be found by drawing the triangle $B^1E^1D^1$ at the side of the bin as in Fig. 326.

One form of Suspension Bunker with column supports for boiler rooms is shown in Fig. 327. This bunker is intended to hold 6600 lbs. per ft. of length. Assume coal as weighing 50 lbs. per cu. ft., and the sag of the bunker as 60 per cent of the span. The bunker will be assumed as carrying the required load when surcharged, and the angle of repose of the coal will be taken 30 degrees. Determine the bin area by using the formula, $A = 0.57\,S^2$. The required area is $\frac{6600}{50} = 132$ sq. ft. From this $132 = 0.57\,S^2$ and $S = 15.3$ ft. If the bin is made 16 ft. wide the sag will be approximately 10 ft. Spacing the columns 20 ft. center to center makes the load of coal upon each column $\frac{(6600 \times 20)}{2} = 66{,}000$ lbs. Adding 4000 lbs. to this to cover the weight of metal, exclusive of columns and under bracing, makes a total of 70,000 lbs. at each column. This load laid off in Fig. 328 gives the stress in the side of the bunker per foot of

length as $\frac{80,000}{20} = 4000$ lbs. The unit stress per square inch in a plate $\frac{1}{4}$ in. thick is $\frac{4000}{(0.25 \times 12)} = 1330$ lbs.

The main columns are subjected to a bending moment of

$$80,000\,[(16 - 6) - 6] = 320,000 \text{ in. lbs.}$$

FIG. 327.

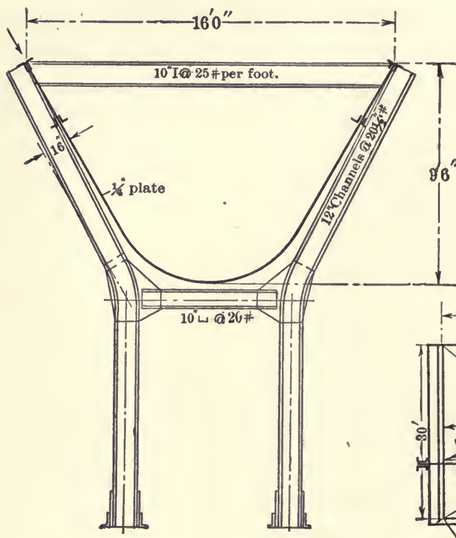


FIG. 328.

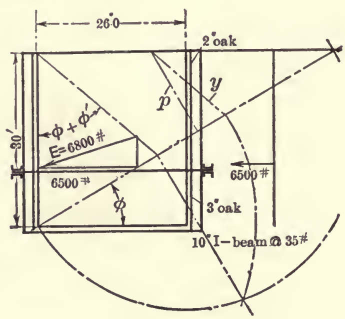


FIG. 329.

Trying two 12-in. channels weighing 20.5 lbs. per ft., we have

Fiber stress due to the bending moment is $\frac{320,000}{(2 \times 21.6)} = 7,400$ lbs. per sq. in.

Fiber stress due to direct loading is $\frac{80,000}{(2 \times 6.02)} = 6,650$ lbs. per sq. in.

Combined stress $= 14,050$ lbs. per sq. in.

The direct load upon the column being 6650 lbs. per sq. in. the

radius of gyration for an assumed length of 11 ft. may be found from the straight-line formula,

$$f = 16{,}000 - 70\left(\frac{l}{r}\right) = 16{,}000 - 70\left(\frac{132}{r}\right),$$

or
$$r = \frac{70 \times 132}{(16{,}000 - 6650)} = 0.99.$$

Two 12-inch channels are to be used back to back. The radius of gyration of a 12-inch channel referred to an axis at the back of the web is

$$r = \sqrt{\frac{I + Ah^2}{A}} = \sqrt{\frac{3.90 + (6.02 \times 0.70^2)}{6.02}} = 1.06.$$

It therefore follows that two 12-in. channels at 20.5 lbs. per ft. either riveted back to back or separated any distance will carry the load of 80,000 lbs.

The horizontal strut under the bin carries a load of 40,000 lbs. Its length is 84 ins. Trying a 10-in. channel at 20 lbs. per ft., whose radius of gyration about an axis parallel to the web is 0.70, we find $\frac{l}{r} = \frac{84}{0.70} = 120$. The allowable stress is $f = 16{,}000 - (70 \times 120) = 7600$ lbs. per sq. in. The required area then is $\frac{40{,}000}{7600} = 5.26$ sq. ins. The combined bending and direct stress on this piece will exceed that allowed by paragraph 93 of the specifications. Considering the design and character of the connections to the columns it will probably be ample; if preferred a four angle latticed strut will meet the requirements but its fabrication would cost more. The angles could be $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$. The strut across the top of the bin carries the conveyor, its truss and the conveyor loading and in addition to these the direct compression due to the bin loading. Assuming the load equivalent to a central load of 3500 lbs. the bending is $M = \frac{W \cdot l}{4} = 3500 \times 16 \times \frac{12}{4} = 168{,}000$ in. lbs. Trying a 10-

in. I beam at 25 lbs. per ft., with an $\frac{I}{e}$ value of 24.6 and an area of 7.34 sq. ins., we have

Fiber stress due to bending moment, $\frac{168,000}{24.6} = 6800$ lbs. per sq. in.

Fiber stress due to direct loading, $\frac{40,000}{7.34} = 5450$ lbs. per sq. in. The combined fiber stress is $6800 + 5450 = 12,250$ lbs. per sq. in. Considering that this beam is held laterally at its center by the conveyor truss this fiber stress would seem permissible. The braces occur every 10 ft.

The side of the bin between columns is carried by a girder. The total load is 80,000 lbs. or 4000 lbs. per ft. The bending moment then is

$$M = \frac{Wl}{8} = 80,000 \times 20 \times \frac{12}{8} = 2,400,000 \text{ in. lbs.}$$

Assuming the distance back to back of flange angles as 3 ft., the distance between the centers of gravity of the two flanges will be about 34 ins.

$$M = A \times f \times h = 2,400,000 = A \times 16,000 \times 34$$

or

$$A = 4.42 \text{ sq. ins.}$$

Two $5 \times 3\frac{1}{2} \times \frac{5}{16}$-in. angles having a gross area of 5.12 sq. ins. and a net section, allowing for one rivet, of 4.58 sq. ins. furnish just a little more than the area required. No allowance has here been made for the web acting as flange. This gives additional security.

Figure 329 shows a bin designed for bituminous coal. The weight of the coal was taken 50 lbs. per cu. ft., the natural slope was taken 30 degrees, while the angle of friction between the sides and the coal was assumed as 18 degrees. The diagram gives $p = 16$ ft. and $y = 17$ ft.

$$E = \tfrac{1}{2} \times w \cdot p \cdot y = \tfrac{1}{2} \times 50 \times 16 \times 17 = 6800 \text{ lbs.}$$

The horizontal component of this force is 6500 lbs. The hori-

zontal pressure per square foot on the sides of the bin at the bottom is $x = 6500 \times \frac{2}{30} = 433$ lbs.

The sides are supported by I beams spaced 6 ft. center to center and horizontal rods are run across the bin at 6-ft. intervals and placed 10 ft. above the bottom of the bin. The stresses were determined graphically as outlined for fixed and continuous beams. The force in the tie rods was 28,800 lbs. and the maximum bending moment on the I beams was found to be 570,000 in. lbs. Bolts $1\frac{7}{8}$ ins. in diameter, having an area of 2.05 sq. ins. at the root of the threads, would have a unit stress of $\frac{28,800}{2.05} =$ 14,000 lbs. per sq. in. and are satisfactory.

Allowing 20,000 lbs. per sq. in. fiber stress in the beam would require an $\frac{I}{e}$ value of $\frac{I}{e} = \frac{M}{f} = \frac{570,000}{20,000} = 28.5$. This would require a 10-in. I beam at 35 lbs. per ft. or a 12-in. I beam at 31.5 lbs. per ft. This bin was tied across the top by the roof framing and was braced transversely at intervals against wind pressure.

Bin Design

Figure 330 illustrates a bin for coal storage over gas producers. The upper portion of the bin was a square of 13 ft. side. The lower part was a curve with a 9-ft. sag. The coal is assumed as weighing 50 lbs. per cu. ft. and its angle of repose has been taken 35 degrees.

The angle of friction between the coal and the sides of the bin has been taken 18 degrees. Owing to the large amount of coal carried in the square portion of the bin the loading cannot be represented as a triangle; therefore an approximate outline of the bin has been assumed in Fig. 330 and the actual curve estimated from this. The left side of the bin has been divided into four strips of equal widths; the weight in each strip per foot of length of bin will be proportional to the middle ordinate, shown in full lines for each section. In the force polygon, Fig. 331, the forces may be represented by these middle ordinates or

by a constant percentage of them. In the figure the scale has been taken as one-fourth of these lines. The forces A^1B^1, B^1C^1, etc., have been laid off on the right-hand side of the bin equal to and symmetrically placed with those on the left-hand side. These forces have been laid off on the load line of the force polygon, Fig. 331, and the vector diagram, on its left-hand side, has been

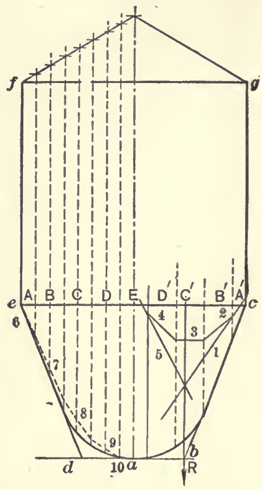


FIG. 330.

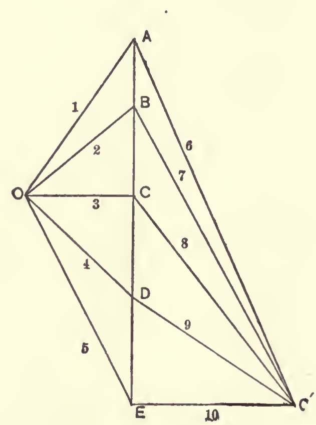


FIG. 331.

used to locate the center of gravity of the load in the right-hand side of the bin. The center of gravity is found in the intersection of the lines 1 and 5 in Fig. 330. R, the total weight of the coal on this side of the bin, acts through this point of intersection. The force in the extreme low point of the bin will be horizontal, and this together with the force in the bin side acting through c must hold R in equilibrium, hence these three forces must pass through a common point b; this gives the direction of bc, which is the theoretical direction of the side of the bin at the point c.

The lines for the other side of the bin have been made similar to those for the right-hand side. The vector polygon drawn on the right-hand side of the load line, Fig. 331, has been used in

drawing the dotted lines in Fig. 330, thus showing the agreement between the actual and the theoretical lines of the bin.

Before the stresses can be read in Fig. 331, the scale must be found.

The mean ordinate in one side of the bin is 21 ft. The bin being 13 ft. wide makes the volume of the bin section for a length of 1 ft. = 21 × 13 = 273 cu. ft. This gives 273 × 50 = 13,650 lbs. of coal per ft. of bin. The load line AE = 6825 lbs. This makes the scale $\frac{6825}{4}$ = 1700 lbs. per in. (approximately).

The force AO^1 = 4.3 × 1700 = 7300 lbs.

The force EO^1 = 1.6 × 1700 = 2720 lbs.

The bin plates will be made $\frac{5}{16}$ in. in thickness to allow for wear and deterioration, and will have 3 × $\frac{1}{2}$-in. wearing strips spaced about 12 ins. center to center on the curved portion of the bin sheets and running the length of the bin. The distance center to center of columns will be assumed as 18 ft. Transverse braces will be placed at the columns and every 9 ft.

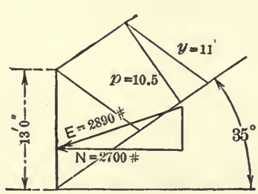


Fig. 332.

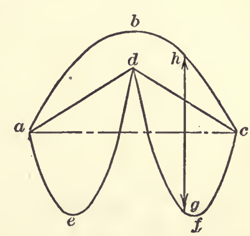


Fig. 333.

The pressure against the vertical sides of the bin is determined in Fig. 332.

$$E = \tfrac{1}{2} \times w \cdot p \cdot y = \tfrac{1}{2} \times 50 \times 10.5 \times 11 = 2890 \text{ lbs.}$$

It should be noted that for so narrow a bin this does not give the accurate pressure but it is the limiting pressure to which this force on the side of the bin could extend. A closer approximation might have been made by assuming a bin, say, 16 ft. high instead of 13 ft. but with a horizontal fill. The horizontal

component of this force of 2890 lbs. is 2700 lbs. Two-thirds of this, or 1800 lbs., act at each of the lower points *e* and *c*, while the other third, 900 lbs., acts at each of the upper points *f* and *g*. These forces act in the opposite direction to the pull from the suspended portion of the bin.

Had the bin been level, substitution in the formula for the pressure on the side of the bin would have given 1060 lbs. or 700 lbs. at the points *e* and *c*; the net pressure here then would be $2700 - 700 = 2000$ lbs. per ft. of length of bin.

The compression in the lower member of the brace due to the total load between the columns is $2000 \times 18 = 36{,}000$ lbs. If the value of $\frac{l}{r}$ in these braces is to be limited to 120 then $r = 13 \times \frac{12}{120} = 1.3$, and this requires two $6 \times 3\frac{1}{2}$-in. angles having their long legs placed back to back. Before the weight is selected the possibility of additional loading from wind should be considered.

In exposed places this might reach 30 lbs. per sq. ft. of vertical projection. Along the bin at the point *e* it would equal $(6.5 + 9) \times 30 = 465$. This makes the total load at the column $(465 \times 18) + 36{,}000 = 44{,}370$ lbs.

The unit stress on the angle is

$$f = 16{,}000 - \left(70 \times \frac{l}{r}\right) = 7600 \text{ lbs. per sq. in.}$$

The area of the angles is $\frac{44{,}370}{7600} = 5.83$ sq. ins.

The minimum angles should therefore prove ample, unless larger angles are used to provide for corrosion and possible injury from the coal passing over them.

The channel at *e* is acted upon by the wind and the pull from the side of the bin. Considering first the wind load, it will act as a uniform load on the span of 18 ft. Its deflection under this load will put a central load on the channel at *c*. If *P* is the load transferred to the channel at *c* from the channel at *e*, the de-

flection due to the wind load W is $\frac{5}{384} \times \left(\frac{W \cdot l^3}{E \cdot I}\right)$, while the deflection due to the central load is $\frac{1}{48} \times \left(\frac{P \cdot l^3}{E \cdot I}\right)$. Since the deflections of the two channels are equal,

$$\frac{5}{384}\frac{Wl^3}{EI} - \frac{1}{48}\frac{Pl^3}{EI} = \frac{1}{48}\frac{Pl^3}{EI}$$

and $$\frac{5W}{384} = \frac{P}{24} \quad \text{or} \quad P = \frac{120}{384}W = 0.31\,W.$$

The maximum bending moment may now be found by adding algebraically the bending moments due to these several loads. Both sides of the bin should be tried although here the maximum bending occurs on the windward side. The moment due to the wind load of 465 lbs. per ft. is

$$M = W \cdot \frac{l}{8} = 465 \times 18 \times 18 \times \tfrac{12}{8} = 226{,}000 \text{ in. lbs.}$$

The bending moment due to the central load is

$$M = P \cdot \frac{l}{4} = 0.31 \times 465 \times 18 \times 18 \times \tfrac{12}{4} = 140{,}000 \text{ in. lbs.}$$

The bending moment due to the uniform load of 2000 lbs. per ft. on the 9-ft. span, that is, between the braces, is

$$M = W \cdot \frac{l}{8} = 2000 \times 9 \times 9 \times \tfrac{12}{8} = 243{,}000 \text{ in. lbs.}$$

The resultant bending moments are given by Fig. 333, and are the intercepts between the curve *abc* and the lower curves *aedfc*. The curve *abc* gives the moments due to the wind load acting on the 18-ft. span and measured from the base line *ac*. The moments due to the central load on the 18-ft. span are then deducted by laying off the triangle *adc*. The bending moments due to the uniform load on the 9-ft. span are then plotted below *ad* and *dc*, thus giving the algebraic sum of these bending moments. The maximum moment scales 340,000 in. lbs. and would require a channel having an $\frac{I}{e}$ value of $\frac{I}{e} = \frac{M}{f} = \frac{340{,}000}{16{,}000} = 21.3$,

which calls for a 12-in. channel at 20.5 lbs. per ft. The fiber stress in this channel would then be $\frac{340,000}{21.4} = 15,900$ lbs. per sq. in.

In a similar way the maximum bending on the upper channels is found to be approximately 139,000 in. lbs. and requires a section modulus of $\frac{139,000}{16,000} = 8.7$.

The sides act as girders to transfer the loads to the columns. The uniform load due to fill is 13,650 lbs. per ft., and assuming the weight of the bin as approximately 1000 lbs. per ft. makes the total load per foot 14,650 lbs. The load per foot on each girder is $\frac{14,560}{2} = 7325$ lbs.

The bending moment on the girder is

$$M = \frac{W \cdot l}{8} = 7325 \times 18 \times \frac{18}{8} = 296,660 \text{ ft. lbs.}$$

If the girder is assumed as 13 ft. deep the flange force is

$$\frac{296,660}{13} = 22,800 \text{ lbs.}$$

When it is considered that in the lower channel the splice plate to which the curved portion of the bin is attached assists in carrying this stress it will be seen that this force of 22,800 lbs. will add very little to the fiber stress in the 12-in. channel.

If 10-in. channels at 15 lbs. per ft. are tried for the upper channels the fiber stress due to the bending is

$$f = \frac{M \cdot e}{I} = \frac{139,000}{13.4} = 10,400 \text{ lbs. per sq. in.}$$

The fiber stress due to the flange force of 22,800 lbs. is

$$\frac{22,800}{4.46} = 5100.$$

The combined fiber stress is $10,400 + 5100 = 15,500$ lbs. per sq. in.

The stresses may be increased by wind acting on the ends of the bin or building and also by thrust from the traveling crane, or the forces may be provided for by additional bracing.

The stiffeners on the sides *ef* and *cq* of the bin carry a triangular load of 2700 lbs. per ft. of length of the bin; hence if the stiffeners are spaced 2 ft. 3 ins. center to center, their total load is 2700 $\times$ 2.25 = 6070 lbs. The moment on a beam due to such a load is $M = 0.128 \times P \cdot l$; $M = 0.128 \times 6070 \times 13 \times 12 = 121{,}000$ in. lbs.

The section modulus required is

$$\frac{I}{e} = \frac{M}{f} = \frac{121{,}000}{16{,}000} = 7.6,$$

calling for 8-in. channels at $11\frac{1}{4}$ lbs. per ft.

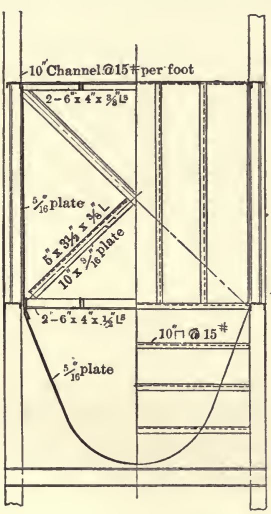


FIG. 334.

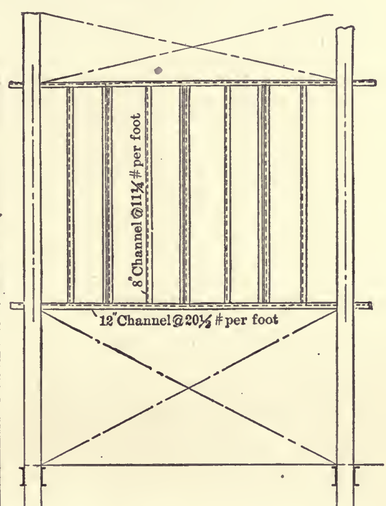


FIG. 335.

The maximum stress in the $\frac{5}{16}$-in. plates along the side of the bin will occur at *e*. Since the load of 2700 lbs. is assumed as distributed as a triangle of pressure, it follows that if p_e is the pressure per square foot at *e*, then $2700 = \frac{13\,p_e}{2}$ and $p_e = 415$ lbs. per sq. ft.

The pressure per square inch is $\frac{415}{144} = 2.88$ lbs.

The maximum fiber stress is $f_m = \frac{1}{2}\phi\frac{a^2b^2}{a^2+b^2}\times\frac{p}{t^2}$,

$$f_m = \frac{1}{2}\times 1.0\times\frac{27^2\times 156^2}{27^2+156^2}\times\frac{2.88}{0.31^2} = 10{,}600 \text{ lbs. per sq. in.}$$

A cross section and end view of this bin is shown in Fig. 334, while the elevation is given in Fig. 335. The diagonal bracing shown in the half section has not been drawn in the elevation. It would be located at the columns and at the center line midway between them. In general only the bin proper has been shown, as the design of the wind bracing and columns would have been complicated by the character of the building and it was not desired to consider this design here.

CHAPTER XIX

SHOP FLOORS

The purpose of a floor is primarily to carry such loads as men, machines and materials. In addition they should preferably be poor conductors of heat as this adds greatly to the comfort of men standing on them.

They should also be durable, resisting wear from the traveling of men and the transportation of goods upon them. They should furthermore be impervious to moisture. In special cases additional qualities may be desirable; thus in many floors it is essential that they shall be nearly dustless, while in other cases a somewhat soft surface may be desired.

Earth and **cinder** make the cheapest floor. Owing to dust they are only adapted to places like forge-shops, where hot metals placed upon them or heavy pieces dropped on them would only injure other floors. The soil below the floor level should be drained if damp and the floor clay or cinder then placed in layers upon a rolled bed; each layer should be thoroughly tamped or rolled. Where the moisture is excessive this floor may be made impervious by a layer of tar concrete placed from 1.5 to 2 ft. below the floor surface.

Cement Concrete Floors. — In a floor of this type it is necessary to have a substantial foundation; its depth will vary with the character of the soil from 1 to 2 ft. The soil having been excavated to the desired depth the surface should be well tamped or rolled. A layer of broken stone, cinder or gravel 6 ins. to 10 ins. thick is placed upon this and thoroughly tamped. In some floors a layer of from 3 ins. to 6 ins. of finely crushed stone is placed upon this and well rammed, this being in turn covered with 2 ins. to 4 ins. of concrete upon which a wearing coat of

cement mortar from $\frac{1}{2}$ in. to 1 in. thick is placed. In other designs the concrete is placed upon the first foundation layer. This concrete is 3 ins. to 4 ins. thick and upon it a wearing coat of 1 in. of cement mortar is placed.

The concrete is usually a mixture of 1 part cement, 2 to 3 parts of sand, and 5 to 6 parts of stone or gravel. The cement-mortar is 1 part cement to 2 parts of sand or may be richer in cement. Where the soil affords a sufficiently good foundation 5 ins. to 6 ins. of concrete placed immediately upon it may prove satisfactory.

Tar Concrete Floors. — Where a wooden wearing surface is desired such surface may be laid upon the concrete base just

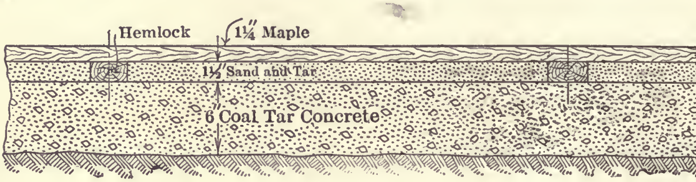


FIG. 336.

described, in which case the surface of the concrete under the wood should be coated with coal tar or asphalt; otherwise the wood deteriorates from dry rot.

Another plan (Fig. 336) makes the foundation of a coal tar or an asphalt concrete. After the top soil is removed the surface is leveled and rolled and a layer of from 4 ins. to 6 ins. of broken stone or gravel thoroughly mixed with tar is placed and well rolled. A 1-in. layer of sand thoroughly saturated with tar is put on this concrete and carefully rolled. While the tar is still hot hemlock planking is put on the sand and pushed into place. The wearing surface, which is preferably maple strips, tongued and grooved, is placed at right angles to and upon this hemlock. In some cases the foundation is made like that described for a concrete floor. The design may also be modified by using hemlock sleepers instead of the planking.

In this case the hemlock strips are placed on about 3-ft. to 4-ft. centers, being imbedded in the sand and tar, and the tongued and grooved maple flooring is then nailed to the sleepers.

Wooden Block Floors. — Wooden blocks set on end make an excellent floor. These blocks are about 4 ins. to 5 ins. thick and may be either cedar, maple, beech, oak or pine. When not treated with a preservative they should be set in coal tar. Also when not treated the blocks should be set with spaces between them to allow for swelling due to any moisture they may absorb. These spaces between the blocks should be filled with tar and sand. These blocks may be placed upon the tar-concrete foundation just described; see Fig. 336.

Asphalt Floors. — Asphalt makes a satisfactory floor in many places, but it is not suited to machine shops or places where much oil is likely to reach it. Asphalt gives a soft surface, is waterproof, dustless and a poor conductor of heat. It therefore makes a very comfortable floor for the men to stand upon. It is easily dented but the dents will gradually smooth out owing to the tendency of the asphalt to flow under pressure.

The foundation is similar to that for a concrete wearing surface. The asphalt is generally made an inch thick. A wash of molten tar and asphalt should be put on the surface of the concrete before placing the asphalt. Where asphalt is placed upon a concrete slab it forms an additional dead load and should not be assumed as resisting any of the compression in the slab.

Brick Floors. — Brick floors are specially adapted to places where a floor is liable to local injury, as no other floor is so readily repaired, it being possible to remove and replace a few bricks without disturbing a large section of the floor. It has therefore found favor in railroad buildings such as round houses. For brick paving the foundation consists of from 4 to 6 ins. of cinder placed in layers and tamped or rolled upon the excavated surface which had also been previously compacted. A layer of sand from 1 to 2 ins. thick is placed and rolled upon the cinder and then the brick are laid upon the sand. The bricks should be

hard, homogeneous and impervious to water. They should be set on edge and joints should be broken by a lap of at least 3 ins. None but whole bricks should be used except when necessary as fillers. After laying, sand should be thoroughly worked into the spaces between the bricks.

To make this floor waterproof the brick may be grouted with tar and pitch, after which the sand may be worked into the spaces as before.

Wooden Floors. — These are commonly made by embedding stringers, timber strips, in a foundation similar to those described for concrete floors and then nailing the wearing surface of the floor on the stringers.

One wooden floor had a foundation of 8 ins. of cinders, the stringers were spaced 3 ft. centers and 3-in. planking was placed upon these.

The stringers and under surface of the planks were coated with lime to preserve them. Present practice prefers tar to lime to preserve wood. In other cases the foundation is a layer of concrete in which the stringers are embedded; the under surface of the wood should be coated with tar as before.

Ground Floors in General. — Where much water is used on floors they should be given sufficient pitch to properly drain the water and suitable drains or gutters must be provided.

Where heavy trucking is done tracks must be laid. These may be either tracks sunk in the floor or flat iron plates placed in the floor level with the wearing surface of the floor. The former method is objectionable on account of breaking into the surface of the floor. The flat plates should be corrugated to prevent their becoming too smooth.

Upper Floors

Floors above ground level usually consist of two parts, one affording the necessary strength to sustain the floor loads, the other supplying the wearing face. The first part may be wood, steel, concrete, brick or tile, or may be a combination of some

of these. The wearing surface may be any of the wearing materials used upon the ground floors.

Wooden Floors. — A wooden floor is commonly carried upon wooden joists or steel floor beams. The depth of the floor timbers furnishing the strength will vary with the distance

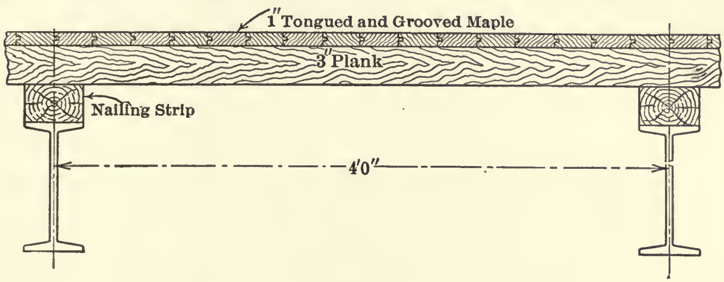


FIG. 337.

center to center of the joists or floor beams and with the floor loads and will range from 2 or 3-in. planking as in Fig. 337 to 2 × 8-in. timbers placed on edge and nailed together as in Fig. 338. These planks should be continuous over at least 2 supports. The joints should be staggered, that is not broken continuously

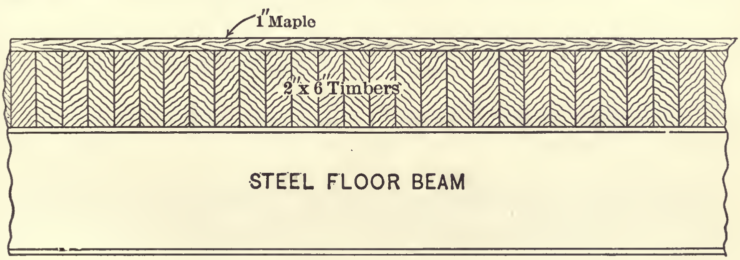


FIG. 338.

on the same line. The wearing surface will usually be 1 to 1.5-in. maple or other hardwood flooring. Frequently 3 thicknesses of rosin-sized paper are placed between the wearing and the supporting wood. Where the timbers are on edge a layer ($\frac{1}{2}$ in.) of tar cement (tar and sand) may be placed between the two sections.

Steel Floors. — The forms shown in Figs. 339 and 340 run to considerable weight and are suited to heavy loads. The fill usually consists of a cinder concrete which need not be richer than 1 part cement, 2 parts sand and 6 parts cinder. This is carried

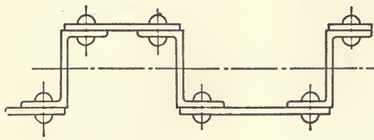


FIG. 339.

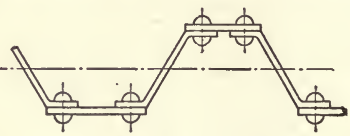


FIG. 340.

a few inches above the top of the trough section. Sleepers are embedded in this fill and the wearing boards nailed to them.

There are other trough sections made of much lighter metal, gauges No. 16 to No. 24. These sections are shown in Figs. 341 to 343. These are made in both black and galvanized steel. The exposed side requires painting.

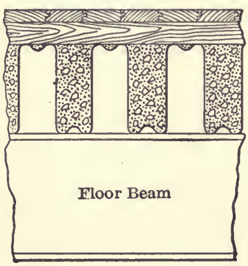


FIG. 341.

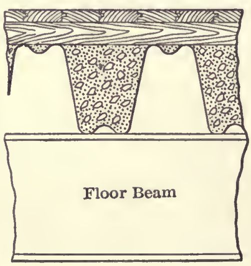


FIG. 342.

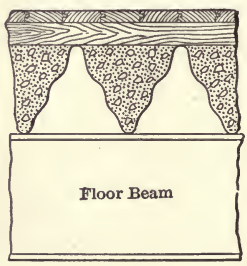


FIG. 343.

Brick Arch. — Another heavy floor construction is shown in Fig. 344. It consists of a common brick arch having a rise of preferably not less than one-eighth of the span.

The tie rods should be placed at frequent intervals along the beams, especially along the outside beams, to prevent too great stress in the beams from lateral pressure. The horizontal thrust or pressure in pounds per lineal foot of arch is given in the Pencoyd handbook as

$$P = \frac{1.5\,WS^2}{R}.$$

W = load in pounds per square foot on the floor.
S = span of arch in feet.
R = rise of arch in inches.

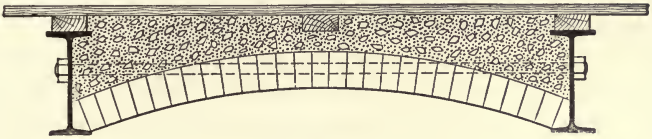


FIG. 344.

The fill above the arch may be a lean cinder concrete, say, 1–2–6.

The wearing face may be any of those previously mentioned but is commonly maple.

FIG. 345.

The arch instead of being made of common brick may be any of the several types of hollow clay tile (Fig. 345), or may be a concrete arch placed on corrugated steel, expanded metal or

FIG. 346.

wire cloth; see Fig. 346. These floors are made suited to lighter floor loads than the first-described arch.

The principal purpose of the corrugated steel, expanded metal or wire cloth is to carry the concrete during setting. The

strength of these floors is due largely to the concrete. The thrust may be calculated by the formula given for brick arches.

There are several forms of hollow-tile arches. The tile are made to give a flat arch when in place. The fill above the tile is a lean cinder concrete, Fig. 345.

Reinforced-concrete Floors (Fig. 347). — These floors may be concrete slabs placed across steel beams as shown in the figure, or across reinforced-concrete beams, in which case the slab forms the flange of the T beam, the rectangular beam then becoming its stem. The design of these reinforced-concrete slabs and T beams has been fully considered in Chapter XIV.

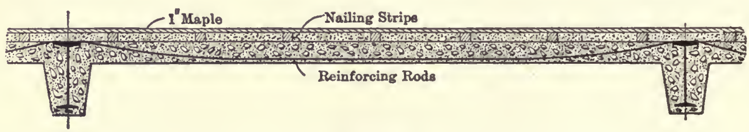


FIG. 347.

The reinforcement may be either plain steel rods, twisted or deformed rods or any of the woven wires or expanded metals intended for such reinforcement. When rods are used some reinforcing steel should be placed at right angles to the main reinforcing rods. These slabs may have any of the previously described wearing surfaces.

Concrete Wearing Surfaces. — This surface should be made to be as nearly waterproof as possible. The stone should not exceed $\frac{1}{2}$ in.; it should be hard, either trap, granite, hard blue limestone or similar rock; no soft rocks are suitable. This concrete is frequently 1 part cement, 1 part sand, and 1 part stone; this should be laid upon the slab before the cement of the slab has fully set. Where the slab has fully set its face should be thoroughly cleaned and then covered with a neat cement slush upon which the wearing surface is then laid.

Floors may be made practically waterproof by the thorough trowelling of the surface. It may also be waterproofed by coating it with pure linseed oil thinned with turpentine or naphtha.

Some concrete floors are painted but this, being merely a surface effect, soon wears off.

Iron Floors. — Foundry floors, steel-mill charging floors and rolling-mill floors are frequently either steel or iron. These may be cast-iron plates, steel plates and in the case of steel-mill charging floors rolled-steel channels placed side by side upon their flange edges.

Where a smooth surface is not one of the requirements of the floor, corrugated plates give a better footing. Several types of corrugated plates and of plates with holes in them are made by the various manufacturers of structural steel.

Steps. — Concrete steps, when made of hard stone, wear well and give satisfaction. Corrugated-iron steps may be used and furnished with one of the patented edges to prevent slipping.

CHAPTER XX

WALLS AND ROOFS

THE ordinary forms of walls include: (1) Wooden walls or sides. (2) Corrugated-steel sides. (3) Brick, or brick with a concrete face. (4) Stone. (5) Solid-concrete walls in steel framing. (6) Hollow-concrete blocks. (7) Reinforced concrete with expanded metal or other reinforcement.

1. **Wooden walls or sides** are serviceable where the nature of the work carried on in the building requires considerable additional ventilation during part of the year, as the wooden sides may be readily removed. The wooden sides may be attached to either timber, metal or stone construction.

2. **Corrugated-steel Sides.** — These are more substantial than the wooden ones. The common sizes of corrugated-steel sheets are given under roof coverings and the comments made there apply in a general way to the sides. The metal for the sides is ordinarily a little lighter than that on the roof, being commonly No. 20 or No. 22 gauge.

The sheets may be secured to girts by clips or metal straps or may be nailed to studding. Where the building must be kept warm a lining of still lighter corrugated steel may be placed inside the first sheets, an air space being left between the two.

3. **Brick.** — The required thickness of brick walls for any community is given by its "Building Code." An average statement of such requirements for a ten-story building would give about the following wall thicknesses.

These sizes are approximate, being intended to be multiples of brick dimensions. They ordinarily apply to buildings not exceeding 120 ft. in length and 25 ft. in width. Where the walls exceed 120 ft. between lateral supports or the distance between

side walls is more than 25 ft. the thicknesses are commonly increased 4 ins. for each additional 120 ft. or part thereof in the length or each additional 25 ft. or part thereof in the width of the building.

Numbers of stories.	Estimated height in feet to roof.	Thickness of wall in inches.
10	14	12–16
9 and 8	40	16–20
8, 7 and 6	80	20–24
5, 4 and 3	120	24–28
2 and 1	150	28–32

Brick foundation walls or footings usually exceed the walls immediately upon them by at least 4 ins. The pressure per square inch upon the brick should not exceed 125 to 150 lbs.

Where heavy concentrated loads, such as trusses or crane loads, are transferred through walls it is frequently more economical to buttress the wall at these points. In this case the design should be treated similarly to a pillar and the resultant pressure should fall within the middle third of the buttress. The walls between the buttresses can then be made lighter.

Concentrated loads may also be carried by steel columns, either built in the wall or placed clear of the wall and inside the building.

Where the walls are supported by columns, they need not exceed 12 ins. in thickness, while if the columns are not in the wall it should be made thicker.

Where walls are cut into considerably for doors and windows their thicknesses should be increased.

4. **Stone walls** are usually made thicker than brick walls would be made for the same situation. Few factory buildings are now made of stone unless very favorably located to suitable quarries.

5. **Solid-concrete Walls.** — These are frequently light walls placed between the columns and steel framing of a building. The objections to a solid-concrete wall are the probability of

moisture passing through the concrete or the wall sweating on the inside, also the liability of such walls to crack when not reinforced to prevent it. Concrete walls of this type are usually made thinner than brick walls would be made for the same location.

6. **Hollow-concrete Blocks.** — These are made in numerous forms. Being hollow they are poorer conductors of heat than solid walls and are therefore not liable to have moisture condensed on the inside. Where desired, plaster can be placed directly on the blocks without furring and lathing.

7. **Reinforced-concrete Walls.** — These may be made by running across the girts small $\frac{3}{4}$-in. channels, having flanges about

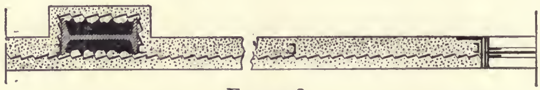


FIG. 348.

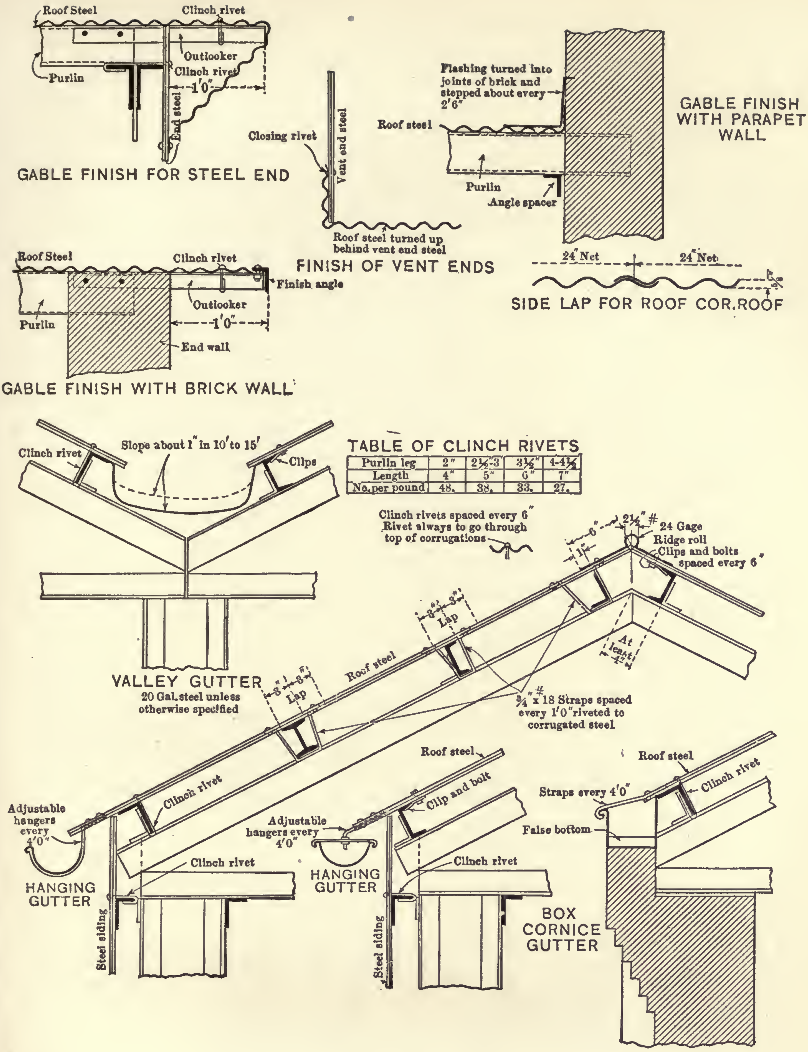
$\frac{3}{8}$ in. wide, and spaced from 12 to 16 ins. center to center. Metal lathing is fastened to these channels, and upon this is placed a coating of concrete on both sides until the wall is made about $2\frac{1}{2}$ ins. thick; see Fig. 348.

When, on account of heating or dampness, a hollow wall is desired, an inside wall similar to the outside one but plastered

FIG. 349.

only on one side may be added, as in Fig. 349; an air space is left between the two wall sections.

Through the courtesy of the American Bridge Company the following illustrations, Figs. 350 to 352, show the corrugated-steel details recommended by them, and Figs. 353 to 359 give their details of various types of window frames and sashes.



Purlin leg	2"	2½"-3"	3½"	4-4½
Length	4"	5"	6"	7"
No. per pound	48.	38.	33.	27.

FIG. 350.

Corrugated steel for roofing is rolled from a sheet 30 inches wide in the flat, 27½ inches when rolled one edge up and one edge down. Laid with 1½ inches of corrugated lap will cover 24 inches of roof.

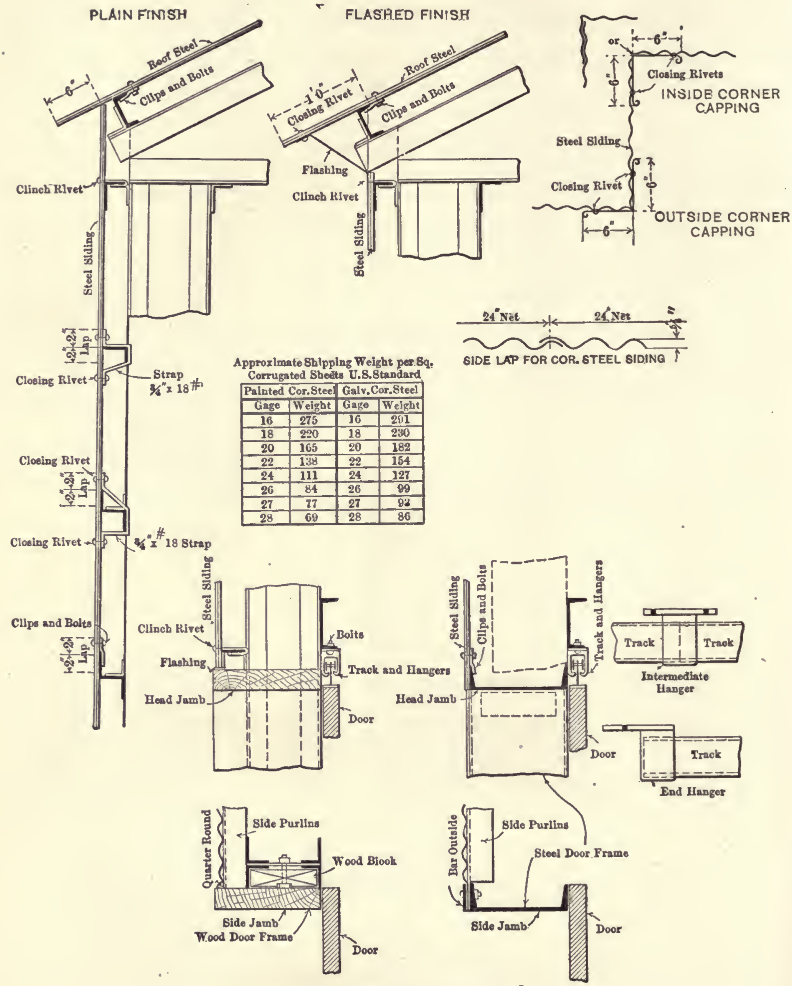
When ordering state distinctly that the sheeting is for roofing, that it is to be 27½ inches wide after corrugating, and that the corrugations are to be ⅝ inch deep. State whether the sheeting is to be plain or corrugated, and also if it is to be painted. Specify the gauge.

Wherever possible order sheets in even feet lengths and to span two purlin spaces. Allow 6 inches for end laps in roofs of 6 inches pitch and 8 inches for end laps in roofs of 4 inches pitch. In roofs of less than 4 inches pitch allow 8 inches for end laps and lay with Slater's cement.

SIZES OF GUTTERS AND CONDUCTORS.

Span of roof, feet.	Gutters, inches.	Conductor, inches.
Up to 50	6	4 every 40 feet
50 to 70	7	5 every 40 feet
70 to 100	8	5 every 40 feet

These are made of No. 24 galvanized steel unless otherwise specified. Hanging gutters should slope at least 1 inch in 15 feet.



Approximate Shipping Weight per Sq. Corrugated Sheets U.S. Standard

Painted Cor. Steel		Galv. Cor. Steel	
Gage	Weight	Gage	Weight
16	275	16	291
18	220	18	230
20	165	20	182
22	138	22	154
24	111	24	127
26	84	26	99
27	77	27	93
28	69	28	86

FIG. 351.

Corrugated steel for siding is rolled from steel sheets 28 inches wide in the flat, and is 26 inches wide when rolled with both edges down. When laid with a side lap of one corrugation it will cover 24 inches of side. When ordering state distinctly that the sheeting is for siding, that it is to be 26 inches wide after corrugating, and that the corrugations are to be ⅝ inch deep. State whether the sheeting is to be galvanized or black, and if it is to be painted; also give gauge. Sheets should be ordered in even feet lengths and to span two purlin spaces, wherever possible. Allow 4 inches for end laps. If side laps are to be riveted space closing rivets about 12 inches apart. Provide roller guides and door stops to hold doors securely in place when open or shut.

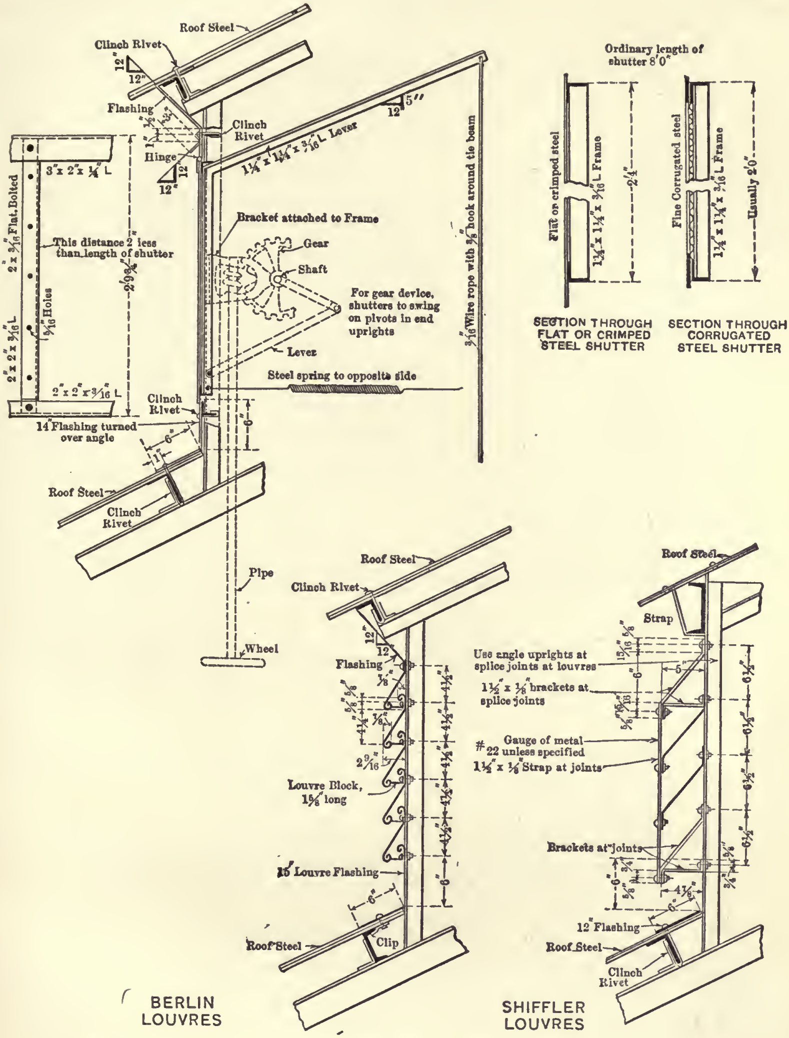


FIG. 352.

Shutters are made from 6 feet to 10 feet long, all standard width. Two hinges are used on shutters up to 8 feet long, while larger sizes are given three hinges.

Louvres of the Berlin type are made of No. 24 gauge steel. The maximum length is 4 feet 1½ inches. The end lap is from ¼ inch to ½ inch.

Steel for these louvres should be ordered 11 inches wide. The uprights have 5⁄16-inch diameter holes for ¼-inch oval screw-head bolts, ¾ inch long.

Schiffler Louvres. The maximum lengths of these are 7 feet 0 inches, without lap. The steel should be ordered 11 inches wide. The holes are 5⁄16 inch in diameter for ¼-inch diameter bolts with oval screw heads and 1 inch long.

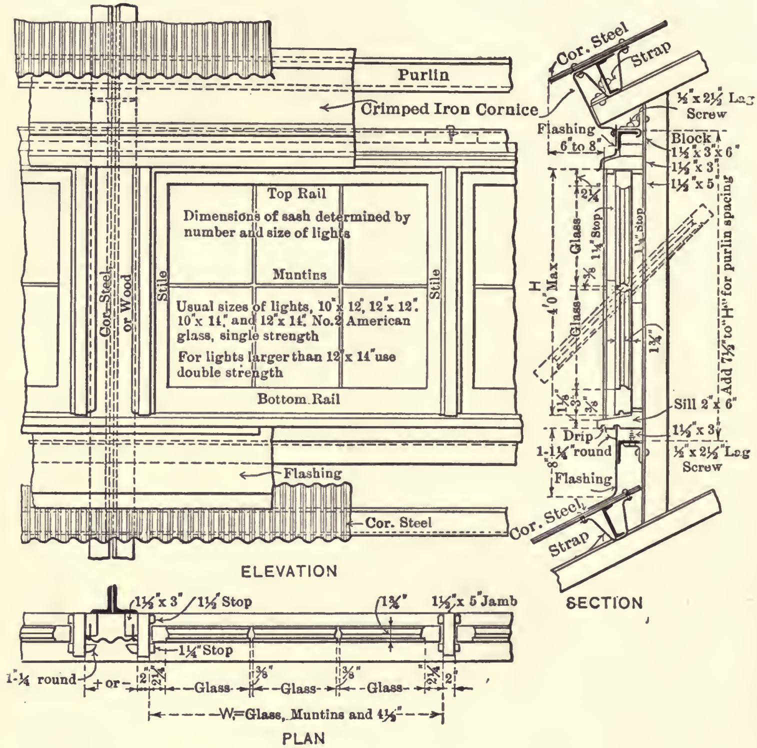


FIG. 353.

The design shown in Fig. 353 is for fixed sash for monitors; for swing monitor sash cut stops off as shown by dotted lines, and omit head stop on the inside. Make frames and sash of white pine, excepting spiking and blocking pieces, which should be spruce, hemlock or Norway pine, planed on all exposed sides. For swing sash order two trunnions for each sash, and call for lever operating device.

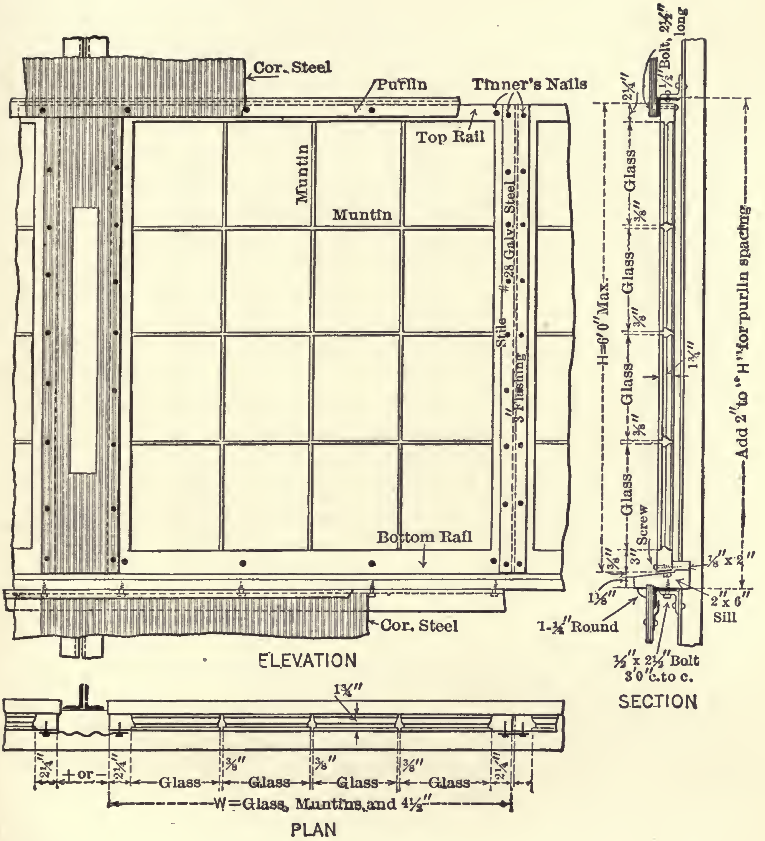


FIG. 354.

The design shown in Fig. 354 is for continuous fixed sash in corrugated steel sides. Make sash and sill of white pine, planed on all exposed sides.

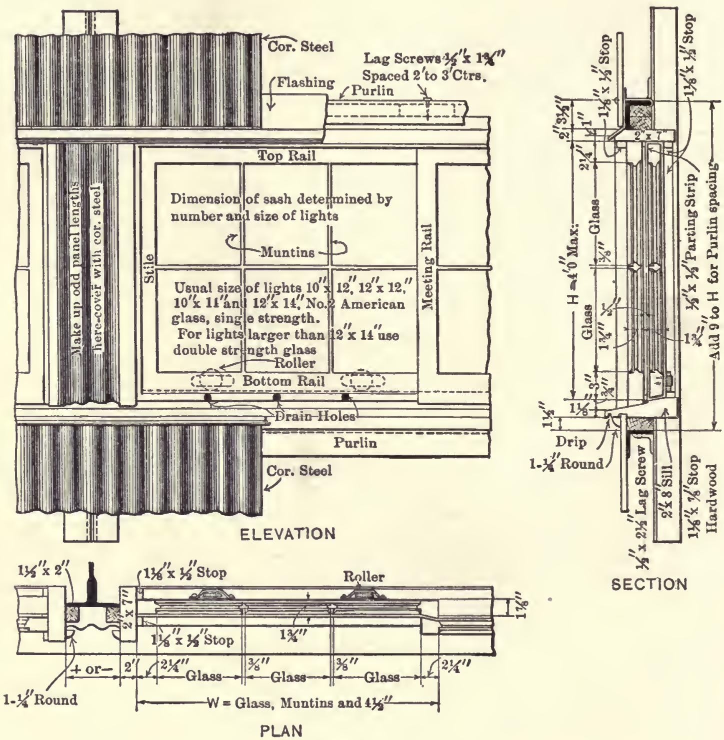


FIG. 355.

The design shown in Fig. 355 is for sliding sash in corrugated steel sides. The stop used for roller track is to be hard wood, other wood the same as recommended for Fig. 353.

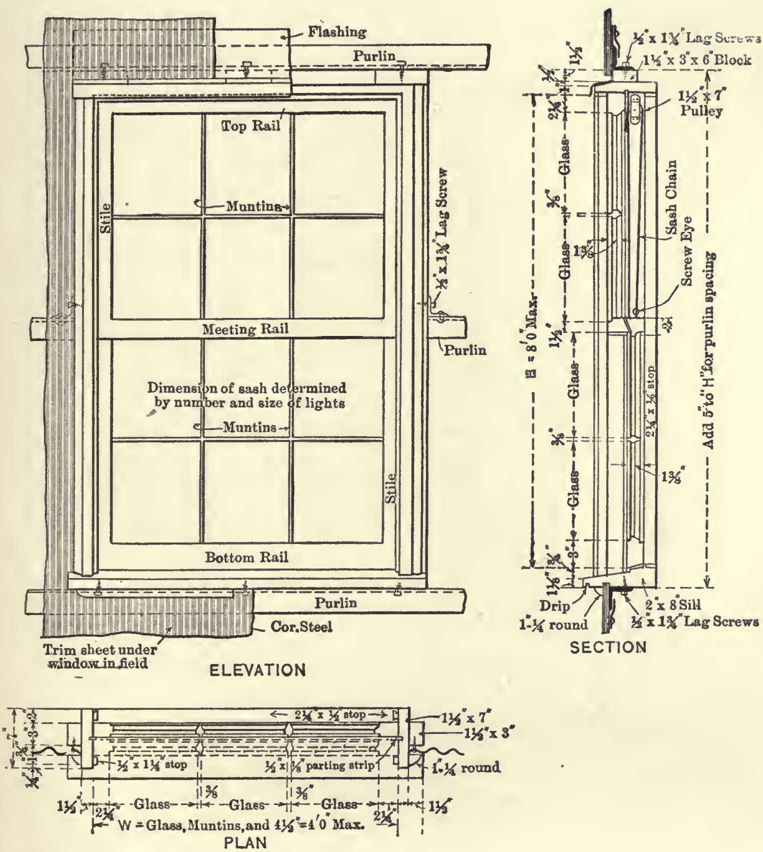


FIG. 356.

The design shown in Fig. 356 is for a window frame with counterbalanced sash in corrugated steel sides. Neither dimension of sash exceeds 4 feet 0 inches. The wood is recommended the same as for Fig. 353.

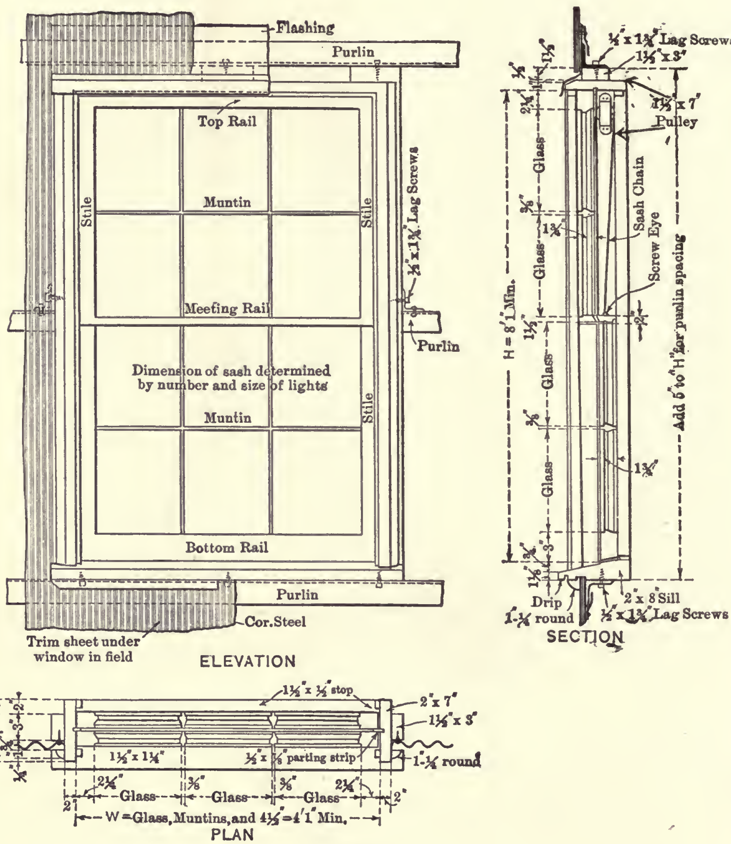


Fig. 357.

The design shown in Fig. 357 is for a window frame with counterbalanced sash in corrugated steel sides. In this design one dimension is not less than 4 feet 1 inch. The wood used is similar to that specified for Fig. 353.

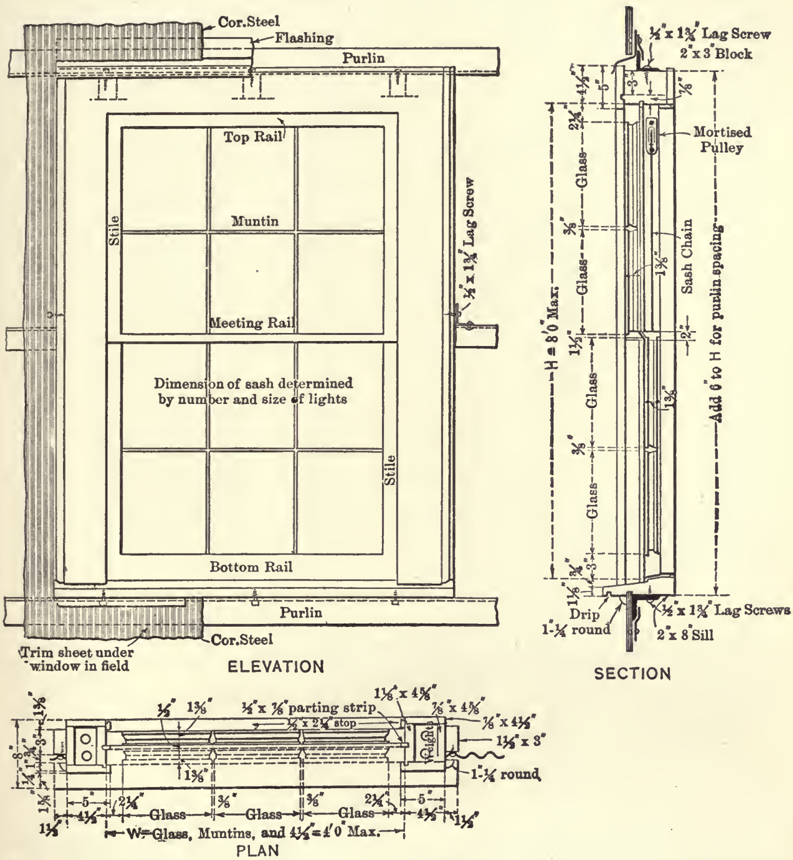

Cor. Steel
Flashing
Purlin
Top Rail
Muntin
Stile
Meeting Rail
Dimension of sash determined by number and size of lights
Stile
Bottom Rail
Purlin
Cor. Steel
Trim sheet under window in field
ELEVATION
Mortised Pulley
Sash Chain
Glass
Add 6" to H for purlin spacing
H = 8'0" Max.
Drip
1-¼ round
2"x 8" Sill
2"x 3" Block
SECTION
½"x ⅞" parting strip
½" x 2¼" stop
Weights
1-¼ round
W = Glass, Muntins, and 4½" = 4' 0" Max.
PLAN

Fig. 358.

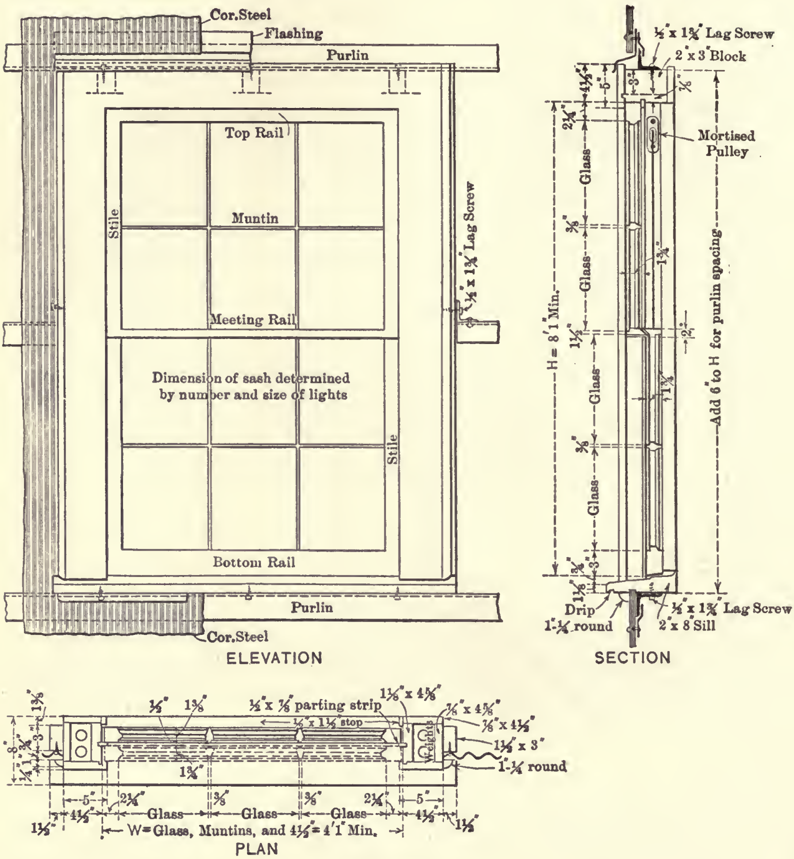
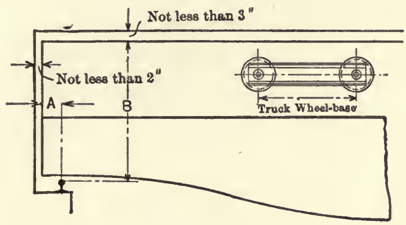


FIG. 359.

The designs shown in Figs. 358 and 359 are for window frames with double hung weighted sash in corrugated steel siding. In Fig. 358 neither dimension of the sash exceeds 4 feet 0 inches, while in Fig. 359 one dimension is not less than 4 feet 1 inch. The sills and casings should be made of white pine; the jambs and parting strips should be made of hard pine; while spiking pieces and blocks should be made of hemlock or Norway pine, planed on all exposed sides.

CRANE CLEARANCES AND WEIGHTS



Capacity, tons (2000 lbs.).	Span.	A.	B.		Wheel base.		Weight of bridge, 2 girders.	Weight of trolley.	Rail.
	Ft.	Inches.	Ft.	Ins.	Ft.	Ins.	Pounds.	Pounds.	Lbs. per yard.
	40	8	4	9	8	0	8,600– 13,000	4,500	
5	60	8	4	9	8	0	16,500– 21,000	to	40
	80	8	4	9	8	0	28,000– 33,000	5,000	
	40	8	5	0	8	6	12,300– 16,000	6,000	
10	60	8	5	0	8	6	20,000– 24,000	to	40
	80	8	5	0	9	0	32,000– 37,000	8,000	
	40	8	5	6	9	0	14,000– 23,000	9,000	
15	60	8	5	6	9	6	23,000– 33,000	to	50
	80	9	5	6	10	0	37,000– 48,000	10,000	
	40	9	6	0	.....		23,000– 28,000	10,000	
20	60	9	6	0	.....		30,000– 37,000	to	50
	80	9	6	0	.....		40,000– 52,000	12,000	
	40	9	6	6	10	0	19,000– 34,000	12,000	
25	60	9	6	6	10	6	29,000– 45,000	to	60
	80	9	6	6	11	0	45,000– 61,000	15,000	
	40	9	7	0	11	0	34,000– 37,000	14,000	
30	60	9	7	0	11	6	44,000– 49,000	to	60
	80	9	7	0	12	0	58,000– 66,000	17,000	
	40	9	8	0	12	6	43,000– 49,000	16,000	
40	60	9	8	0	13	0	60,000– 63,000	to	80
	80	9	8	0	13	3	70,000– 82,000	20,000	
	40	9	8	6	12	6	48,000– 57,000	24,000	
50	60	9	8	6	13	0	66,000– 73,000	to	100
	80	9	8	6	13	6	85,000– 95,000	30,000	
	40	10	9	0	15	0	78,000		
60	60	10	9	0	15	3	70,000– 95,000	32,000	
	80	10	9	0	15	6	100,000–126,000		
	40	10	9	6	15	6	101,000		100
75	60	10	9	6	15	6	80,000–120,000	40,000	to
	80	10	9	6	15	6	120,000–144,000		
	40	12	10	0	15	6	134,000		150
100	60	12	10	0	15	6	94,000–161,000	56,000	
	80	12	10	0	15	6	125,000–187,000		

The maximum wheel loads are approximately 25 per cent of the bridge weight plus 50 per cent of the combined weights of the trolley and crane capacity.

The dimensions of windows depend upon the light required and the size of the panes used. In shop construction the greatest possible amount of light is wanted but without sun. On this account a northern light is preferred. In one-story buildings this leads to saw-tooth or similar construction.

The usual sizes of common glass can be obtained in any handbook on Building Construction. The regular sizes of glass range from 6 × 8 ins. to 44 × 72 ins.; the dimensions up to 16 ins. varying by inches, while those above 16 ins. vary by 2 ins. and are the even inches only.

Single-thick glass is about $\frac{1}{16}$ in. thick.

Double-thick glass is about $\frac{1}{8}$ in. thick.

Wired glass has the advantage of not falling out when cracked, thus affording increased fire protection. Translucent fabric is a substitute for glass that has some advantages. It cuts out a little light but when a sufficient amount is used it affords perfect lighting. It is impervious, elastic and burns with difficulty.

The subject of methods of lighting is an interesting one and should be given careful consideration in buildings of any importance. According to Ketchum it is common to specify that 10 per cent of the exterior surface of ordinary mill buildings and 25 per cent of machine shops and similar buildings should be glazed, this as a rule being divided between windows and skylights.

Roof Coverings

Corrugated-steel Roofing. — This is one of the commonest kinds of roofing materials. It is made by rolling the plain sheets with corrugations.

These sheets range from No. 16 to No. 28, U. S. Standard gauge (0.063 in. to 0.016 in.). The corrugations range from $\frac{3}{16}$ in. to 2.5 ins. A few companies roll sheets with corrugations of 5, 3 and 2 ins.

The sizes most frequently used are gauge Nos. 20 and 24, with $2\frac{1}{2}$-in. corrugations, $\frac{5}{8}$ in. deep. The lengths of sheets range from 6 ft. to 12 ft. A common length is 8 ft.

Roofs covered with corrugated steel should not have a slope of less than 3 ins. in 12 ins. According to Kidder's Handbook they should be supported on about the following spans:

Gauge.	Span in feet.	Gauge.	Span in feet.
24	2–2.5	18	4–5
22 and 20	2–3	16	5–6

The lap at the end of the sheet should range from 3 to 6 ins. according to the slope.

Slope 1 to 3.............. Lap 3 ins.
Slope 1 to 4.............. Lap 4 ins.
Slope 1 to 8.............. Lap 5 ins.

The side laps vary from one to two corrugations; a lap of $1\frac{1}{2}$ corrugations makes a very good joint; see Fig. 360.

FIG. 360.

Corrugated roofing is secured to the building at the purlins by nails when nailing strips are used, and by hoop-iron clips or bands when secured directly to the purlins; see Figs. 350 to 352. The riveting and nailing should be done at the top of the corrugations.

The crippling load in pounds per square foot is approximately

$$W = \left(\frac{100{,}000 \times t \times d}{L^2}\right).$$

t = thickness of the metal in inches.

d = depth of corrugations in inches.

L = length of span in feet.

The following safe loads in pounds per square foot are one-quarter of the calculated crippling loads for sheets with $2\frac{1}{2}$-in. corrugations, $\frac{5}{8}$ in. deep.

Gauge No.	Span in feet.			
	3	4	5	6
20	65	36	23	16
22	53	30	19	13
24	43	24	15	11
26	32	18	12	8

Purlins should be spaced for a roof load of not less than 30 lbs. per sq. ft.; spacing for lighter loads results in injury to the corrugated-steel joints when the roof is walked upon. The sizes most frequently used are No. 20 for the roof and No. 22 for the sides. The corrugated steel should be ordered in lengths sufficient to cover two purlins. The laps should be painted before being riveted. Galvanized steel will take paint better after weathering a while.

When the buildings are lined, $1\frac{1}{4}$ by $\frac{3}{8}$-in. corrugated steel is frequently used. When lined the corrugated steel is better nailed to nailing strips.

The greatest objection to the corrugated-steel roofing is the fact that it sweats. In heated buildings or in rooms with moist air the moisture condenses on the under side of the roof and drops to the floor, possibly striking machinery or materials it may injure. A common method of overcoming this condensation is to cover the purlins with galvanized poultry netting, running the netting from the eaves on one side, over the purlins and down to the eaves on the other side. The netting is secured to the eaves and the several widths of netting are woven together along the edges. Upon this netting are then laid one or two layers of asbestos paper, and on this one or two layers of tar or other paper impervious to moisture; finally the corrugated steel is placed on these. The netting is 60 ins. wide and weighs 10 lbs.

per square of 100 sq. ft. The asbestos paper is about $\frac{1}{16}$ in. thick. The asbestos paper prevents the tar or other paper from taking fire, thus making an excellent fireproof roof.

Gauge.	Weight of corrugated steel, in pounds per 100 sq. ft.	
	Black.	Galvanized.
20	165	182
22	138	154
24	111	127
26	84	99

Slate Roofing. — Slate makes a very satisfactory roof covering. The best slates have a somewhat metallic appearance, do not absorb water and are strong. Generally the strongest slates are the best. The sizes range from 6 × 12 ins. to 16 × 26 ins. and in thickness from $\frac{1}{8}$ in. to $\frac{1}{4}$ in. The most frequently used sizes run from 6 × 12 ins. to 12 × 18 ins., and are generally $\frac{3}{16}$ in. thick. In slate roofs the slope should not be less than 1 in 4. The usual lap is a double lap of 3 ins.; that is, the upper 3 ins. of the first slate are covered by the third slate; see Fig. 361.

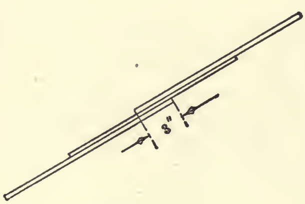


Fig. 361.

As slate breaks easily under shock, the roof must be designed to deflect but slightly under loading.

The slates are commonly laid on sheathing; this may be either plain or tongued-and-grooved boards, their thickness varying with the purlin spacing and the load. The sheathing is covered with tar paper or with waterproof paper or felt. The slates may be laid on roofing laths; these are 2 to 3 ins. wide and 1 in. to $1\frac{1}{4}$ ins. thick; they are spaced on the rafters to suit the nails in the slates. At gutters, valleys and ridges the slates should be laid in cement and are sometimes entirely laid that way.

The following are examples of slate roofing:

Trusses 14 ft. 6 ins. center to center. No purlins. Sheathing 3-in. yellow-pine plank. The planks were 29 ft. long and joints were broken on alternate trusses.

Another roof whose purlins were 5 ft. 8 ins. center to center had 2-in. yellow-pine sheathing.

Although slate is an expensive roofing material, it can reasonably be expected, with ordinary repairs, to last from 25 to 30 years. While not resisting great heat (it cracks and disintegrates, and then exposes the sheathing to the fire), slate itself does not ignite, and thus makes a fairly good fireproof roof.

Clay tiles make a thoroughly fireproof roof. They are, however, very heavy and expensive. Several makes of tiles are also molded in glass, thus permitting skylights to be very readily placed. Their artistic effect is good.

Concrete Roofs. — Concrete, reinforced by expanded metal or steel rods, is also much used for roofs. The purlins may be spaced from 5 to 7 ft., or they may be omitted altogether, the concrete slabs being placed directly on the trusses or on concrete beams or rafters. When the slope is steep, slate may be nailed directly to the concrete; this should be done within a couple of weeks after placing the concrete; or if the slope is slight a standard slag roof may be used.

The following is an example of a reinforced-concrete roof: The roof slab was $4\frac{1}{2}$ ins. thick; it was reinforced with $\frac{1}{2}$-in. square bars, spaced 5 ins. center to center; there were no longitudinal beams; the transverse girders were 14 ft. center to center. Wooden nailing strips were embedded in the concrete slabs; these secured a five-ply waterproofing course upon which a standard slag roof was placed.

Slag or Gravel Roofing. — This roof covering may be placed on either wooden sheathing or on a reinforced-concrete roof. Such a roof when on wooden sheathing is commonly made by covering the wood with dry felt paper and on this placing from three to five layers of tarred or asphaltic felt. The layers of

paper are lapped the same as slate, exposing from 6 to 10 ins. of each sheet.

Another method is to lay rosin-sized sheathing paper weighing not less than 6 lbs. per 100 sq. ft.; upon this lay two ply of tarred felt lapping 17 ins. and weighing not less than 14 lbs. per 100 sq. ft. single thickness. These are then covered with a spreading of pitch, and finally three additional layers of felt are laid and similarly coated with pitch. On this layer of pitch the slag or gravel is spread; if slag, it should be crushed smaller than $\frac{5}{8}$ in. but larger than $\frac{1}{4}$ in. Gravel should be screened and both slag and gravel should be clean and dry. When laid on cement roofing the rosin sheathing paper should be omitted but the cement should be coated with pitch. Although a slag or gravel roof is not fire proof, tests have shown its fire-resisting qualities to be good. It protects the wooden sheathing better than slate.

Slope of Roofs

The slope of a roof depends upon that required by the roofing material used. The following is taken from Kidder's Handbook.

Roofing material.	Slope of rafter, inches per foot.
Slate	8
Tiles (interlocking)	7
Tin shingles (painted)	6
Cedar shingles	6
Corrugated steel	3
Ready roofing	1

In the case of slate the permissible slope will vary with the weight and size of the slate.

Largest sizes of slate	Slope 5 ins. per ft.
Medium sizes of slate	Slope 6 ins. per ft.
Smallest sizes of slate	Slope 8 ins. per ft.

Roofs having a slope of $\frac{1}{2}$ in. to $\frac{5}{8}$ in. per ft. constitute what are termed flat roofs, and are generally covered with tar and gravel, asphalt, ready roofing or tin with lock and soldered joints.

Pitch roofs should be given a slope not exceeding 2 ins. per ft.

Rafters or their equivalents should be spaced not over 5 ft. when ordinary sheathing of 1-in. boards is used.

The following are examples of roofs:

Pennsylvania Steel Co. — Purlins 6 ft. center to center. Sheathing $1\frac{7}{8}$-in. tongued and grooved hemlock. Purlins 4 × 10-in. yellow pine. Slag roofing, trusses 20 ft. center to center.

A Bridge Shop Roof. — Composition roofing. Purlins spaced about 7 ft. 6 ins. center to center. Sheathing 2 ins. thick.

Slate Roof on Tar Paper. — Purlins spaced about 4 ft. center to center. Sheathing $1\frac{1}{8}$ in.

Slate roof on 2-in. sheathing. Purlins spaced about 8 ft. center to center.

Slate roof on $2\frac{1}{2}$-in. plank. Purlins about 8 ft. 6 ins. center to center.

Five-ply Slag Roofing. — Sheathing $1\frac{1}{4}$-in. spruce on purlins spaced about 5 ft. 6 ins. center to center.

Asphalt Roofing. — Purlins spaced 5 ft. 6 ins. center to center. Sheathing $1\frac{3}{8}$-in. tongued and grooved boards.

CHAPTER XXI

SPECIFICATIONS FOR STRUCTURAL STEEL WORK

MATERIALS

1. Process of Manufacture. — Steel may be made by either the open-hearth or the Bessemer process.

NOTE. — For the more important work rolled steel is preferably made by the basic open-hearth process, as this process permits the elimination of the greater portion of the phosphorus and a small percentage of the sulphur contained in the pig iron and scrap from which the steel is made. Steel for castings is commonly made by the acid open-hearth process.

2. Chemical and Physical Requirements. —

Chemical and physical requirements.	Structural steel.	Rivet steel.	Steel castings.
Phosphorus, maximum	0.04%	0.04%	0.05%
Sulphur, maximum	0.05%	0.04%	0.05%
Ultimate tensile strength in lbs. per sq. in. See No. 3	Desired 60,000	Desired 50,000	Not less than 65,000
Elongation, min. per cent in 8 ins., see Fig. 1. See No. 4	$\frac{1{,}500{,}000}{\text{Ult. ten. str.}}$	$\frac{1{,}500{,}000}{\text{Ult. ten. str.}}$	....
Elongation, min. per cent in 2 ins., see Fig. 2	22%	....	18%
Character of fracture	Silky	Silky	Silky or fine granular
Cold bends without fracture	180° flat; See No. 5	180° flat; See No. 6	90°, $d=3\,t$

NOTE. — The effect of phosphorus is to make the steel cold short, brittle when cold; while the sulphur makes it hot short, brittle when hot.

3. Allowable Variations. — Tensile tests of steel will be considered satisfactory if showing an ultimate strength within 5000 pounds of that desired.

4. Modifications in Elongation. — For material less than $\frac{5}{16}$ inch thick a deduction of $2\frac{1}{2}$ per cent will be allowed in the elongation for each $\frac{1}{16}$ inch the material is under $\frac{5}{16}$ inch. For material more than $\frac{3}{4}$ inch thick a de-

duction of 1 per cent may be allowed for each $\frac{1}{8}$ inch the material exceeds $\frac{3}{4}$ inch. For pins and rollers exceeding 3 inches in diameter the elongation in 8 inches may be 5 per cent below that given in No. 2.

5. **Bending Tests.** — These tests may be made either by pressure or by blows. Plates, shapes and bars less than 1 inch thick shall bend as specified in No. 2.

Full-sized material for eyebars and other steel 1 inch thick and over, tested as rolled, shall bend cold 180 degrees around a pin whose diameter is twice the thickness of the bar. It must show no fracture on the outside of the bend.

Angles $\frac{3}{4}$ inch and less in thickness shall open flat, angles $\frac{1}{2}$ inch and less in thickness shall bend shut, cold, under blows of a hammer, without signs of fracture. This test will be made only when required by the inspector.

6. **Nicked Bends.** — Rivet steel, when nicked and bent around a bar the same size as the rivet rod, shall give a gradual break and a fine, silky, uniform fracture.

7. In order that the ultimate strength of full-sized annealed eyebars may meet the requirements of paragraph 65 the ultimate strength in test specimens may be determined by the manufacturers; all other tests than those for ultimate strength shall conform to the requirements in paragraph 2.

8. The yield point, as indicated by the drop of the beam, shall be recorded in the test reports.

9. **Chemical Analyses.** — Chemical determinations of the percentages of carbon, phosphorus, sulphur and manganese shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt of steel and a correct copy of such analysis shall be furnished to the engineer or his inspector. Check analyses shall be made from the finished material, if called for by the purchaser, in which case an excess of 25 per cent above the required limits will be allowed.

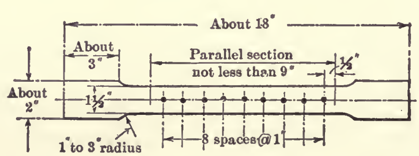


Fig. 1.

10. **Form of Specimens.** — (a) *Plates, Shapes and Bars.* — Specimens for tensile and bending tests for plates, shapes and bars shall be made by cutting coupons from the finished product, which shall have both faces rolled and both edges milled to the form shown in Fig. 1; or with both

edges parallel; or they may be turned to a diameter of $\frac{3}{4}$ inch for a length of at least 9 inches, with enlarged ends.

(b) *Rivets.* — Rivet rods shall be tested as rolled.

(c) *Pins and Rollers.* — Specimens shall be cut from the finished rolled or forged bar in such a manner that the center of the specimen shall be 1 inch from the surface of the bar. The specimen for the tensile test shall be turned to the form shown by Fig. 2. The specimen for bending test shall be 1 inch by $\frac{1}{2}$ inch in section.

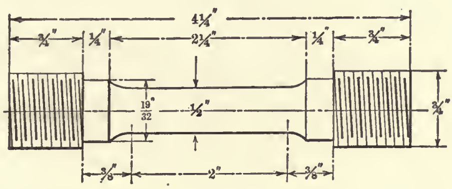


FIG. 2.

11. **Steel Castings.** — The number of tests will depend on the number and importance of the castings. Specimens shall be cut cold from coupons molded and cast on some portion of one or more castings from each melt or from the sink heads, if the heads are of sufficient size.

The coupon or sink head, so used, shall be annealed with the casting before it is cut off. Test specimens shall be of the form prescribed for pins and rollers.

12. **Annealed and Unannealed Specimens.** — Material which is to be used without annealing or further treatment shall be tested in the condition in which it comes from the rolls. When material is to be annealed or otherwise treated before use, the specimens for tensile tests representing such material shall be cut from properly annealed or similarly treated short lengths of the full section of the bar.

13. **Number of Tests.** — At least one tensile and one bending test shall be made from each melt of steel as rolled. In event of the material rolled from one melt varying in thickness by $\frac{3}{8}$ inch or more a test shall be made from the thickest and the thinnest material rolled.

14. **Finish.** — Finished material shall be free from injurious seams, flaws, cracks, defective edges, or other defects, and have a smooth, uniform, workmanlike finish. Plates 36 inches and under in width shall have rolled edges.

15. **Stamping.** — Every finished piece of steel shall have the melt number and the name of the manufacturer stamped or rolled upon it. Steel for pins and rollers shall be stamped on the end. Rivet and lattice steel and

other small parts may be bundled with the above marks on an attached metal tag.

16. **Defective Material.** — Material, which, subsequent to the above tests at the mills, and its acceptance there, develops weak spots, brittleness, cracks, or other imperfections, or is found to have injurious defects, will be rejected at the shop and shall be replaced by the manufacturer at his own cost.

17. **Allowable Variation in Weight.** — A variation in cross section or weight in the finished members of more than $2\frac{1}{2}$ per cent from that specified shall be sufficient cause for rejection.

18. **Cast Iron.** — Except where chilled iron is specified, castings shall be made of tough gray iron, with sulphur not over 0.10 per cent. They shall be true to pattern, out of wind (perfectly straight or flat) and free from flaws and excessive shrinkage. If tests are demanded they shall be made on the "Arbitration Bar" of the American Society of Testing Materials, which is a round bar $1\frac{1}{4}$ inches in diameter and 15 inches long. The transverse test shall be on a supported length of 12 inches with the load at the middle. The minimum breaking load so applied shall be at least 2900 pounds, with a deflection of at least $\frac{1}{10}$ inch before rupture.

Workmanship

19. **General.** — All parts forming a structure shall be built in accordance with approved drawings. The workmanship and finish shall be equal to the best practice in modern bridge works.

20. **Straightening Material.** — Material shall be thoroughly straightened in the shop, by methods that will not injure it, before being laid off or worked in any way.

21. **Finish.** — Shearing shall be neatly and accurately done, and all portions of the work exposed to view shall be neatly finished.

22. **Rivets.** — The size of rivets called for in the plans shall be understood to mean the actual size of the cold rivets before heating.

23. **Rivet Holes.** — When general reaming is not required, the diameter of the punch for material not over $\frac{5}{8}$ inch thick shall be not more than $\frac{1}{16}$ inch, nor that of the die more than $\frac{1}{8}$ inch, larger than the diameter of the rivet. The diameter of the die shall not exceed the diameter of the punch by more than one-fourth the thickness of the material punched. Material over $\frac{5}{8}$ inch thick, except that for minor details, and all material where general reaming is required, shall be sub-punched and reamed as per paragraphs 49, 50 and 51, or drilled from the solid. Holes in the flanges of rolled beams and channels used in the floors of railroad bridges shall be

drilled from the solid. Those in the webs of same shall be so drilled or sub-punched and reamed.

NOTE. — Mr. Schneider in his specifications replaces the $\frac{5}{8}$ inch occurring in paragraph 23 by $\frac{3}{4}$ inch.

24. **Punching.** — Punching shall be accurately done. Slight inaccuracy in the matching of holes may be corrected with reamers. Drifting to enlarge unfair holes will not be allowed. Poor matching of holes will be cause for rejection at the option of the inspector.

NOTE.— Drifting is driving a taper pin through holes that fail to match properly; this distorts the hole and injures the material.

25. **Assembling.** — Riveted members shall have all parts well pinned up and firmly drawn together with bolts before riveting is commenced. Contact surfaces shall be painted (see paragraph 52).

26. **Lattice Bars.** — Lattice bars shall have neatly rounded ends, unless otherwise called for.

27. **Web Stiffeners.** — Stiffeners shall fit neatly between the flanges of girders. Where tight fits are called for, the ends of the stiffeners shall be faced and brought to a true contact bearing with the flange angles.

28. **Splice Plates and Fillers.** — Web splice plates, and fillers under stiffeners, shall be cut to fit within $\frac{1}{8}$ inch of flange angles.

29. **Connection Angles.** — Connection angles for floor girders shall be flush with each other and correct as to position and length of girder. In case milling is required after riveting, the removal of more than $\frac{1}{16}$ inch from their thickness shall be cause for rejection.

30. **Riveting.** — Rivets shall be driven by pressure tools wherever possible. Pneumatic hammers shall be used in preference to hand driving.

31. **Rivets.** — Rivets shall look neat and finished, with heads of approved shape, full and of equal size. They shall be central on shank and shall grip the assembled pieces firmly. Recupping and calking will not be allowed. Loose, burned or otherwise defective rivets shall be cut out and replaced. In cutting out rivets great care shall be taken not to injure the adjacent metal. If necessary they shall be drilled out.

31 (a). **Heating Rivets.** — Rivets shall be heated to a light cherry red, in a gas or oil furnace. The furnace must be so constructed that it can be adjusted to the proper temperature.

NOTE. — Paragraph 31 (a) is inserted by Mr. Schneider but is not generally included by others.

32. **Field Bolts.** — Wherever bolts are used in place of rivets which transmit shear, the holes shall be reamed parallel and the bolts turned to a driving fit. A washer not less than $\frac{1}{4}$ inch thick shall be used under the nut.

33. **Members to be Straight.** — The several pieces forming one built-up member shall be tight and fit closely together, and finished members shall be free from twists, bends or open joints.

34. **Finish of Joints.** — Abutting joints shall be cut or dressed true and straight and fitted closely together, especially where open to view.

In compression joints, depending on contact bearing, the surfaces shall be truly faced, so as to have even bearings after they are riveted up complete and perfectly aligned.

35. **Field Connections.** — Holes for floor-girder connections shall be sub-punched and reamed with twist drills to a steel template 1 inch thick. Unless otherwise allowed, all other field connections shall be assembled in the shop and the unfair holes reamed; when so reamed the pieces shall be match-marked before being taken apart.

36. **Eyebars.** — Eyebars shall be straight and true to size, and shall be free from twists, folds in the neck or head, or any other defect.

Heads shall be made by upsetting, rolling or forging. Welding will not be allowed. The form of the heads will be determined by the dies in use at the works where the eyebars are made, if satisfactory to the engineer, but the manufacturer shall guarantee the bars to break in the body with a silky fracture, when tested to rupture. The thickness of the head and neck shall not vary more than $\frac{1}{16}$ inch from that specified.

37. **Boring Eyebars.** — Before boring each eyebar shall be properly annealed and carefully straightened. Pinholes shall be in the center line of the bars and in the center of the heads. Bars of the same length shall be bored so accurately that, when placed together, pins $\frac{1}{32}$ inch smaller in diameter than the pinholes can be passed through the holes at both ends of the bars at the same time.

38. **Pinholes.** — Pinholes shall be bored true to gauges, smooth and straight, at right angles to the axis of the member and parallel to each other, unless otherwise called for. Wherever possible, the boring shall be done after the member is riveted up.

39. **Variation in Pinholes.** — The distance center to center of pinholes shall be correct within $\frac{1}{32}$ inch, and for pins up to 5 inches in diameter the diameter of the hole shall not exceed the diameter of the pin by more than $\frac{1}{50}$ inch; for larger pins this difference shall not exceed $\frac{1}{32}$ inch.

40. **Pins and Rollers.** — Pins and rollers shall be accurately turned to gauges and shall be straight, smooth and entirely free from flaws.

41. **Pilot Nuts.** — At least one pilot and driving nut shall be furnished for each size of pin for each structure, and field rivets 10 per cent in excess of the number of each size actually required.

NOTE. — Pilot and driving nuts are nuts used to guide and protect truss pins during driving on erection.

42. **Screw Threads.** — Screw threads shall make tight fits in the nuts and shall be U. S. standard, except above the diameter of $1\frac{3}{8}$ inches, when they shall be made with six threads per inch.

43. **Annealing.** — Except in minor details, steel which has been partially heated shall be properly annealed.

44. **Steel Castings.** — All steel castings shall be annealed.

45. **Welds.** — Welds in steel will not be allowed.

46. **Bed Plates.** — Expansion bed plates shall be planed true and smooth. Cast wall plates shall be planed top and bottom; the cut of the planing tool shall correspond with the direction of expansion.

47. **Shipping Details.** — Pins, nuts, bolts, rivets and other small details shall be boxed or crated.

Additional Specifications when General Reaming and Planing are Required

48. **Planing Edges.** — Sheared edges and ends shall be planed off at least $\frac{1}{4}$ inch.

49. **Reaming.** — Punched holes shall be made with a punch $\frac{3}{16}$ inch smaller in diameter than the nominal size of the rivets and shall be reamed to a finished diameter of not more than $\frac{1}{16}$ inch larger than the rivet.

50. **Reaming after Assembling.** — Wherever practicable, reaming shall be done after the pieces forming one built member have been assembled and firmly bolted together. If necessary to take the pieces apart for shipping and handling, the respective pieces reamed together shall be so marked that they may be reassembled in the same position in the final setting up. No interchange of reamed parts will be allowed.

51. **Removing Burrs.** — The burrs on all reamed holes shall be removed by a tool countersinking about $\frac{1}{16}$ inch.

Painting

51 (a). **Shop Painting.** — Steel work, before leaving the shop, shall be thoroughly cleaned and given one good coating of pure linseed oil, or such paint as may be called for, well worked into all joints and open spaces.

52. In riveted work, the surfaces coming in contact shall each be painted before being riveted together.

53. Pieces and parts which are not accessible for painting after erection, including tops of stringers, eyebar heads, ends of posts and chords, etc., shall be given two coats of paint before leaving the shop.

54. Steel work to be entirely embedded in concrete shall not be painted.

55. Painting shall be done only when the surface of the metal is perfectly dry. It shall not be done in wet or freezing weather, unless protected under cover.

56. Machine-finished Surfaces. — These shall be coated with white lead and tallow before shipment or before being put out into the open air.

57. Field Painting. — After the structure is erected, the metal work shall be painted thoroughly and evenly with an additional coat of paint, mixed with pure linseed oil, of such quality and color as may be selected.

Succeeding coats of paint shall vary somewhat in color, in order that there may be no confusion as to the surfaces that have been painted.

Inspection and Testing

58. Facilities for Inspection. — The manufacturer shall furnish all facilities for inspecting and testing the weight, quality of material and workmanship at the shop where the material is manufactured. He shall, if required, furnish a suitable testing machine for testing full-sized members.

The manufacturer shall prepare all test pieces for the machine, free of cost.

59. Access to Shop. — When an inspector is furnished by the purchaser, he shall have full access, at all times, to all parts of the shop where material under his inspection is being manufactured.

60. The purchaser shall be furnished complete shop plans, and must be notified well in advance of the start of work in the shop, in order that he may have an inspector on hand to inspect material and workmanship.

61. Shipping Invoices. — Complete copies of shipping invoices shall be furnished to the purchaser with each shipment.

62. Mill Orders. — The purchaser shall be furnished with complete copies of mill orders, and no material shall be rolled and no work done before he has been notified as to where the orders have been placed, so that he may arrange for the inspection.

63. Inspector's Mark. — The inspector shall stamp with a private mark each piece accepted. Any piece not so marked may be rejected at any time, at any stage of the work. If the inspector, through an oversight or otherwise, has accepted material or work which is defective or contrary to specifications, this material, no matter in what stage of completion, may be rejected by the purchaser.

Full-sized Tests

64. Full-sized Tests. — Full-sized parts of the structure may be tested at the option of the purchaser. Such tests on eyebars and similar members, to prove the workmanship, shall be made at the manufacturer's expense, such members shall be paid for by the purchaser, at contract price, if the tests are satisfactory. If the tests are not satisfactory, the members represented by them will be rejected. The expense of testing members, to prove their design, shall be paid for by the purchaser.

65. Eyebar Tests. — In eyebar tests, the minimum ultimate strength shall be 55,000 pounds per square inch. The elongation in 10 feet, including fracture, shall be not less than 15 per cent. Bars shall break in the body and the fracture shall be silky or fine granular. The elastic limit as indicated by the drop of the mercury shall be recorded. Should a bar break in the head and develop the specified elongation, ultimate strength and character of fracture, it shall not be cause for rejection, provided not more than one-third of the total number of bars break in the head.

SPECIFICATIONS FOR STEEL MILL BUILDINGS

66. Dimensions. — By height shall be understood the distance from the under side of the lower chord of the roof truss to the tops of the foundations.

The width and length of the building shall be measured to the outsides of the framing or sheathing.

67. Spans. — The spans of trusses, beams and girders for the purpose of calculations shall be assumed as the distance center to center of bearings for supported spans, while they will be considered as the distance back to back of framing angles when the trusses, beams or girders are framed into columns.

Loads

68. Dead Loads. — The dead load is the weight of all permanent construction and fixtures. The calculated weights shall be based upon those hereafter given.

(a) **Weight of Trusses.** — The weight of a truss may be estimated as

$$W = \frac{P \cdot L}{\left(300 + 6 \cdot L + \frac{P \cdot D}{3}\right)},$$

where

W = weight of truss per square foot of building area,

L = span of truss, in feet,

D = distance center to center of trusses, in feet,

P = load per square foot on the truss.

(b) **Weight of Purlins.** — The weight of purlins per square foot of horizontal projection of the roof may be taken as

$$W_1 = \frac{\sqrt{P_1} \times D}{45} - \frac{1}{4},$$

where

W_1 = weight of purlins per square foot of building,

D = distance center to center of the trusses,

P_1 = load per square foot on purlins.

(c) **Weight of Roof Coverings.** — The weights of roof coverings not including sheathing per square foot of actual roof surface will average

Tin....................	1 lb.	Corrugated steel, No. 18....	3.0 lbs.
Slate, $\frac{3}{16}$-inch...........	6.6 lbs.	Corrugated steel, No. 20....	2.3 lbs.
Slate, $\frac{1}{8}$-inch............	4.4 lbs.	Felt and gravel.............	7–9 lbs.
Terra-cotta, 1 inch thick..	6.0 lbs.		

(d) **Weight of Sheathing.** —

Wooden sheathing, per foot, board measure............	3.5 lbs.
Concrete, 1 inch thick, per square foot................	10–12 lbs.

NOTE. — A foot board measure is a piece 12 inches square and 1 inch thick.

LIVE LOADS

69. (a) **Live Loads.** — These shall not be less than the following uniform loads:

Warehouses.....................	120 pounds per square foot.
Foundry charging floors.........	300 pounds per square foot.
Power houses, uncovered floors...	200 pounds per square foot.

In all cases special concentrations, such as engines, turbines, boilers, chimneys, etc., should be considered from actual weights.

(b) **Crane Loads.** — Where available the actual weights and dimensions of traveling cranes should be used; otherwise the table of weights given on page 364 and taken from the Specifications of Mr. C. C. Schneider may be used.

(c) **Distribution of Wheel Loads.** — Wheel loads transferred to beams or girders through rails may be considered as distributed a distance equal to the depth of the girder or beam but not exceeding 30 inches. (Ostrup.)

(d) **Lateral Loading.** — Besides the vertical loading, beams and girders carrying traveling cranes shall be designed so that their upper flanges shall in addition resist a lateral bending due to one-twentieth of the crane's capacity acting at each bridge-truck wheel.

(e) **Impact.** — An addition of 25 per cent of the live-load bending moments and shears shall be made to all beams, girders and columns carrying traveling cranes.

70. **Flat Roofs.** — Flat roofs liable to be loaded with people should be designed as floors. This would subject them to a possible additional load of 40 pounds per square foot.

71. **Wind Loads.** — The normal wind pressure on a roof shall be based upon a horizontal wind pressure of 20 pounds per square foot and its magnitude shall be estimated by Duchemin's formula,

$$W_n = \frac{2 \sin A}{1 + \sin^2 A} \times W_h,$$

where W_n is the normal component of the wind pressure in pounds per square foot.

W_h is the horizontal pressure in pounds per square foot on a vertical surface.

A is the slope of the roof with the horizontal in degrees.

For $W_h = 20$, W_n has the following values.

A	W_n	A	W_n
5	3.5	33.7 (slope 2 to 3)	17.0
10	6.7	40	18.2
20	12.3	45	18.8
21.8 (slope 2 to 5)	13.1	50	19.3
26.6 (slope 1 to 2)	14.9	60	19.8
30	16.0	90	20.0

NOTE. — The available data relating to wind pressures on structures is exceedingly unsatisfactory. This relates both to the horizontal pressure corresponding to any wind velocity and the effect on the large surfaces exposed by ordinary buildings. Of the several formulæ proposed for estimating the normal wind pressure upon inclined surfaces Duchemin's gives values somewhat higher than Hutton's or the straight-line formula and, as it agrees with recent small surface tests, is preferred by many.

Hutton's formula, which was formerly generally accepted, is

$$W_n = W_h \sin A^{1.842 \cos A - 1}.$$

The straight-line formula, which gives results closely approximating those given by Hutton's formula, is

$$W_n = \frac{W_h \times A}{45}.$$

In this last formula A is expressed in degrees and the formula only applies to angles of slope not exceeding 45 degrees. The specifications of the American Bridge Co. require that "the wind pressure shall be assumed acting horizontally, at 20 pounds per square foot, on the vertical projections of roof surfaces." Still other specifications assume a horizontal wind pressure of 30 pounds per square foot reduced to normal pressure by Duchemin's formula.

Observations made by M. E. Bret on the top of the Eiffel Tower and recorded in "Le Genie Civil," March 12, 1910, led him to conclude that horizontal wind pressure seldom exceeded 20 pounds per square foot, and that the relation between pressure and velocity should read $p = 0.0029\,v^2$, where p = pressure per square foot in pounds and v = the velocity of the wind in miles per hour.

It is generally accepted that wind pressures per square foot are lower over large areas than over small ones, and that these pressures are lower near the ground than at higher altitudes.

Although the experiments of Stanton, Smith, Boardman and others seem to indicate the existence of suction acting upon the leeward sides of buildings, walls and roofs, their tests were upon too small scales to warrant the immediate changing of usual practice.

WEIGHTS AND DIMENSIONS OF TYPICAL TRAVELING CRANES

Capacity.	Span.	Wheel base.	Maximum wheel load.	Side clearance.	Vertical clearance.	Weight of rail for	
Tons.	Feet.	Ft. Ins.	Pounds.	Inches.	Ft. Ins.	Runway girder.	Beams.
5	40	8 6	12,000	10	6 0	40	40
5	60	9 0	13,000	10	6 0	40	40
10	40	9 0	19,000	10	6 0	45	40
10	60	9 6	21,000	10	6 0	45	40
15	40	9 6	25,000	10	7 0	50	50
15	60	10 0	29,000	10	7 0	50	50
20	40	10 0	33,000	12	7 0	55	50
20	60	10 6	36,000	12	7 0	55	50
25	40	10 0	40,000	12	8 0	60	50
25	60	10 6	44,000	12	8 0	60	50
30	40	10 6	48,000	12	8 0	70	60
30	60	11 0	52,000	12	8 0	70	60
40	40	11 0	64,000	12	9 0	80	60
40	60	12 0	70,000	12	9 0	80	60
50	40	11 0	72,000	14	9 0	100	60
50	60	12 0	80,000	14	9 0	100	60

NOTE. — The rail weights are in pounds per yard. The side clearance is from the center of the rail, while the vertical clearance is from the top of the rail.

72. Wind Pressure on the Sides and Ends of a Building. — The wind pressure on the sides and ends of a building shall be taken at 20 pounds per square foot.

NOTE. — Other specifications assume 20 pounds per square foot on buildings not exceeding 30 feet to the eaves and this is increased up to 30 pounds per square

foot for buildings measuring 60 feet or more to the eaves. Still other specifications for positions not severely exposed substitute 15 pounds per square foot for the 20 pounds specified in the first sentence of paragraph 72, and where the distance from the ground to the eaves exceeds 25 feet the normal pressure on the roof is that corresponding to a horizontal pressure of 20 pounds per square foot.

73. **Wind Pressure on Framework.** — The wind pressure shall be estimated for the wind acting horizontally in any direction upon the total exposed surface of the framework. Such pressure shall be assumed at 30 pounds per square foot of exposed surface. (This applies during construction.)

74. **Snow Loads.** — For latitudes of about 40 degrees the snow load may be assumed at 25 pounds per square foot of horizontal projection of the roof for flat roofs and those inclined up to angles of 20 degrees; above this slope the load may be decreased uniformly, so that at 45 degrees it would become zero. (The slope corresponding to 20 degrees is about one in three.)

These loads should be increased for higher latitudes and reduced for lower latitudes. In tropical countries snow loads may be neglected.

The amount and character of the precipitation in any locality may also affect the loading, which would naturally be lower in arid sections.

NOTE. — According to Mr. C. C. Schneider, in climates corresponding to that of New York, ordinary roofs up to spans of 80 feet may be designed for the following minimum equivalent loads per square foot of exposed roof surface. These loads then replace the dead loads, wind loads and snow loads given above.

		Lbs.
Gravel or composition roofing	On boards, slope 1 to 6, or less	50
	On boards, slope exceeding 1 to 6	45
	On 3-inch flat tile or cinder concrete	60
Corrugated sheets, on boards or purlins		40
Slate, on boards or purlins		50
Slate, on 3-inch flat tile or cinder concrete		65
Tile on steel purlins		55
Glass		45

Where no snow is to be expected the above loads may be reduced 10 pounds, excepting that no roof should be designed for a load less than 40 pounds per square foot.

74a. **Trusses Forming Parts of Bents.** — Truss members shall be designed for those combinations of dead load, snow load and wind load

which shall require maximum sections. The three possible combinations shall be assumed,

1. Dead load plus maximum snow load.
2. Dead load plus wind load.
3. Dead load plus minimum snow load plus wind load.

The minimum snow load is generally assumed as one-half the maximum snow load. It is commonly considered that the full snow load will not act simultaneously with the maximum wind load.

NOTE. — Where trusses are placed upon columns, forming bents, and the structure is stiffened to resist the wind pressure by the introduction of knee-braces between the trusses and the columns, the stresses, determined upon the basis of an equivalent uniform load, give no indication of the effect of the wind pressure upon the members of the trusses or upon the columns. When designed in this way careful consideration should be given to the effect of wind upon the structure, particularly regarding the possibility of compression in pieces subjected to tension for uniform loads. Such members should preferably be made of two angles having a suitable l/r value.

75. **Minimum Loads on Purlins and Roof Coverings.** — Purlins and roof coverings shall not be designed for normal loads under 30 pounds per square foot.

76. **Loads on Foundations.** — The areas of foundation piers shall be proportional to their respective dead loads; in no case, however, shall the combined live and dead loads on a pier exceed the permissible pressure on the soil.

NOTE. — The desire is to obtain uniform settlement. In making the foundations proportional to the dead loads it is considered that as the dead loads act continuously the settlement will depend more on dead loads than on the possibly very intermittent action of the live loads.

UNIT STRESSES AND PROPORTION OF PARTS

SUBSTRUCTURE

77. **Pressures on Soils.** — The pressures on the soil at the base of the foundation shall not exceed the following in tons of 2000 pounds.

Clay, soft	1
Ordinary clay, or dry sand mixed with clay	2
Dry sand and dry clay	3
Hard clay and firm coarse sand	4
Firm gravel and coarse sand	6
Rock, according to condition	15–200

78. Compressive Stresses in Masonry. — The following compressive stresses will be permitted in masonry structures:

	Lbs. per sq. in.
Common brick, in Portland cement mortar	170
Hard-burned brick, in Portland cement mortar	200
Rubble masonry, in Portland cement mortar	150
Sandstone masonry, first class	280
Limestone masonry, first class	350
Granite masonry, first class	420
Portland cement concrete, 1–2–4	400
Portland cement concrete, 1–2–5	300

79. Wall-plate Pressure. — According to Mr. C. C. Schneider, the pressure of beams, girders, wall plates, columns, etc., on masonry shall not exceed the following:

	Lbs. per sq. in.
On brick work in cement mortar	300
On rubble masonry in cement mortar	250
On Portland cement concrete (1–2–4)	600
On first-class sandstone, dimension stone	400
On first-class limestone	500
On first-class granite	600

80. Bearing Power of Piles. — Piles shall not be spaced closer than 30 inches center to center. The maximum load on any pile shall not exceed 40,000 pounds, or 600 pounds per square inch of average cross section. When driven to rock or equivalent bearing through loose or wet soil, which gives them no lateral support, the limiting load shall be determined by reducing 600 pounds, the maximum allowable compression, by a suitable column formula.

81. Walls shall be built in accordance with the local "building code" when that is available; otherwise they may be made the thicknesses given on page 333.

82. Pillars. — (a) When concentrically loaded and having a height not exceeding twelve times their least dimension, pillars may be loaded until the fiber stresses reach the figures given in paragraph 78.

(b) When eccentrically loaded the resultant pressure must pass within the kern of the section and the maximum pressure should not exceed that given in paragraph 78.

NOTE. — For the explanation of kern of a section see page 247.

Unit Stresses in Steel Work

83. **Permissible Stresses.** — The resulting stresses due to dead load, snow load, wind load and impact shall not exceed the following limiting values, excepting where permitted in accordance with paragraph 91.

84. **Tension,** net section, rolled steel, 16,000 pounds per square inch.

85. **Direct Compression,** rolled steel and steel castings, 16,000 pounds per square inch.

86. **Bending Stresses,** on extreme fibers of rolled shapes, built sections, girders and steel castings, net section, 16,000 pounds per square inch.

On extreme fibers of pins, 24,000 pounds per square inch.

87. **Shearing Stresses.** —

	Lbs. per sq. in.
On shop rivets and pins	12,000
On bolts and field rivets	10,000
On plate girders and beams, cross section, average	10,000

88. **Bearing Stresses.** —

	Lbs. per sq. in.
On shop rivets and pins	24,000
On field rivets and pins	20,000

89. **Pressure on Expansion Rollers.** — The pressure per linear inch on expansion rollers shall not exceed 600 pounds per inch of roller diameter.

90. **Axial Compression in Columns.** — The load per square inch of gross section for columns axially loaded shall neither exceed $16{,}000 - \left(C\frac{l}{r}\right)$ pounds, nor 14,000 pounds. Here l = the length of the member in inches; r = the corresponding radius of gyration of the section in inches; and C has the following values:

Condition of ends.	C.
Both ends hinged or butting	70
Both ends fixed	35
One end fixed, the other hinged	47
One end fixed, the other free	140

Note. — The specifications issued Dec. 1, 1912, by the American Bridge Co. permit loads upon columns, concentrically loaded, as given by the formula $19{,}000 - \left(100 \times \frac{l}{r}\right)$ but not to exceed 13,000 pounds per square inch and applying to values of $\frac{l}{r}$ up to 120.

For values of $\frac{l}{r}$ from 120 to 200 they give the following table:

$\frac{l}{r}$.	Allowable load, lbs. per sq. in.	$\frac{l}{r}$.	Allowable load, lbs. per sq. in.
120	7000	160	5000
130	6500	170	4500
140	6000	180	4000
150	5500	190	3500
...	....	200	3000

91. Stresses in Bracing and Members with Combined Stresses. — In bracing and members subjected to combined stresses resulting from wind and other loadings the permissible working stresses previously given may be increased 25 per cent, but the section shall not be less than that otherwise required by the other loadings excluding the wind load.

92. Eccentric Loading. — In designing columns provision must be made for eccentric loading.

93. Combined Stresses. — Members subjected to combined axial and bending stresses shall be designed so that the greatest fiber stress due to the combined stresses shall not exceed the fiber stress permitted in the member.

94. Alternate Stresses. — Members subjected to a reversal of the stresses shall be proportioned for that stress giving the greater section. The connections shall be designed to carry the sum of the stresses.

95. Net Sections. — In finding net sections, the rivet holes shall be assumed as $\frac{1}{8}$ inch larger than the nominal diameter of the rivet.

96. Limiting Lengths of Columns. — Columns assumed as having hinged or butting ends shall have unsupported lengths not exceeding 125 times their least radius of gyration for main members nor more than 150 times their least radius of gyration for wind bracing or secondary members.

Note. — The recent specifications of the American Bridge Co. place these lengths at 120 and 200 times the least radius of gyration.

97. Limiting Lengths of Tension Pieces. — Riveted tension pieces shall have lengths not exceeding 200 times their radii of gyration about their horizontal axes, when used in horizontal or inclined positions. The length for this calculation shall be considered that of its horizontal projection.

98. **Rolled Sections used as Beams.** — Rolled sections used as beams shall be proportioned by their moments of inertia.

99. **Plate Girders.** — Plate girders shall be designed upon the assumption that one-eighth of the web acts as flange area.

NOTE. — For buildings and similar structures it is frequently specified that the bending stresses shall be resisted by the flanges and that the web shall resist shear only.

NOTE. — The recent specifications of the American Bridge Co. permit the design of girders by their moments of inertia, thus making paragraph 99 apply to both girders and rolled sections.

100. **Web Thickness of Plate Girders.** — The web thickness shall be not less than $\frac{1}{160}$ of the unsupported distance between flange angles.

101. **Web Stiffeners.** — Web stiffeners shall be placed on both sides of the web, with a close bearing against upper and lower flange angles, at the ends and inner edges of bearing plates, at all local and concentrated loads; where the thickness of the web plate is less than $\frac{1}{60}$ of the unsupported distance between flange angles, stiffeners shall be placed at intervals along the girder about equal to the girder depth but not exceeding 5 feet.

NOTE. — The recent specifications of the American Bridge Co. place this limiting distance at 6 feet.

102. **Compression Flange.** — The area of the compression flange must at least equal the area of the tension flange at the same section of the girder. Where the unsupported distance along the girder exceeds 15 times the width of the flange the fiber stress in the compression flange must be that determined by considering the flange as a column and buckling laterally.

NOTE. — The practice on this point varies considerably among designers. Mr. C. C. Schneider limits the fiber stress in the compression flanges at all times to that given by the following formulæ. When the flange section consists of angles and plates the fiber stress is limited by $16{,}000 - \left(200 \times \frac{l}{b}\right)$, and where the flange is a channel section by $16{,}000 - \left(150 \times \frac{l}{b}\right)$. Here l is the distance along the flange between points of support and b is the width of the flange plate, both being in inches.

The requirements as stated by the specifications of the American Bridge Co. are that the flange must be braced at intervals not exceeding 40 times the width of the flange plates, and that the limiting fiber stress for unsupported flange lengths exceeding 10 times the widths of the flanges shall be that given by $19{,}000 - \left(300 \times \frac{l}{b}\right)$. The symbols have the same significance as those given above.

Other specifications permit an unsupported flange length of 12, 16 and even 20 times the width of the flange plates before requiring any reduction in the fiber stress.

103. Depths of Rolled Beams. — The depths of rolled beams where used for floors shall be not less than $\frac{1}{20}$ of the span; when used for purlins they may be as small as $\frac{1}{30}$ of the span. Floor beams when subjected to considerable vibration or shock should be made not less than $\frac{1}{15}$ of their span.

NOTE. — The purpose of this requirement is to limit the deflection.

104. Permissible Stresses on Cast Iron. —

Tension................	2,500 lbs. per sq. in.
Compression............	12,000 lbs. per sq. in.
Shear..................	1,500 lbs. per sq. in.

105. Permissible Stresses on Timber. — The timber in the structure may be designed with the following fiber stresses given in pounds per square inch.

	Bending stress.	Bearing on end.	Columns under 10 diameters.		Shear along fibers.	Weight per cu. ft.
			Compression.	Bearing across fibers.		
White oak........................	1200	1400	1000	500	200	50
Long-leaf yellow pine............	1500	1500	1000	350	100	38
White pine and spruce	900	900	700	200	100	24
Hemlock..........................	700	700	500	200	100	25

106. Timber Columns. — The allowable fiber stress on columns exceeding 10 diameters in length may be obtained by the formula

$$P = C - \left(\frac{C \times l}{100 \times d}\right),$$

where C is the unit stress given above for short columns; l is the length and b the least width of the column, both in inches.

Details of Construction

107. General. — Adjustable members will be, if possible, avoided in all parts of the structure.

108. Symmetrical Sections. — All sections shall preferably be made symmetrical.

109. Minimum Number of Rivets. — Excepting lattice bars all connections shall have at least two rivets.

110. Minimum Thickness of Material. — Excepting for lining or fillers no material shall be less than $\frac{1}{4}$ inch thick.

NOTE. — Some designers place this minimum thickness at $\frac{3}{16}$ inch for material protected from the weather and not affected by injurious gases. Where the conditions might be expected to cause rapid deterioration of the material it is frequently made in excess of the $\frac{1}{4}$ inch first stated.

111. **Connections.** — Connections shall be made strong enough to develop the full strength of the members.

112. **Floor Beams.** — Floor beams shall ordinarily be rolled sections.

113. **Trusses.** — Trusses shall preferably be riveted structures. Heavy trusses of long span may be designed as pin-connected structures.

Roof trusses shall preferably permit of placing purlins at panel points only. Where this would prove uneconomical the upper chord may be designed to resist combined flexure and compression and the purlins then placed where convenient.

114. **Bracing.** — Lateral, longitudinal and transverse bracing in all structures is preferably composed of rigid members, and should be designed of sufficient strength to withstand the wind pressure and any other forces acting on it both during and after erection.

Trusses should be braced in pairs in the plane of their lower and upper chords.

115. **Column Splices.** — Column splices should be designed to resist the bending as well as the direct stresses.

116. **Intersecting Members.** — Intersecting members should be located so that the axes through their centers of gravity shall intersect in a point.

117. **Separators.** — When rolled beams are placed side by side to act as a single girder they shall be secured together by bolts and separators at intervals not exceeding 5 feet. Beams of greater depth than 10 inches should have two bolts in each separator. Parallel beams carrying arched floors, whether of the ordinary or flat arch type, shall have tie bolts spaced not farther apart than 8 times the depth of the beams. These tie bolts should be placed well below the centers of the beams.

118. **Width of Flange Plates.** — Flange plates shall be limited in width so as not to extend more than 6 inches beyond the line of rivets securing the flange plates to the flange angles, nor should this dimension exceed 8 times the thickness of the thinnest flange plate.

119. **Web Splices.** — All web splices must have a plate on each side of the web, and these plates must be able to transmit through their rivets the full stress coming on the splice at that section.

RIVETING

120. **Riveting.** — The minimum distance between centers of rivet holes shall be three diameters of the rivet; it is preferred, however, that this distance should be not less than 3 inches for $\frac{7}{8}$-inch rivets, $2\frac{1}{2}$ inches for $\frac{3}{4}$-inch rivets, 2 inches for $\frac{5}{8}$-inch rivets, and $1\frac{3}{4}$ inches for $\frac{1}{2}$-inch rivets.

The maximum pitch in the line of the stress for members composed of plates and shapes will be 6 inches for $\frac{7}{8}$-inch and $\frac{3}{4}$-inch rivets, $4\frac{1}{2}$ inches for $\frac{5}{8}$-inch rivets and 4 inches for $\frac{1}{2}$-inch rivets.

121. **Rivet Spacing in Angles.** — For angles built into sections with two gauge lines, the rivets being staggered, the maximum pitch in each line shall be twice that given in paragraph 120.

122. **Riveting Plates.** — Where two or more plates are in contact, they shall be held together by rivets whose distance apart in either direction does not exceed 12 inches.

The pitch of rivets in the direction of the stress shall not exceed 6 inches, nor 16 times the thickness of the thinnest outside plate.

The spacing at right angles to the stress shall not exceed 50 times the thickness of the thinnest plate. (Some specifications make this 40 instead of 50.)

123. **Rivet Spacing from Edges.** — The minimum distance of a rivet from a sheared edge shall be $1\frac{1}{2}$ inches for $\frac{7}{8}$-inch rivets, $1\frac{1}{4}$ inches for $\frac{3}{4}$-inch rivets, $1\frac{1}{8}$ inches for $\frac{5}{8}$-inch rivets, and 1 inch for $\frac{1}{2}$-inch rivets. The minimum distances from rolled edges shall be $1\frac{1}{4}$, $1\frac{1}{8}$, 1 and $\frac{7}{8}$ inch, respectively.

The maximum distance of a rivet from an edge shall be 8 times the thickness of a plate.

124. **Calculations of Rivet Strength.** — All calculations of rivet values both in shear and in bearing shall be based upon their nominal diameters.

The grips of rivets should preferably not exceed 4 diameters of the rivets.

NOTE. — The grip is the total thickness of the pieces held by the rivet. This limitation is made to insure the rivet completely filling the rivet hole when driven.

125. **Maximum Rivet Diameters in Angles.** — In main members the diameters of rivets in angles shall not exceed one-quarter the widths of the legs in which they are driven. In other places these diameters may be increased $\frac{1}{8}$ inch.

126. **Pitch of Rivets at Ends of Compression Pieces.** — The pitch of rivets at the ends of compression pieces shall not exceed 4 times the rivet diameter for a distance equal to $1\frac{1}{2}$ times the maximum width of the member.

127. **Tie Plates.** — Latticed sides of compression members shall have tie plates as close to the ends as practicable, and at all intermediate points at which the lacing may be omitted. The lengths of these tie plates shall at least equal the distance between the rivet lines securing them to the flanges. The intermediate plates may be but one-half this length. The thickness of these tie plates must be at least $\frac{1}{50}$ of the distance between the lines of rivets securing the plate to the flanges.

128. Lattice Bars. — Lattice bars shall be proportioned to resist the shear corresponding to the allowance for bending made in the column formula, paragraph 90. The minimum thickness of lattice bars shall be $\frac{1}{40}$ of their lengths for single lattice bars and $\frac{1}{60}$ of their lengths for double lattice bars riveted at their centers.

The minimum widths of lattice bars shall be as follows:

For 15-inch channels, or built sections with $3\frac{1}{2}$ and 4-inch angles,	$2\frac{1}{2}$ inches ($\frac{7}{8}$-inch rivets).
For 12, 10 and 9-inch channels, or built sections with 3-inch angles,	$2\frac{1}{4}$ inches ($\frac{3}{4}$-inch rivets).
For 8 and 7-inch channels, or built sections with $2\frac{1}{2}$-inch angles,	2 inches ($\frac{5}{8}$-inch rivets).
For 6 and 5-inch channels, or built sections with 2-inch angles,	$1\frac{3}{4}$ inches ($\frac{1}{2}$-inch rivets).

NOTE. — The specifications of the American Bridge Co. require the lattice bars to be designed to carry a shear of 2 per cent of the direct stress on the column.

The inclination of the lattice bars with the axis of the member shall not ordinarily be under 45 degrees. When the distance between the rivet lines in the flange is more than 15 inches, if a single riveted bar is used the latticing shall be double, and the bars riveted at their intersection.

Lattice bars with two rivets shall generally be used in flanges wider than 5 inches.

129. Pitch of Lattice Connections along the Member. — The ratio of the pitch of lattice connections along a flange to the least radius of gyration of that side of the member shall not exceed the ratio of length to least radius of gyration for the whole column.

130. Joints. — In general all joints in riveted work, whether for tension or compression members, shall be fully spliced. Joints in compression members when the abutting faces are finished for bearing may be spliced sufficiently to hold the connecting members accurately in place.

131. Pins. — A pin shall have a diameter not under $\frac{3}{4}$ of the width of the widest bar held by it. Pins must be turned true to size and straight; they must be driven with pilot nuts.

132. Pinholes. — Pinholes shall be reinforced by plates where necessary. At least one plate shall be as wide as the projecting flanges will allow; where angles are used, this plate shall be on the same side as the angles. The plates must contain sufficient rivets that their portion of the pin pressure may be distributed to the full cross section of the member.

Pins must be sufficiently long to insure a full bearing of all parts connected upon the turned-down body of the pin. Members should be packed on pins to produce the least bending moment on the pin. Vacant spaces

along the pin should be filled and the members held against lateral movement.

133. **Temperature Range.** — Expansion and contraction shall be provided for corresponding to a temperature range of 150 degrees Fahrenheit.

134. **Expansion Rollers.** — The minimum diameter of expansion rollers shall be 4 inches.

135. **Anchor Bolts.** — Columns, when resisting tensile stresses at their bases, shall be anchored by bolts to the foundations. Anchor bolts shall be long enough to secure a weight of masonry at least one and one-half times the tension in the anchor. The minimum size of anchor bolts shall be $1\frac{1}{4}$ inches.

Materials. — See paragraphs 1 to 18 inclusive.

Workmanship. — See paragraphs 19 to 57 inclusive.

Inspection. — See paragraphs 58 to 65 inclusive.

SPECIFICATIONS FOR A DECK-PLATE GIRDER RAILWAY BRIDGE

136. **Material.** — All material to be rolled steel as specified in paragraphs 1 to 10 inclusive. Cast iron or steel castings will be permitted only in machinery of movable bridges and in special cases for shoes and bearings.

137. **Plate Girders.** — Plate girders are recommended for spans from 20 feet to 100 feet.

138. **Spacing of Girders.** — Deck plate girders shall generally be spaced 6 feet 6 inches.

139. **Floor.** — The floor shall consist of 8-inch by 8-inch cross ties separated 6 inches. They shall be notched to fit on the flanges on which they should have a full and even bearing.

140. **Guard Rails.** — Guard timbers 6 by 8 inches shall be placed on each side of the track. Their inner faces shall be not less than 3 feet 3 inches from the center of the track. These guard timbers shall be notched one inch over each tie and shall be fastened to every third tie and at each splice by a $\frac{3}{4}$-inch bolt. Splices shall be over floor timbers with half-and-half joints of 6 inches lap. The floor and guard timbers must be continued over piers and abutments.

Loads

141. **Dead Load.** — In estimating the weight of the structure, for use in calculating the strains, the timber shall be assumed as weighing $4\frac{1}{2}$ pounds per foot B. M. (board measure, 144 cubic inches). The weight of the rails, spikes and joints shall be taken at 160 pounds per lineal foot of track.

142. **Live Load.** — All bridges shall be designed to carry, in addition to their own weight and that of the floor, a moving load on each track con-

sisting of two engines coupled at the head of a uniformly distributed train load, placed so as to give the greatest strain in the structure.

This loading is frequently specified Cooper's E–40 as a minimum.

NOTE. — The loading Cooper's E–60 is given on page 72. For the other Cooper's loadings the distances between the wheels remain the same as the E–60 loading. The driving-wheel loads are 1000 times the number designating the loading; thus for E–60 the load on the drivers is 60,000 pounds, while for E–40 it is 40,000 pounds. The pilot-wheel load is assumed 50 per cent of the load on the drivers, while the tender-wheel loads are 65 per cent of the driving-wheel loads.

143. Impact Allowance. — The effect of impact and vibration shall be added to the above-mentioned maximum live-load stresses, and shall be determined by the formula,

$$I = S\left(\frac{300}{300 + L}\right),$$

where I = impact to be added to the live-load stresses;
S = computed live-load stress;
L = length of loaded distance which produces the maximum stress in the member.

Impact shall not be added to stresses produced by longitudinal, centrifugal and lateral wind forces.

144. Wind and Lateral Loading. — All bridges shall be designed for a lateral force on the loaded chord of 200 pounds per lineal foot plus 10 per cent of the specified train load on one track, and 200 pounds per lineal foot on the unloaded chord, these forces being considered as moving.

NOTE. — This loading provides for the wind load and vibration and impact due to lateral swaying of the train caused by unbalancing of the locomotives, etc.

UNIT STRESSES

145. Unit Stresses in Structural Steel. — All parts of the structure shall be so designed that on structural and rivet steels the sums of the maximum stresses shall not exceed the following unit stresses, excepting as specified in paragraph 91.

Character of stress.	Allowable unit stress, lbs. per sq. in.
Axial tension, net section	16,000
Flexural stress, extreme fiber stress, net section	16,000
Flexural stress, on pins, extreme fibers	24,000
Axial compression, gross section	$S = 16{,}000 - \left(70\frac{l}{r}\right).$

S = allowable unit stress;
l = unsupported length of member, in inches;
r = least radius of gyration of the member, in inches.

146. Stresses on Rivets. — The following are the allowable working stresses in pounds per square inch on bolts and rivets in shear and bearing.

Character of bolt or rivet.	Unit stress.	
	Shear.	Bearing.
Hand-driven field rivets and turned bolts.......	9,000	18,000
Power-driven field rivets......................	11,000	22,000
Shop-driven rivets and pins	12,000	24,000
Plate girder webs, gross section.................	10,000	......
Expansion rollers, per linear inch, where d = the diameter of the roller in inches..............	......	$600 \times d$

147. Pressure on Foundations. — The pressures on masonry piers shall not exceed the following loads in pounds per square inch.

	Pounds per sq. in.
First-class sandstone or limestone masonry, and Portland cement concrete, 1–2–4, including impact..................	400
First-class granite masonry, and Portland cement mortar, including impact.....................................	600

See also paragraph 79.

148. Pressure on Soils. — See paragraph 77.

149. Combined Stresses. — See paragraph 93.

150. Girder Design. — The depth shall preferably be not less than $\frac{1}{12}$ of the span. When made shallower the flange stresses should be reduced so that the deflection will not exceed that of a girder having its depth $\frac{1}{12}$ of the span. Girders may be designed either by calculating the moments of inertia of their net sections or by assuming the flange areas as concentrated at their centers of gravity; in this case $\frac{1}{8}$ of the gross web section may be considered as flange area; when this is assumed the web splices must be designed for carrying bending as well as shear.

151. Flange Design. — The flanges shall be designed so that the area of the cover plates shall not exceed 60 per cent of the total flange area. See also paragraph 118.

152. Compression Flange. — The area of the compression flange must at least equal the area of the tension flange at the same section of the girder. Where the unsupported distance along the girder exceeds 16 times the width of the flange the fiber stress in the compression flange must be limited by

that determined by the following formulæ. Where the flange consists of plates, $f = 16{,}000 - \left(200 \times \frac{l}{b}\right)$; where the flange is a channel section, $f = 16{,}000 - \left(150 \times \frac{l}{b}\right)$; here

f is the allowable fiber stress in pounds per square inch;
l is the distance between lateral supports along the girder, in inches;
b is the width of the flange in inches.

153. **Flange Rivets.** — The rivets securing the upper flange angles to the web plate must be calculated to resist the change in horizontal shear, together with such vertical shear due to wheel concentrations and floor loads as may be transferred to the web through the flange angles.

When the rails are placed directly on the upper flange the wheel loads shall be assumed as distributed over 30 inches; if ties transfer this load to the girder, the load may then be considered as distributed over three ties.

154. **Web Design.** — The web shall be designed for the total maximum shear, including dead load, live load and impact; the fiber stress on the gross section shall not exceed 10,000 pounds per square inch. See also paragraph 100.

155. **Web Splices.** — See paragraph 150.

156. **Web Stiffeners.** — There shall be web stiffeners, generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than $\frac{1}{60}$ of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a maximum limit of six feet and not greater than the depth of the web; $d = \frac{t}{40}(12{,}000 - s)$.

d = the clear distance between stiffeners, in inches;
t = the web thickness, in inches;
s = the unit shearing stress in the web, in pounds per square inch.

157. **Bracing.** — Lateral bracing shall be placed as close as possible to the plane of the upper chord when clearing the ties.

Spans exceeding 70 feet shall have lateral bracing near the plane of both top and bottom chords. Cross frames shall be placed at the ends and at intermediate points separated not more than 20 feet.

158. **Minimum Sizes of Materials.** — Excepting for fillers the minimum thickness of material shall be $\frac{3}{8}$ inch. Rivets shall not be under $\frac{3}{4}$ inch diameter.

159. **Rivet Spacing.** — See paragraphs 120 to 126.

160. **Expansion.** — See paragraphs 133 and 134.

All spans exceeding 80 feet in length shall have hinged bolsters at both ends, and at one end turned rollers, running between planed surfaces.

Rollers shall not be under 4 inches in diameter. Bridges with spans under 80 feet shall have one end free to move upon planed surfaces.

161. **Camber.** — The camber shall be $\frac{1}{16}$ inch for every 10 feet.

NOTE. — Camber is an upward curvature given the bridge and should theoretically be equal and opposite to that caused by the deflection of the bridge under the load for which it was designed. It, therefore, serves to bring the nominally horizontal members into actually horizontal positions when subjected to their full load. Under full load then the several members will be in the same relative positions as those for which their stresses were determined.

SPECIFICATIONS FOR PORTLAND CEMENT CONCRETE AND REINFORCED CONCRETE

162. **Cement.** — Cement shall be Portland, and shall meet the requirements of the standard specifications of the American Society for Testing Materials.

163. **Fine Aggregates.** — These shall consist of sand, crushed stone, or gravel screenings graded from fine to coarse, and when dry passing a screen having holes $\frac{1}{4}$ inch in diameter; it shall preferably be of siliceous material, clean, coarse, free from vegetable loam or other deleterious matter, and not more than 6 per cent shall pass a sieve having 100 meshes per linear inch.

164. **Test.** — Mortars composed of one part Portland cement and three parts fine aggregate by weight when made into briquettes shall show a tensile strength of at least 70 per cent of the strength of 1–3 mortar of the same consistency made with the same cement and standard Ottawa sand.

165. **Coarse Aggregates.** — These shall consist of crushed stone or gravel, graded in size, which is retained on a screen having holes $\frac{1}{4}$ inch in diameter; it shall be clean, hard, durable and free from all deleterious matter. Aggregates containing soft, flat or elongated particles shall not be used.

166. **Maximum Size of Coarse Aggregate.** — The maximum size of the coarse aggregate shall be such that it will not separate from the mortar in laying and will not prevent the concrete fully surrounding the reinforcement or filling all parts of the forms.

Where concrete is used in mass the maximum size of the coarse aggregate may, at the option of the engineer, be such as to pass a 3-inch ring. For reinforced concrete, sizes usually are not to exceed one inch in any direction, but may be varied to suit the character of the reinforcement.

167. Water. — Water used in mixing concrete shall be free from oil, acid, alkalies, or vegetable matter.

Reinforcing Steel

168. Metal Reinforcement. — The metal reinforcing steel shall be manufactured from new billets and shall meet the requirements of the following specifications and be free from rust, scale, or coatings of any character which would tend to reduce or destroy the bond.

169. Process of Manufacture. — See paragraph 1.

170. Chemical and Physical Requirements. —

Requirements.	Structural steel.	High carbon steel.
Phosphorus, max. Basic	0.04%	0.085%
Phosphorus, max. Acid	0.06%	0.075%
Sulphur, maximum	0.05%	....
Ultimate tensile strength, lbs. per sq. in.	Desired 60,000	Desired 85,000
Elongation, min. per cent in 8 ins., Fig. 1	$\frac{1,500,000}{\text{Ult. ten. str.}}$	$\frac{1,400,000}{\text{Ult. ten. str.}}$
Elongation, min. per cent in 2 ins., Fig. 2	22%	....
Character of fracture	Silky	....
Cold bends without fracture	180° flat	180° $d=4t$

171. Yield Point.—The yield point, as indicated by the drop of the beam, shall be not less than 60 per cent of the ultimate strength.

172. Allowable Variations. — See paragraph 3.

173. Chemical Analyses. — See paragraph 9.

174. Forms of Specimens. — See paragraph 10.

175. Number of Tests. — At least one tensile and one bending test shall be made from each melt of steel as rolled.

176. Modifications in Elongation. — See paragraph 4.

177. Bending Tests. — Bending tests may be made by pressure or blows. Shapes and bars less than one inch thick shall bend as called for in paragraph 170.

178. Bending Tests of Thick Material. — See paragraph 5.

179. Finish. — See paragraph 14.

180. Stamping. — See paragraph 15.

181. Defective Material. — See paragraph 16.

Concrete

182. Proportions. — The materials to be used in concrete shall be of uniform quality and so proportioned as to secure as nearly as possible a maximum density.

183. Unit of Measure. — The unit of measure shall be the barrel, which shall be taken to be 3.8 cubic feet. Four bags containing 94 pounds of cement each shall be considered the equivalent of one barrel. Fine and coarse aggregates shall be measured separately as loosely thrown into the measuring receptacle.

184. Relation of Fine and Coarse Aggregate. — The fine and coarse aggregate shall be used in such relative proportions as will insure maximum density.

185. Relation of Cement and Aggregates. — For reinforced-concrete construction a density proportion based on one to six shall be used, *i.e.*, one part of cement to a total of six parts of fine and coarse aggregates measured separately.

186. Mixture for Massive Concrete. — For massive or rubble concrete a density proportion based on one to nine shall be used.

187. Mixing. — The ingredients of concrete shall be thoroughly mixed to the desired consistency, and the mixing shall continue until the cement is uniformly distributed and the mass is uniform in color and homogeneous.

188. Measuring Proportions. — Methods of measurement of the proportions of the various ingredients, including the water, shall be used, which will secure separate uniform measurements at all times.

189. Machine Mixing. — When the conditions will permit a batch mixer of a type which insures uniform mixing of the materials throughout the mass shall be used.

190. Hand Mixing. — When it is necessary to mix by hand the mixing shall be on a watertight platform and especial precautions shall be taken to turn the materials until they are homogeneous in appearance and color.

(a) Tight platforms shall be provided of sufficient size to accommodate men and materials for the progressive and rapid mixing of at least two batches of concrete at the same time. Batches shall not exceed one cubic yard each, and smaller batches are preferable, based upon a multiple of the number of sacks of cement to the barrel.

(b) Spread the fine aggregates evenly upon the platform, and the cement upon the fine aggregates; mix these thoroughly until of an even color. Add all the water necessary to make a thin mortar and spread again; add the coarse aggregates, which, if dry, should first be thoroughly wet down. Turn the mass with shovels or hoes until thoroughly incorporated and all the aggregates are thoroughly covered with mortar; this will probably require the mass to be turned four times.

(c) Another approved method, which may be permitted at the option of the engineer in charge, is to spread the fine aggregates, then the cement, and mix dry, then the coarse aggregates; add water and mix thoroughly as above.

191. Water. — The materials shall be mixed wet enough to produce a concrete of such a consistency as will flow into the forms and into the metal reinforcement, and which, on the other hand, can be conveyed from the place of mixing to the forms without separation of the coarse aggregate from the mortar.

192. Retempering. — Remixing concrete with water after it has partially set will not be permitted.

193. Placing Concrete. — (a) After the addition of water, it shall be handled rapidly from the place of mixing to the place of final deposit, and under no circumstances shall concrete be used that has partially set before final placing.

(b) The concrete shall be deposited in such a manner as will permit the most thorough compacting, such as can be obtained by working with a straight shovel, or slicing tool kept moving up and down until all the ingredients have settled in their proper place by gravity and the surplus water has been forced to the top.

(c) In depositing concrete under water, special care shall be exercised to prevent the cement from floating away, and to prevent the formation of laitance.

NOTE. — Laitance is a whitish gelatinous substance, of about the same composition as cement but having little or no hardening properties. It is caused by the action of water on the surface of the concrete.

(d) Before placing concrete the forms shall be thoroughly wetted and the space to be occupied by the concrete freed from débris.

(e) When work is resumed, concrete previously placed shall be roughened, thoroughly cleaned of foreign material and laitance, drenched and slushed with a mortar consisting of one part of Portland cement and not more than two parts of fine aggregate.

(f) The faces of concrete exposed to premature drying shall be kept wet for a period of at least seven days.

194. Freezing Weather. — The concrete shall not be mixed or deposited at a freezing temperature, unless special precautions, approved by the engineer, are taken to avoid the use of materials containing frost or covered with ice crystals, and to provide means to prevent the concrete from freezing after being placed in position and until it is thoroughly hardened.

195. Rubble Concrete. — Where the concrete is to be deposited in massive work, clean stones, thoroughly embedded in the concrete as near together as is possible and still entirely surrounded by concrete, may be used at the option of the engineer.

196. Forms. — Forms shall be substantial and unyielding and built so that the concrete shall conform to the designed dimensions and contours,

and so constructed as to prevent the leakage of mortar. These forms shall not be removed until authorized by the engineer.

197. **Forms for Important Work.** — For important work, the lumber used for face work shall be dressed to a uniform thickness and width, and shall be sound and free from loose knots, secured to the studding or uprights in horizontal lines.

198. **Less Important Work.** — For backings and other rough work undressed lumber may be used.

199. **Round Corners.** — Where corners of masonry and other projections liable to injury occur, suitable moldings shall be placed in the angles of the forms to round or bevel them off.

200. **Re-using Lumber.** — Lumber once used in forms shall be cleaned before using again.

201. **Wetting Forms.** — In dry but not freezing weather the forms shall be drenched with water before the concrete is placed against them.

Details of Construction

202. **Splicing Reinforcement.** — Whenever it is necessary to splice the reinforcement by lapping, the length of lap will be decided by the engineer on the basis of the safe bond stress in the reinforcement at the point of splice. Splices shall not be made at the points of maximum stress.

203. **Joints in Concrete.** — Concrete structures, wherever possible, shall be cast in one operation, but when this is not possible the work shall be stopped, so that the resulting joint shall have the least effect on the strength of the structure.

204. **Placing Girders and Slabs.** — Girders or slabs shall not be placed over freshly formed walls or columns without permitting a period of at least two hours to elapse to provide for settlement or shrinkage in the supports. Before resuming work the top of the supports should be thoroughly cleansed of foreign matter and laitance.

205. **Temperature Changes.** — In massive work, such as retaining walls, abutments, etc., built without reinforcement, joints shall be provided approximately every 50 feet throughout the length of the structure to care for the temperature changes. To provide against the structures being thrown out of line by unequal settlement, each section of the wall may be tongued and grooved into the adjoining section. To provide against unsightly cracks, due to unequal settlement, a joint shall be made at sharp angles.

206. **Surface Finish.** — The desired finish of the surface shall be determined by the engineer before the concrete is placed, and the work shall be so conducted as to make it possible to secure the finish desired.

Plastering of surfaces will not be permitted.

NOTE. — The preceding portion of this specification is taken from those of the American Railway Engineering and Maintenance of Way Association, Proceedings of 1909.

207. Fireproofing. — For ordinary conditions, it is recommended that the metal in columns and girders be protected by a minimum of two inches of concrete; that the metal in beams be protected by one and one-half inches of concrete and that the metal in slabs be protected by at least one inch of concrete. In monolithic columns a section around the column having a width of at least one and one-half inches should be allowed for fireproofing and not estimated as carrying any load. All corners of columns should be rounded, as rounded corners are affected less by fire and are also less liable to other injury.

LOADS

208. Vertical Loads. — The same vertical loads as those given under "Steel Mill Building Specifications" may be followed; see paragraphs 68, 69, 70 and 74. Buildings in cities should be designed in accordance with the local building laws. Where such laws are not available the following floor loads may be assumed:

Office floors..........................	75 lbs. per sq. ft.
Floors for light running machinery.....	75–150 lbs. per sq. ft.
Floors for medium heavy running machinery........................	200 lbs. per sq. ft.
Storage, to be estimated from weight of the materials in each case..........	150–1000 lbs. per sq. ft.

The weight of the concrete shall be assumed as 150 pounds per cubic foot. For Loads on Bridges see paragraphs 141, 142 and 143.

209. Wind Loads. — These will be the same as those given under the previous specifications; see paragraphs 71, 72, 73 and 144.

210. Impact. — Impact may be taken into account by making additions to the live-load stresses.

211. Dimensions for Calculations. — (a) The spans of beams and girders shall be assumed as the distance center to center of supports but need not be taken greater than the clear span plus the depth of the beam or girder.

(b) For slabs supported at the ends the span shall be assumed as the clear span plus the depth of the slab.

(c) For continuous slabs the spans shall be taken as the distances between the centers of supports.

212. Bending Moments. — (a) For continuous beams and slabs the bending moments at the center and at the supports shall be assumed as

80 per cent of the moment on a freely supported beam having the same span and load.

NOTE. — The Specifications of the Joint Committee of the A. S. C. E., A. S. T. M., A. R. E. and M. of W. Assn., etc., give this as 67 per cent for interior spans but 80 per cent for end spans. In the case of beams and slabs continuous for two spans only, the bending moment at central support shall be calculated, using 100 per cent as the factor, while near the middle of the span the factor shall be assumed 80 per cent.

(b) Floor slabs should be designed and reinforced as continuous over the supports. If the length of the slab exceeds one and one-half times its width, the entire load shall be carried by the transverse reinforcement.

Square slabs shall be reinforced in both directions.

NOTE. — The distribution of loads in the two directions upon slabs having a ratio of length to width not exceeding 1 to 1.5 will be approximately as given in the following table.

$\frac{l}{b}$.	r.	$\frac{l}{b}$.	r.
1.0	0.50	1.3	0.75
1.1	0.59	1.4	0.80
1.2	0.67	1.5	0.83

Here

r is the proportion of the load carried by the transverse reinforcement,
l is the length, and
b is the breadth of the slab.

Using the values just specified, each set of reinforcements is to be calculated in the same manner as for slabs having supports on two sides only, but the total amount of reinforcement thus determined may be reduced 25 per cent by gradually increasing the rod spacing from the third point to the edge of the slab.

213. **T Beams.** — In beam-and-slab construction, an effective bond should be provided at the junction of the beam and slab. When the principal slab reinforcement is parallel to the beam, transverse reinforcement should be used, extending over the beam and well into the slab.

Where adequate bond and shearing resistance between the slab and web of beam is provided, the slab may be considered as an integral part of the beam, but its effective width shall be determined by the following rules: (a) It shall not exceed one-fourth of the span length of the beam; (b) its overhanging width on either side of the web shall not exceed four times the thickness of the slab.

In the design of T beams acting as continuous beams, careful consideration must be given to the compressive stresses at the supports.

Beams in which the T form is used only for the purpose of providing additional compression area of concrete should preferably have a width of flange not more than three times the width of the stem and a thickness of the flange not less than one-third of the depth of the beam.

Both in this form and in the beam and slab form, the web stresses and the limitations in placing the longitudinal reinforcements will probably be controlling factors in the design.

214. **Internal Stresses.** — The internal stresses in slabs, beams and girders shall be determined by the formulæ recommended by the Joint Committee of the A. S. C. E., etc., which are given in the body of the book, page 215. Throughout the entire beam the shearing and bonding stresses must be determined and proper provision made to develop the required strength.

215. **Columns.** — The ratio of the unsupported length of a column divided by its least width shall not exceed 15. The effective area of a column shall be considered as the area within the fireproofing, or in the case of hooped columns, or columns reinforced with structural shapes, it shall be taken as the area within the hooping or structural shapes.

The following working fiber stresses may be allowed in columns varying with the class of reinforcement in it.

(a) Columns with longitudinal reinforcement only, such reinforcement not less than 1 per cent nor more than 4 per cent. The unit stress shall be those allowed for axial compression in Working Stresses, paragraph 218.

(b) Columns with reinforcements of bands, hoops or spirals, as hereafter specified: the stresses allowed shall be 20 per cent higher than those given for (a), provided the ratio of the unsupported length of the column to the diameter of the hooped core is not more than eight.

(c) Columns reinforced with not less than 1 per cent and not more than 4 per cent of longitudinal bars, and with hoops or spirals, as hereafter specified: the stresses shall be 45 per cent higher than those given for (a), provided the ratio of the unsupported length of the column to the diameter of the hooped core is not more than eight.

NOTE. — In all cases the longitudinal reinforcement is supposed to carry its proportion of the stress. The hoops or bands are not to be counted on as adding directly to the strength of the column.

Bars composing longitudinal reinforcement shall be straight and shall have sufficient lateral support to be securely held in place until the concrete has set.

Where hooping is used, the total amount of such reinforcement shall not be less than 1 per cent of the volume of the column inclosed. The clear spacing of such hooping shall not be greater than one-sixth of the diameter of the inclosed column,

and preferably not greater than one-tenth; in no case shall it exceed 2.5 inches. The hooping must be circular, and the ends of bands must be united to develop the full strength of the bands. Adequate means must be provided to hold the hoops or bands in place so that the column shall have a straight and well-centered core. As the effect of hooping decreases rapidly with the increase of ratio of length to core diameter the above fiber stresses are limited to columns in which this ratio does not exceed 8.

216. Bending Stresses on Columns. — Bending stresses due to eccentric loading and lateral forces must be provided for by increasing the section until the maximum stress does not exceed the values above specified; and where tension is possible in the longitudinal bars, adequate connection between the ends of the bars must be provided to take this tension.

217. Reinforcing for Shrinkage and Temperature Stresses. — Reinforcement not under one-third of 1 per cent, and able to develop a high bonding strength shall be provided; it shall be placed near the exposed surfaces and be well distributed.

Working Stresses

218. Working Fiber Stresses. — The following working stresses for concrete are based on the compressive strength developed by the concrete after 28 days, when tested in cylinders 8 inches in diameter and 16 inches long. Such tests should show the following ultimate strengths, in pounds per square inch.

Aggregate.	1:1:2	1:1.5:3	1:2:4	1:2.5:5	1:3:6
Granite, trap rock	3300	2800	2200	1800	1400
Gravel, hard limestone and hard sandstone	3000	2500	2000	1600	1300
Soft limestone and sandstone	2200	1800	1500	1200	1000
Cinders	800	700	600	500	400

(a) **Bearing.** — When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 32.5 per cent of the ultimate compressive strength may be allowed.

(b) **Axial Compression.** — For concentric compression on a plain concrete column or pier, the length of which does not exceed 12 diameters, 22.5 per cent of the ultimate compressive strength of the concrete may be allowed. See also paragraph 215.

(c) **Compression in Extreme Fibers.** — The extreme fiber stress on a beam, calculated on the assumption of a constant modulus of elasticity for the concrete under working stresses, may reach 32.5 per cent of the ultimate

compressive strength of the concrete. Adjacent to the supports of continuous beams, these stresses may be increased 15 per cent.

(d) **Shear and Diagonal Tension.** — In calculations on beams in which the maximum shearing stress in a section is used as the means of measuring the resistance to diagonal tension stress, the following allowable values for the maximum vertical shearing stresses are recommended.

1. For beams with horizontal bars only and without web reinforcement, 2 per cent of the ultimate compressive strength may be allowed.

2. For beams thoroughly reinforced with web reinforcement, the value of the shearing stress having been calculated, using the total vertical shear in determining the unit horizontal shear, the working shearing stress should not exceed 6 per cent of the ultimate compressive strength of the concrete. The web reinforcements, exclusive of the bent-up bars in this case, shall be proportioned to resist two-thirds of the external vertical shear.

3. For beams in which part of the longitudinal reinforcement is used in the form of bent-up bars distributed over a portion of the beam in a way covering the requirements of this type of web reinforcement: the limit of the allowable working shearing stress shall be 3 per cent of the ultimate compressive strength of the concrete.

4. Where punching shear occurs, that is, shearing stress uncombined with compression normal to the shearing surface, and with all tension normal to the shearing plane provided for by reinforcement; a shearing stress of 6 per cent of the ultimate compressive strength of the concrete may be allowed.

(e) **Bond.** — The bond stress between plain reinforcing bars and concrete may be assumed as 4 per cent of the ultimate compressive strength of the concrete. In the case of drawn wire, 2 per cent should be the limit.

219. Steel. — The working fiber stress in the steel shall not exceed 16,000 pounds per square inch.

220. Modulus of Elasticity. — The ratio of the moduli of elasticity of steel and concrete will vary with the ultimate compressive strength of the concrete. The following values may be used.

Strength of concrete, lbs. per sq. in.	Ratio of moduli of elasticity, concrete to steel.
2200 and less	$\frac{1}{15}$
Exceeding 2200 but under 2900	$\frac{1}{12}$
Exceeding 2900	$\frac{1}{10}$

CHAPTER XXII

PROBLEMS

The object of these problems is to furnish work for the student paralleling that in the drawing room. Introductory review problems in *Mechanics* are given that lead up to the principal problems in the book. As far as possible, all the problems have been made similar to those occurring frequently in practice. Although the tables of sections in the book give sufficient data for the working of the problems the writer believes that the small additional outlay for one of the handbooks issued by the manufacturers of structural steel is money well spent. Among those issuing these books are the Cambria Steel Company of Johnstown, Pa., and the Carnegie Steel Company and Jones & Laughlin Company of Pittsburgh, Pa.

Center of Gravity

1. Prove that the distance from the base to the center of gravity of a rectangle is $\frac{1}{2}h$.

2. Prove that the distance from the base to the center of gravity of a triangle is $\frac{1}{3}h$.

3. Prove that the distance from the base to the center of gravity of a half-round section is $c = 0.4244\ R$.

Note. — Use the fact established in Problem 1 and make the infinitesimal strips run from the diameter to the circumference.

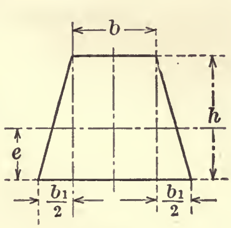


Fig. 1.

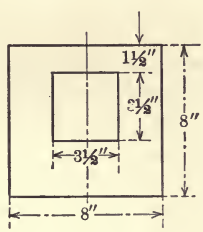


Fig. 2.

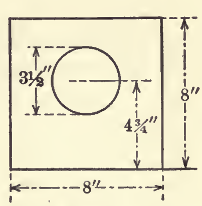


Fig. 3.

4. Prove that in the given trapezoid, Fig. 1,

$$e = \frac{1}{3} \cdot \frac{3b + b_1}{2b + b_1} \times h.$$

5. Calculate the distance from the back to the center of gravity of a 15-in. channel weighing 33 lbs. per ft. See p. 16.

6. In a $6 \times 4 \times \frac{1}{2}$-in. angle calculate the distance of the center of gravity from the back of the short leg.

7. How far from the base is the center of gravity in Fig. 2?

8. How far from the base is the center of gravity in Fig. 3?

9. Prove that the center of gravity of a semicircular arc is $c = \frac{2r}{\pi}$ from the diameter joining its extremities.

10. How far from its base is the center of gravity of a cone? How far from its base is the center of gravity of its lateral surface?

Moments of Inertia

$$I = \int y^2 dA.$$

Given I the moment of inertia, then I' referred to any axis parallel to the principal axis is $I' = I + Ah^2$.

In these problems do not recalculate positions of centers of gravity.

11. Calculate the moment of inertia about the principal axis of a rectangle of altitude h and width b.

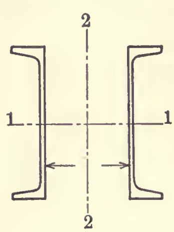


FIG. 4.

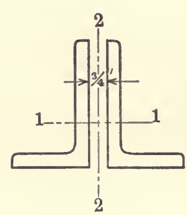


FIG. 5.

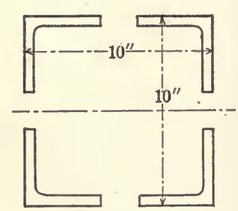


FIG. 6.

12. Calculate the moment of inertia of a triangle about an axis through its axis center of gravity and parallel to its base.

13. Calculate the moment of inertia of a circle for an axis through its center. (Use polar coördinates.)

14. Calculate the moment of inertia of a $6 \times 4 \times \frac{3}{8}$-in. angle referred to an axis through its center of gravity and parallel to its short leg.

15. Calculate the inertia of a 12-in. channel at $20\frac{1}{2}$ lbs. per ft. for an axis through its center of gravity and parallel to its back.

16. Calculate the inertia of two 10-in. channels at 15 lbs. per ft. referred to axis 2–2 when the distance between channel backs is 6.33 ins. (See Fig. 4.)

17. Calculate the inertia about axis 2–2 for two $5 \times 3 \times \frac{1}{2}$-in. angles placed as in Fig. 5.

18. Calculate the inertia for the horizontal axis passing through center of gravity of Fig. 2.

19. Calculate the inertia for axis 1–1 of four $4 \times 4 \times \frac{1}{2}$-in. angles placed as in Fig. 6.

20. Calculate the inertia of a half-round for axis 1–1, Fig. 7.

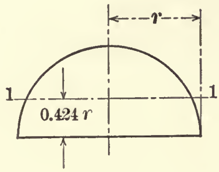


FIG. 7.

FIG. 8.

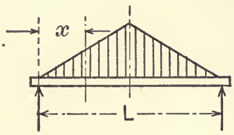


FIG. 9.

21. Calculate the inertia of the girder section referred to axis 1–1, Fig. 8.
   Web $\frac{1}{4}$-in. plate.
   Angles $4 \times 4 \times \frac{3}{8}$ in.
   Flange plates $10 \times \frac{7}{16}$ in.

Assume the section contains two $\frac{7}{8}$-in. diam. holes for $\frac{3}{4}$-in. rivets through horizontal legs of angles and flange plates for which allowance must be made.

Reactions and Bending Moments

It should be recalled that for equilibrium when the forces act on a section or a point that the

Sum of the horizontal forces = 0.
Sum of the vertical forces = 0.
Sum of the moments of forces = 0.

The vertical shear at a section is the algebraic sum of the vertical forces to the left of that section.

22. A cantilever beam of length L ft. carries the load P at its free end. What is the reaction at the support, bending moment at this point and also bending moment a distance X from free end?

23. A cantilever beam of length L ft. carries a uniform load of W lbs. per ft. What is the reaction, the bending moment at the support and also at a distance X ft. from the free end?

24. A cantilever beam carries a load of W lbs., varying uniformly from zero at the free end to a maximum at the support. Required the reaction, the bending moment at the support and at a distance X ft. from the free end.

25. A simple beam of span L ft. carries a central load of W lbs. What are the reactions, the maximum bending moment and the bending moment X ft. to the right of the left support?

26. A simple beam carries a uniform load of W lbs. per ft. Its span is L ft. What are the reactions, the maximum bending moment and the bending moment X ft. to the right of the left support?

27. A simple beam, Fig. 9, carries a load W, varying from zero at supports to a maximum at the middle. Required the reactions, bending moment at the middle and at a distance X ft. from the left support.

28. A simple beam, Fig. 10, carries a load W lbs., varying from zero at the left support to a maximum at the right support. Required the reactions, the maximum bending moment and the distance it occurs from the left support.

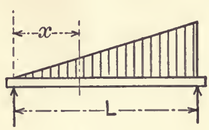


FIG. 10.

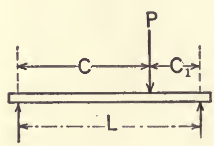


FIG. 11.

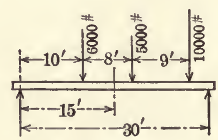


FIG. 12.

29. Required the reactions, maximum bending moment, and bending moment X ft. from the left support in Fig. 11.

30. In Fig. 12 find the reactions and maximum bending moment. What is the bending moment 15 ft. from the left support?

31. In Fig. 13 what are the reactions? What is the bending moment over support? What is the bending moment X ft. to the right of the left support? What is the maximum bending moment and where does it occur?

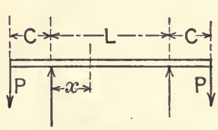


FIG. 13.

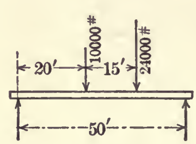


FIG. 14.

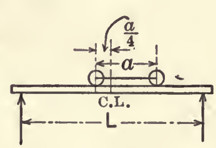


FIG. 15.

32. The given beam in Fig. 14 in addition to the loads shown carries a uniform load of 500 lbs. per ft. of span. What is the maximum bending moment and where does it occur? What is the bending moment 28 ft. from the left support?

33. Prove that in Fig. 15 when two loads P, a constant distance a apart, roll across a girder the maximum bending occurs when a load is a distance $\frac{a}{4}$ from the center of the span.

Selection of Beam Sections

In selecting beams from manufacturers' handbooks it is common practice to use a factor called the section modulus which is tabulated in the handbooks. The section modulus $= \frac{I}{e}$ or moment of inertia divided by the distance from the neutral axis to the extreme fibers.

Since $$M = \frac{fI}{e}, \quad \text{it follows} \quad \frac{M}{f} = \frac{I}{e};$$

hence, first find $\frac{M}{f}$ by dividing the maximum bending moment in inch pounds by the allowable working fiber stress. In the following problems assume the beams sufficiently stiffened laterally. Problems where the ratio of span to flange width must be considered will be taken up later.

34. Select a beam to carry a uniform load of 1500 lbs. per lineal foot on a span of 20 ft.; allow a working fiber stress of 15,000 lbs. per sq. in.

35. A beam of 32-ft. span carries four loads of 5000 lbs. each, spaced 8 ft. apart, while the first is 4 ft. from the left support. Allow 15,000 lbs. fiber stress per sq. in. and select an economical beam section, neglecting the bending due to the beam's weight.

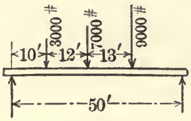


Fig. 16.

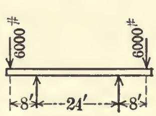


Fig. 17.

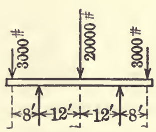


Fig. 18.

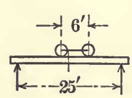


Fig. 19.

36. In Fig. 16 assume also a uniform load including weight of beam as 100 lbs. per lineal foot. Select an economical beam section, allowing a working fiber stress of 12,000 lbs. per sq. in.

37. In Fig. 17 neglect weight of beam, allow 12,000 lbs. fiber stress and select a suitable section.

38. The loading is as shown in Fig. 18. Neglect weight of beam. Allow a working fiber stress of 12,000 lbs. per sq. in. and select an economical beam section.

39. Select I beam for Fig. 19. Assume beam secured laterally. The moving wheel loads are 6 ft. 0 ins. *c–c*. Each wheel load is 10,000 lbs. Use a working fiber stress of 12,000 lbs. per sq. in. Assume dead load including weight of beam as 60 lbs. per ft.

40. Select I beam for the span in Fig. 20. Assume beam secured laterally. The moving wheel loads are 12 ft. 0 ins. *c–c*. Each wheel load is 12,000 lbs. Allowable working fiber stress is 12,000 lbs. per sq. in. Assume dead load including weight of beam as 80 lbs. per ft.

Deflection of Beams

It is not proposed to review the entire discussion of deflection of beams but simply to recall the subject by one or two of the easier cases and then solve a few problems by the use of the formulæ given on page 7. The general equation of the elastic curve for all beams is

$$\frac{d^2y}{dx^2} = \frac{M}{EI};$$

here M = bending moment due to the external forces at a section whose abscissa is X.

I = moment of inertia of beam section.

E = modulus of elasticity of the material.

y = ordinate along which deflection is measured.

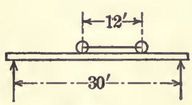


Fig. 20.

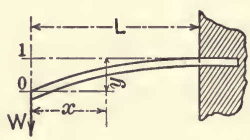


Fig. 21.

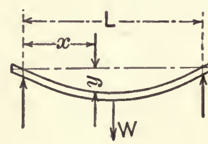


Fig. 22.

41. A cantilever beam, Fig. 21, of uniform section carries a concentrated load W at its free end. What is its deflection if its length is L?

Assume origin of coördinates at o. Here $M = WX$ and the general equation becomes

$$EI\frac{d^2y}{dx^2} = -WX.$$

Integrate this twice and eliminate the constants of integration by means of the following facts. At the fixed end, $\frac{dy}{dx} = 0$ and $X = L$, also $Y = 0$ when $X = 0$.

42. Prove that the deflection of a simple beam of span L and central load W is

$$\Delta = \frac{WL^3}{48\,EI}.$$

Find the form of the equation of the elastic curve for this beam, Fig. 22, and loading. Integrate twice, remembering that

$$\frac{dy}{dx} = 0 \text{ when } X = \frac{L}{2} \text{ and that } X = 0 \text{ when } y = 0.$$

43. A standard 15-in. I beam 42 lbs. per ft. having a span of 30 ft. is to carry a uniform load; its working fiber stress is not to exceed 16,000 lbs.

per sq. in. and the maximum deflection $\frac{1}{360}$ of its span. What load will it carry?

44. A 6-in. I beam $12\frac{1}{4}$ lbs. per ft. extends 5 ft. from a wall as a cantilever beam. Assume that it is amply stiff, laterally. What load will it carry if the working fiber stress is 12,000 lbs. per sq. in. and what will be its maximum deflection?

45. What uniform load will a 12-in. I beam $31\frac{1}{2}$ lbs. per ft. carry on a 20-ft. span if it is merely supported at the ends and the fiber stress is 16,000 lbs. per sq. in.? Compare the deflection and fiber stress of this beam with a beam having fixed ends, carrying the same uniform load.

NOTE. — In practice one generally assumes simple beams, beams merely supported at the reactions. Fixed and continuous beams are seldom designed. Reinforced concrete designers make allowance for fixing and continuity in their beam designs.

Tension Pieces

In the usual tension piece of constant cross section the stress is uniformly distributed over the cross section and is the same for all sections. The section may not be constant, in which case the minimum or net section must be considered.

46. A 2-in. round bar is to have its end upset for U. S. Standard screw thread. What size screw must be cut if the area at the root of the thread exceeds the area at the body of the rod by 18 per cent?

47. A $4 \times 4 \times \frac{3}{8}$-in. angle carries a load of 35,000 lbs. in tension. Assume two $\frac{13}{16}$-in. diameter holes in a cross section. What is the unit stress per square inch on the net section?

Columns

In the case of columns the stress tends to increase any initial flexure due to inaccurate workmanship or vibration and a column, when of sufficient length, fails by bending. It should be noted that tension in a piece reduces any buckling tendency.

Columns Failing by Flexure

The derivation of column formulæ is based upon the general equation of the elastic curve previously used in finding the deflection of beams. In these column formulæ, of which there are several, a quantity depending entirely upon the section of the column occurs; it is $\frac{I}{A}$, and its square root

is called the radius of gyration of the section referred to the same axis as I. Here I is the moment of inertia and A the area of the section.

$$\text{Radius of gyration} = \sqrt{\frac{\text{Inertia of section}}{\text{Area of section}}} = \sqrt{\frac{I}{A}}.$$

48. Two 8-in. channels $11\frac{1}{4}$ lbs. per ft. are spaced such a distance back to back that the moments of inertia referred to their two principal axes are equal. What do the radii of gyration equal?

49. Two $6 \times 4 \times \frac{1}{2}$-in. angles are placed with their long legs parallel and separated $\frac{5}{8}$ in. What are the radii of gyration referred to the principal axes?

50. Two $5 \times 3\frac{1}{2} \times \frac{1}{2}$-in. angles are placed with their short legs parallel and separated $\frac{5}{8}$ in. What are the radii of gyration referred to the two principal axes?

51. Calculate the radius of gyration of a circular section 8 ins. in diameter. Then calculate the radius of gyration of a ring section 8 ins. outside diameter and 6 ins. inside diameter.

52. Prove the following values of radii of gyration:

$$\text{Rectangle, principal axis } r = \frac{d}{\sqrt{12}}.$$

$$\text{Triangle, principal axis } r = \frac{h}{\sqrt{18}}.$$

$$\text{Circle, principal axis } r = \frac{d}{4}.$$

The following formulæ are frequently used for mild-steel column design:

Ends.	Fixed.	Hinged.
Rankine's	$f' = \dfrac{16{,}000}{1 + \dfrac{l^2}{40{,}000\, r^2}}$,	$f' = \dfrac{16{,}000}{1 + \dfrac{l^2}{10{,}000\, r^2}}$.
Straight line	$f' = 16{,}000 - 35\dfrac{l}{r}$,	$f' = 16{,}000 - 70\dfrac{l}{r}$.

The most rational formula for columns is that of Ritter,

$$f' = \frac{f}{1 + \dfrac{f_e}{m\pi^2 E} \times \left(\dfrac{l}{r}\right)^2}.$$

f' = allowable unit working stress, pounds per square inch.
l = length of column, inches.
r = least radius of gyration.
f = maximum desired fiber stress, occurring at the dangerous section of the column and resulting from the flexure of the column.
f_e = fiber stress per square inch at the elastic limit of material.
E = coefficient of elasticity of the material of the column.
m = a constant, depending upon the way the column ends are secured. Its value for the usual end conditions are:

Ends.	Both Fixed.	1 Fixed and 1 Hinged.	Both Hinged.
m	4	2.25	1

For soft or mild steel, taking $E = 30{,}000{,}000$,

$$\pi^2 = 10 \text{ approximately}$$

and

$$f_e = 30{,}000;$$

the formula becomes, for hinged ends,

$$f' = \frac{16{,}000}{1 + \dfrac{1}{10{,}000}\left(\dfrac{l}{r}\right)^2}.$$

53. Compare the allowable working stresses as given by the three formulæ for a mild-steel hinged column, whose length is 20 ft. and least radius of gyration of the column section is $r = 2$.

54. Compare the allowable working stresses as given by the three formulæ for a mild-steel fixed-end column whose length is 20 ft. and least radius of gyration of the column section is $r = 2$.

55. Two 15-in. channels weighing 33 lbs. per ft. are to be made into a latticed column. Assume hinged ends, use Ritter's formula and determine the maximum load which they can be designed to carry if the column is 35 ft. long.

56. What load will a Bethlehem 12-in. H section weighing 78 lbs. per ft. carry? Assume the column, with fixed ends, 25 ft. long and the material mild steel. Use Ritter's formula and compare the result with that given in the Bethlehem Steel Company's handbook, using the formula

$$f' = 16{,}000 - 55\frac{l}{r}.$$

57. Compare the loads that can be carried by two mild steel columns, each 22 ft. long, both having fixed ends, the first being a latticed column made of two 10-in. channels 15 lbs. per ft., spaced for maximum load, the

other being a 10-in. column 54 lbs. per ft., Bethlehem Steel Company's section. Use Ritter's formula in both cases.

In structural design it is usual practice to limit the value of $\frac{l}{r}$ for important work subjected to considerable shock to $\frac{l}{r} \leqq 100$, while for other work the usual limit is $\frac{l}{r} \leqq 120$.

58. Would a $6 \times 6 \times \frac{3}{8}$-in. angle used singly make a good column? Should it be used for a height of 16 ft.? Why? What load could it carry if 10 ft. long, with hinged ends? Use Ritter's formula and assume the material mild steel.

59. Would an 8-in. I beam weighing 18 lbs. per ft. make a good column? Why? Should it be used for a height of 12 ft.? Why? What load would it carry if 8 ft. high? Use Ritter's formula. Assume fixed ends and the material mild steel.

60. Two $6 \times 4 \times \frac{3}{8}$-in. angles are to be placed back to back, separated $\frac{1}{2}$ in. and used for a hinged strut. Take your data from Manufacturer's Handbook and show if it would be more economical to have the long or short legs parallel. What load would it carry if 12 ft. long? Use Ritter's formula and assume the material mild steel.

In the beams in the previous problems no consideration has been taken of their lateral stiffness. In practice it is customary when using a maximum working fiber stress to limit the ratio of unsupported beam length to flange width to from 12 to 20. Where the ratio exceeds these numbers the allowable working fiber stress should be reduced. This is made necessary by the fact that in a vertically loaded beam the upper flange, being subjected to compression, is liable to fail as a column by buckling and, therefore, to secure the same factor of safety, the working stress in this flange must be reduced. There are several ways by which allowance is made for this condition, all of which are empirical and subject to criticism. Attention is called to the reduction of the working fiber stress by the use of a column formula, as in the Cambria Handbook,

$$f = \frac{18{,}000}{1 + \dfrac{l^2}{3000\, b^2}}.$$

f = allowable stress in pounds per square inch.
l = length between lateral supports in inches.
b = width of flange in inches.

This formula is for a maximum desired working stress of 16,000. For limiting values other than 16,000 lbs. reduce the maximum fiber stresses by the percentage that the above formula reduces 16,000 lbs.

Another method is based upon the tests of long beams by the Pencoyd Iron Works and adopted by other companies' handbooks. It assumes the maximum allowable fiber stress applicable to spans of 20 flange widths and a uniform reduction as the ratio of span to flange width increases until a beam whose span equals 70 flange widths is reached when the allowable working fiber stress is limited to one-half that used in the first case.

Still another method is that explained in Chapter VII of this book.

61. What uniform load will a 15-in. I beam weighing 42 lbs. per ft. carry on a 23-ft. span if unsupported laterally, and if the maximum working fiber stress for a beam whose span is 20 flange widths is limited to 16,000 lbs. per sq. in.? Compare the results by the methods given above.

62. What central load can be carried upon a Bethlehem girder beam G–20 weighing 112 lbs. per ft. on a 30-ft. span? The maximum fiber stress for a span of 20 widths is 16,000 lbs. per sq. in. Compare the results obtained by the methods given.

* **63.** A 10-ton crane is to be carried across a span of 30 ft. on rolled beams. The maximum wheel load is 21,000 lbs., the wheel base 10 ft. 6 ins. The maximum desired working stress is 15,000 lbs. per sq. in. reduced to allow for ratio of span to flange width. Select standard Cambria sections and test their ability to resist a lateral pull of one-tenth of the live load divided between the two wheels acting on the beam.

* **64.** A 25-ton crane is to be carried across a 36-ft. span on rolled beams. The maximum wheel load is 40,000 lbs.; the wheel base is 10 ft. 6 ins. All other conditions are the same as in Problem 63, excepting that a Bethlehem Grey Mill section with a wide flange is to be used. Determine the section.

Web Stresses

In addition to the points already considered, in designing beams with heavy concentrations of loads or very short spans the ability of the web to resist vertical shearing, vertical crippling and horizontal shearing must also be taken into account. In the case of plate girders the web is reinforced with stiffening angles to resist this crippling. Rolled steel beams are rarely weak in vertical or horizontal shear; timber beams should be carefully examined for horizontal shear; while in reinforced-concrete construction

* Specifications frequently call for an impact allowance of 25 per cent in the case of traveling cranes.

careful designing to resist the web stresses, including horizontal shear and diagonal tension, is of vital importance.

The horizontal shear at any section of a homogeneous beam is

$$f_s = \frac{V}{Iw} \Sigma ay.$$

f_s = horizontal shear on section per square inch.
I = moment of inertia of entire section.
V = total vertical shear.
w = width of web.
Σay = statical moment of the area of the cross section on one side of the neutral axis, or summation of areas multiplied by their respective distances from the neutral axis.

65. Show that for a timber beam of width b and depth d

$$f_s = \frac{3\,V}{2\,bd}.$$

66. If a 15-in. I beam weighing 42 lbs. per ft. is to be used on varying spans and a fiber stress of 16,000 lbs. is allowed in tension and 12,000 lbs. in shear, what is the minimum span for which the horizontal shear may be neglected?

67. Allowing a tensile fiber stress of 1500 lbs. per sq. in. and a shearing fiber stress with the grain of 120 lbs. per sq. in., what uniform load can be carried by a yellow pine timber 12 ins. deep, 3 ins. wide on a 12-ft. span?

68. Allowing a tensile fiber stress of 900 lbs. per sq. in. and a shearing fiber stress with the grain of 100 lbs. per sq. in., what uniform load will a spruce timber 16 ins. deep $\times$ 4 ins. wide carry on a 16-ft. span?

69. What is the maximum span upon which a yellow pine timber 12 ins. deep can be used to carry a uniform load if its fiber stress is to be limited to 1200 lbs. per sq. in. and its deflection to $\frac{1}{360}$ of its span? Assume its modulus of elasticity as 1,500,000 lbs. per sq. in.

70. What central load will a white oak timber 16 ins. deep $\times$ 4 ins. wide carry on an 18-ft. span if stressed to 1500 lbs. per sq. in.? What would be its maximum deflection if the modulus of elasticity were taken at 1,500,000 lbs. per sq. in.?

Combined Flexure and Direct Stress

Sometimes beams are subjected to direct compression or tension in addition to flexural stresses. It is usually sufficient to take the algebraic sum of the flexural and direct stresses at the extreme fibers of the beam. When greater accuracy than this is required, Johnson's formula, as modified by

Merriman, can be used. According to this formula the fiber stress due to bending, when direct compression or tension acts upon a piece in addition to bending, is

$$f' = \frac{Me}{I \pm \frac{n}{m}\frac{Pl^2}{E}}.$$

The $-$ is for a compression force.

The $+$ is for a tensile force.

The combined stress is $f' \pm \dfrac{P}{A}$.

P = compression or tensile force in pounds.

l = span in inches.

Simple beam uniform load $= \dfrac{n}{m} = \dfrac{1}{9.6}$.

Simple beam central load $= \dfrac{n}{m} = \dfrac{1}{12}$.

A = area of cross section in square inches.

An exact method is given by Merriman but its application is too difficult to be generally used.

71. A yellow pine beam 10 ft. long and 12 ins. square is subjected to a compression force acting along its length of 50,000 lbs. while carrying a uniform load of 20,000 lbs. What is the fiber stress? E = 1,500,000.

What is the maximum fiber stress, using the formula

$$f' = \frac{Me}{I \pm \frac{n}{m}\frac{Pl^2}{E}}.$$

72. A horizontal steel tension bar has a cross section of $9 \times 1\frac{1}{4}$ ins. It is subjected to a unit stress in tension of 10,000 lbs. per sq. in. The bar is 20 ft. long. Determine the maximum resulting fiber stress due to the combined action of its weight and its tensile load. The bar runs horizontally with its $1\frac{1}{4}$-in. edge down. E = 30,000,000

$$\frac{n}{m} = \frac{1}{9.6}.$$

73. In Fig. 23 the top plate is $20 \times \frac{3}{8}$ in., the sides are 15-lb. channels at 40 lbs. per foot each. Assume the gross cross section loaded with 7500

lbs. per sq. in. Chord 25 ft. long. $E = 30,000,000$. Determine the maximum fiber stress due to direct loading and its own weight. Use formula

$$f' = \frac{Me}{I \mp \frac{n}{m}\frac{Pl^2}{E}}.$$

74. In Fig. 24 the top plate is $24 \times \frac{7}{8}$ in. The sides are, two upper angles $4 \times 4 \times \frac{9}{16}$ in., two plates $24 \times \frac{13}{16}$ in. and two bottom angles $6 \times 4 \times \frac{3}{4}$ in. Area of combined section is 82.3 sq. ins. Inertia of section is 6969. Assume chord 20 ft. long. Uniform load, including live load

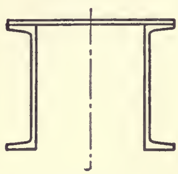


FIG. 23.

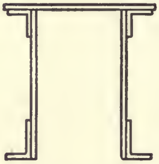


FIG. 24.

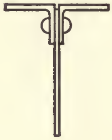


FIG. 25.

and weight of section, is 4000 lbs. per foot of span. Compression load 400,000 lbs. acting along the center of gravity of the section. What is the total maximum fiber stress? $E = 30,000,000$.

$$f' = \frac{Me'}{I - \frac{n}{m}\frac{Pl^2}{E}} \qquad \frac{n}{m} = \frac{1}{9.6}.$$

75. What is the maximum fiber stress in Fig. 25? Two angles $4 \times 3 \times \frac{5}{16}$ in., one plate $10 \times \frac{5}{16}$-in. Compression due to direct load acting at center of gravity of section is 10,000 lbs. per sq. in. on gross section. (In this problem neglect rivet holes.) Length of piece 8 ft. 0 ins. Neglect weight and assume a central load due to purlin of 1600 lbs. What is the maximum fiber stress?

$$f' = \frac{Me}{I \pm \frac{n}{m}\frac{Pl^2}{E}}.$$

$$E = 30,000,000.$$

$$\frac{n}{m} = \frac{1}{12}.$$

Riveting

The following considerations are useful in designing riveted joints for structural work.

1. The rivet strength is calculated upon the nominal diameter of the rivet notwithstanding the fact that the hole in which the rivet is driven is generally $\frac{1}{16}$ in. greater than the rivet.

2. The unit shearing strength of steel rivets is taken about $\frac{3}{4}$ the unit tensile strength of the steel and the unit bearing strength of rivets is taken at double their unit shearing strength.

3. Rivets should not be used in tension.

4. Use table of rivet values.

76. A 24-in. I beam weighing 80 lbs. per ft. carries a uniform load on a 19-ft. span. First find what load it will carry if the extreme fiber stress is 16,000 lbs. per sq. in. and then find how many $\frac{3}{4}$-in. rivets will be required to secure two $4 \times 4 \times \frac{3}{8}$-in. angles to the web of the beam and transfer the reaction due to the given load. Compare your result with the standard framing. Allow 10,000 lbs. per sq. in. shear.

77. Two $4 \times 4 \times \frac{5}{16}$-in. angles are fastened back to back to a $\frac{3}{8}$-in. plate. How many $\frac{3}{4}$-in. rivets will be required in the angles if the net section of the material is stressed 12,000 lbs. per sq. in.? Allow one rivet hole $\frac{7}{8}$ in. diam. through angles and plate. Allow 7500 lbs. per sq. in. in shear.

78. The long leg of a $5 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angle is fastened to a $\frac{3}{8}$-in. plate. Assume one rivet hole $\frac{7}{8}$ in. in diameter, in a section. Allow a fiber stress of 12,000 lbs. per sq. in. on the net section and determine how many $\frac{3}{4}$-in. rivets are required. Allow 7500 lbs. per sq. in. in shear.

Timber Columns

The U. S. Dept. of Agriculture has made a large number of tests upon timbers and the following is their formula suggested for timber columns.

$$p = F \times \frac{700 + 15c}{700 + 15c + c^2}.$$

To find the safe load divide p by the required factor, where

p = ultimate strength in pounds per square inch.
F = crushing strength of timber.

$$c = \frac{L}{d}, \quad \frac{\text{Length (inches)}}{\text{small diameter (inches)}}.$$

79. What load can be safely carried by a yellow pine post 10 ins. square and 20 ft. high? Use a factor of 4 and consider the ultimate crushing strength 5000 lbs. per sq. in.

80. A cedar post is 22 ft. high and is 10 $\times$ 12 ins. in cross section. What load can it safely carry if its ultimate crushing strength is assumed at 3500 lbs. and a factor of safety of 5 is desired?

81. A spruce column is 18 ft. high and 9 ins. square. What load can it safely carry if its ultimate crushing strength per sq. in. is taken as 4000 lbs. and a factor of safety of 4 is desired?

Plate Girders

In designing girders of greater depth than rolled sections (24 and 36 ins.) the following method is easier and is sufficiently accurate. In Fig. 26,

- A = area of one flange in square inches.
- h = distance between the centers of gravity of the two flanges in inches. Where the flange is made up of angles and plates this distance is frequently assumed as the distance back to back of the angles.
- f = mean fiber stress in the flange in pounds per square inch.
- M = external bending on the section measured in inch pounds.

Then $$M = A \times f \times h.$$

This formula assumes the web as resisting shear only. When the web is assumed to resist bending also the formula becomes

$$M = \left(A + \frac{a}{8}\right) \times f \times h.$$

Here a is the area of the web plate in square inches.

82. Derive the two formulæ for girders,

$$M = A \times f \times h \quad \text{and} \quad M = \left(A + \frac{a}{8}\right) \times f \times h.$$

83. Find the net flange area required at the middle of the following girder. Span 60 ft. Depth of girder back to back of flange angles is 6 ft. 0 ins. Uniform dead load is 600 lbs. per ft. (this covers ties, rails and metal of girder, etc.). Uniform live load is 2250 lbs. per lineal foot. Increase the live-load bending 80 per cent to allow for impact due to a moving train load. Allow a fiber stress of 16,000 lbs. per sq. in. Assume that the web takes shear only.

84. A girder spans 80 ft. Its depth, back to back of flange angles, is 7 ft. 0 ins. Uniform dead load is 700 lbs. per ft. (this covers ties, rails, metal of girder, etc.). Uniform live load is 2000 lbs. per lineal foot. Increase the live-load bending 80 per cent to allow for impact due to a moving train load. Allow a fiber stress of 16,000 lbs. per sq. in. Assume that one-eighth of the web acts as flange area and that the web is $\frac{3}{8}$ in. thick. What

additional area is required to complete the net flange section 30 ft. from the left abutment?

85. A girder which spans 50 ft. is to be made 5 ft. 0 ins. deep, back to back of flange angles. It carries two wheel loads 12 ft. 0 ins. center to center of 80,000 lbs. each. Assume a dead load including girder weight of 200 lbs. per ft. Increase the live-load bending 20 per cent to cover impact. Allow a fiber stress of 16,000 lbs. per sq. in. Assume the web $\frac{5}{16}$ in. thick and that one-eighth of it is considered as being flange. Determine what additional area is required to complete the net flange area at the point of maximum bending.

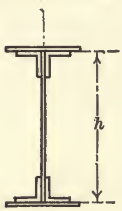


FIG. 26.

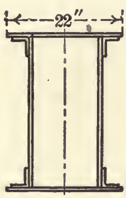


FIG. 27.

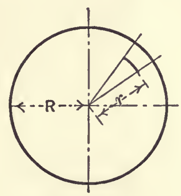


FIG. 28.

86. A bridge which spans 60 ft. is to be built for a crane. Assume that each girder weighs 15,000 lbs. and that this forms a uniform load. The trolley wheels are 6 ft. apart and each wheel load is 16,000 lbs. The girder at the center is 48 ins. deep, back to back of angles. On account of stiffening the crane laterally the flange plates are made 22 ins. wide. Assume the section like Fig. 27. Assume the web as taking shear only. The fiber stress in the compression flange is to be 9000 lbs., while that in the tension flange can be 12,000 lbs. per sq. in. Determine the dimensions of the plates.

SHAFTING

Considering the differential area, Fig. 28, the fiber stress is proportional to its distance r from the center so that if p is the fiber stress at a distance R from the center the stress p_r at this distance is

$$p_r = \frac{p \times r}{R}.$$

The force acting on an area dA then is $dF = p_r \times dA$.

The moment of this force about the center then is

$$dM_t = \frac{p \times r^2}{R} dA.$$

Integrating $$M = \frac{p}{R}\int r^2 dA.$$

It will be noticed that this is in the form of the general equation for torsion.

$$M_t = \frac{p \times J}{R},$$

where J is the polar moment of inertia.

87. Allowing a limiting shearing working stress of 7000 lbs. per sq. in. upon a steel round 2 ins. in diameter, what twisting moment measured in inch pounds will the round carry? What would be the force applied at a radius of 1 ft.?

88. Allowing a limiting shearing working stress of 7000 lbs. per sq. in., what twisting moment in inch pounds would be carried by a 2 in. square shaft? Acting at a radius of 1 ft. what force would this correspond to?

89. Find the ratio of the twisting moments carried by a round shaft and a square shaft of the same sectional area and the same limiting fiber stress.

90. What horse power will a shaft of diameter d transmit when making N revolutions per minute and subjected to a working fiber stress of p lbs. per sq. in. at its circumference?

Torsional Deflection

It is sometimes desirable to limit the torsional deflection of a shaft, in which case the angle of deflection in degrees is given by

$$\Delta = \frac{M_t \times L}{1660 \times d^4}.$$

M_t = twisting moment in inch pounds.
L = length of shaft in feet.
d = diameter of shaft in inches.

91. A shaft 2 ins. in diameter and 60 ft. long is subjected to a twisting moment which produces an extreme fiber shearing stress of 6000 lbs. per sq. in. Through what angle is one end of the shaft twisted ahead of the other?

Combined Bending and Twisting

Shafting must generally be designed for combined torsion and bending. There are several formulæ but according to Guest's law the following may be used for mild steel.

$$M_e = \sqrt{M^2 + T^2}.$$

Here M_e = equivalent bending moment.
M = bending moment.
T = twisting moment.

Having found the equivalent bending moment, the shaft diameter can be determined by placing these values in the equation

$$M_e = \frac{pI}{e}.$$

I = moment of inertia referred to axis.
e = distance from center of gravity to extreme fibers.

92. In Fig. 29 a force of 2000 lbs. acts on the teeth of a pinion 5 ins. in diameter. The distance from the center line of the pinion to the center of the adjacent bearing is 6 ins. What diameter of shaft will be required if 9000 lbs. per sq. in. is permitted in flexure?

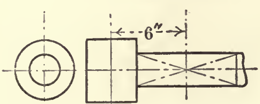


FIG. 29.

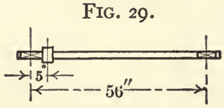


FIG. 30.

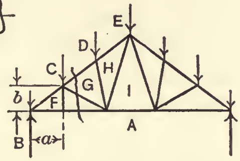


FIG. 31.

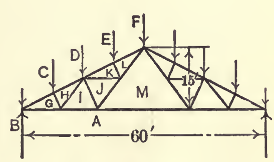


FIG. 32.

93. In Fig. 30 a force of 2500 lbs. acts upon the teeth of a pinion $4\frac{1}{2}$ ins. in diameter. The pinion is inside the bearings as shown. Using a flexural fiber stress of 9000 lbs. per sq. in. what diameter of shaft is required?

Stresses in Structures Determined Algebraically

In Fig. 31 to determine the stress in any piece, as FA, cut the truss at a section containing this piece and then equate the internal and external moments about any point that most conveniently gives the desired result. Consult Chapter IV.

In Fig. 32 BC = 2400 lbs., $CD = DE = EF$ = etc. = 4800 lbs.

94. Calculate algebraically the stress in GC and AG in Fig. 32.

95. Determine algebraically the stresses in GH and HD in Fig. 32.

96. Calculate the stresses in HI and IA in Fig. 32.

97. Calculate the stresses in FL and LM in Fig. 32.

Problems 94 to 97 should be checked graphically.

In Fig. 33 $BC = CD = DE = EE'$, etc. = 31,250 lbs.

98. Calculate algebraically the stresses in BF and FA in Fig. 33.

99. Calculate algebraically the stresses in FG and GC in Fig. 33.

100. Calculate algebraically the stresses in GH and HI in Fig. 33.

Problems 98 to 100 should be checked by method of coefficients.

Graphical Analysis of Truss Stresses

101. In using the common graphical method of determining truss stresses, what assumption is made concerning the construction of the truss? What are the three conditions necessary for equilibrium in a structure acted on by forces?

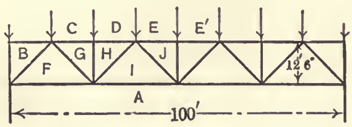


Fig. 33.

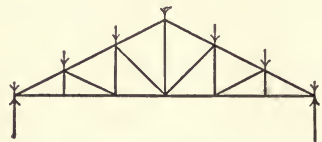


Fig. 34.

102. What is a force triangle? A force polygon? What can you say concerning the force polygon representing a number of concurrent forces in equilibrium? What general direction will the forces in the polygon take?

103. Make a sketch of any simple truss and show how force polygons may be used to determine the magnitude and character of the stresses in the structures when the external forces are known.

104. Take Fig. 34, letter the truss and make a careful free-hand sketch of the stress diagram, then find the character of the stresses.

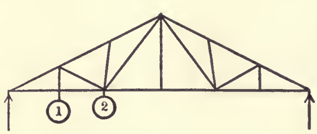


Fig. 35.

Fig. 36.

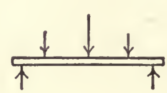


Fig. 37.

105. Take Fig. 35, letter the truss and make a careful free-hand sketch of the stress diagrams, first with a live load L at 1 and then at 2. Determine the character of the stress.

106. What is an equilibrium polygon? When a number of nonconcurrent forces are in equilibrium what graphical conditions are fulfilled?

107. In Fig. 36 show by a free-hand sketch how the resultant of the three forces acting on the piece can be determined in direction, magnitude and located in relation to the piece.

108. In Fig. 37 show by a free-hand sketch how the reactions due to the three parallel forces acting on the beam can be determined.

109. In Fig. 38 a truss is acted on by wind forces. Both ends of the truss are fixed. Show how the reactions can be found.

110. In Fig. 39 the truss is acted on by wind pressure. The right-hand end is on rollers while the other end is fixed. Show by a free-hand sketch

how the magnitude of the right reaction and the magnitude and direction of the left reaction can be found.

111. Explain how an equilibrium polygon may be used as a bending-moment diagram and prove that the statement is correct.

112. Make a free-hand sketch showing by an equilibrium polygon how the bending can be found on the beam in Fig. 40.

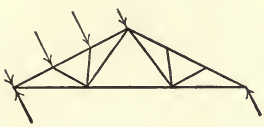


FIG. 38.

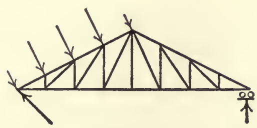


FIG. 39.

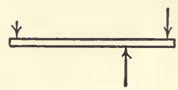


FIG. 40.

The equilibrium polygon may also be used in determining the stresses as follows: In finding truss stresses algebraically the internal and external moments were equated. The number of calculations can be reduced by using an equilibrium polygon for the external bending moment, the single diagram serving for all points.

113. In Fig. 41 show how the stresses AI, HI and HE may be determined by combining both graphical and algebraic methods as suggested.

114. In Fig. 42 determine the dead-load stresses in members AJ, JK and IF. The bridge has the following dimensions: Span, 150 ft.; height, center to center chords, 30 ft.; uniform dead load, 2000 lbs. per ft., carried at lower apex points.

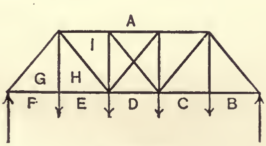


FIG. 41.

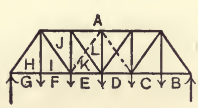


FIG. 42.

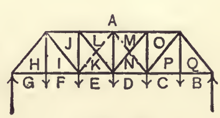


FIG. 43.

115. Show how a bending-moment diagram may be determined graphically to give the maximum moments on all sections of a girder for two loads, a constant distance a apart, moving across the girder.

116. Show how a bending-moment diagram may be determined graphically to give the maximum moments on all sections of a girder, due to a number of loads (use 6), a constant distance apart, moving across the girder.

117. Show how to construct a diagram giving the maximum shears at all points on a girder due to a locomotive and train load passing across it. Prove that the diagram gives the required results.

118. In Fig. 43 show how the maximum stresses due to moving wheel loads crossing the bridge may be determined for members AH, HI and IJ. Use a combined graphical and algebraic method.

Stresses in Crane Frames

Show how to determine the stresses in the following crane frames. Explain the methods fully.

1. Find direct stresses due to live load at maximum radius.
2. Find direct stresses due to live load at minimum radius.
3. Draw bending-moment diagrams for members subjected to bending. If necessary draw a stress diagram for position of live load producing maximum bending in any member.
4. Draw stress diagram for dead load.

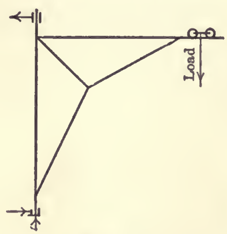


FIG. 44.

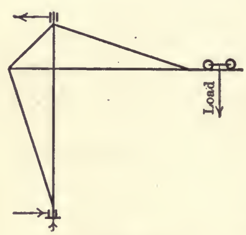


FIG. 45.

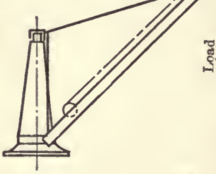


FIG. 46.

119. Make the necessary stress and moment diagrams for the frame given in Fig. 44.

120. Make the necessary stress and moment diagrams for the frame given in Fig. 45.

121. Make the necessary stress and moment diagrams for the frame given in Fig. 46.

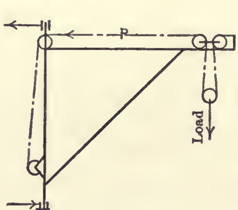


FIG. 47.

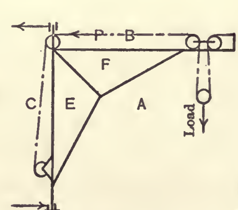


FIG. 48.

122. Make the necessary stress and moment diagrams for the frame given in Fig. 47, taking the chain or rope pull into account.

123. Make the necessary stress and moment diagrams for the frame given in Fig. 48, taking the chain or rope pull into account.

124. In Fig. 48 show how to stiffen FB so that the carriage can travel back close to the post. Also show how channels used for EA could be stiffened to reduce the $\frac{l}{r}$ value and permit the load chain to travel back close to the post.

Plate-girder Design

Consult Chapter XI for information concerning these problems.

125. Assume a half dozen wheel loads and their distances apart and explain how the diagram of maximum bending moments would be drawn. Show how to include the bending due to the girder weight in this diagram.

126. A girder has a span of 65 ft. Assume that it carries a load equivalent to a uniform load of 2000 lbs. per ft. Calculate the bending at the center and draw the bending-moment diagram.

Lengths of Flange Plates

In plate girders the flange force varies from zero at the supports to a maximum near the center of a girder. If the girder is of approximately constant depth the bending-moment diagram can represent the flange force by changing the scale; thus,

$$M = A \times p \times h$$

and the flange force

$$F = A \times p = \frac{M}{h};$$

h being constant F varies as M. This diagram can now be used to determine the lengths of the several plates and angles composing the flange.

127. Assume any girder. Show how to determine the lengths of the several flange plates.

Flange Riveting

128. For stationary loads the diagram representing the forces acting in the girder flanges can be used to determine the riveting of the flange angles to the web and also the several flange plates together. Show how to do this.

129. A girder spans 60 ft. Assume the depth back to back of angles as 6 ft. The flange at the middle is composed of two $6 \times 6 \times \frac{5}{8}$-in. angles, and two $15 \times \frac{5}{8}$-in. plates. The web is $\frac{3}{8}$ in. thick, rivets are $\frac{7}{8}$ in. in diameter.

Assume the bending as due to a uniform load on the girder of 6000 lbs. per ft. of span. By the method suggested in Problem 128, find the number of rivets required between the web and flange angles for the panel extending from 6 ft. to 12 ft. from the left pier. Allow 11,000 lbs. shear and 22,000 lbs. bearing on the rivets.

130. Take the same data as in Problem 129, and determine the number of rivets in the panel extending from 24 ft. to 30 ft. from the left pier.

131. Explain how to construct a diagram of maximum shears for a locomotive and train load passing across a bridge.

132. Estimate the reaction from the data given in Problem 129 (this includes dead load, live load and impact) and determine the shearing fiber stress in the web plate which is 6 ft. deep and $\frac{3}{8}$ in. thick.

133. Explain the object of web stiffeners. What is the usual spacing of web stiffeners in ordinary railroad practice? How thick must the web be, in relation to the girder depth, to permit the omission of stiffeners?

134. Show by means of a diagram of maximum shears that the maximum vertical shear at any point due to a moving uniform load occurs when the girder is loaded from this point to the more remote of the two reactions.

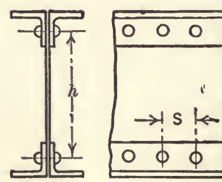


FIG. 49.

Instead of the method previously indicated for obtaining the rivet spacing between the flange and the web, the following is the method more generally used as it is applicable to moving loads. In Fig. 49

$$s = \frac{R \times h}{V}.$$

s = rivet pitch, inches.
R = rivet value, pounds.
h = distance between gauge lines of flange angles, inches.
V = vertical shear, generally taken as the mean of the panel in which the rivets are to be spaced.

135. Assume the same general data as given in Problem 129 but consider the load a moving one. Use the method just given and determine the rivet spacing in the lower flange for the section of the girder extending from 12 ft. to 18 ft. to the right of the left support. Assume that the web takes shear only. Assume that $h = 64\frac{3}{4}$. Use $\frac{7}{8}$-in. rivets, allowing 11,000 lbs. in shear and 22,000 lbs. in bearing.

136. Given $\frac{3}{8}$-in. web, $6 \times 4 \times \frac{7}{16}$-in. flange angles, $h = 55$ ins. Rivet diameter $\frac{7}{8}$ in., shearing stress 10,000 lbs. per sq. in., bearing stress 20,000

lbs. per sq. in., vertical shear 75,000 lbs. What rivet spacing is required at this section? Assume that the web takes shear only.

When concentrated loads are transferred to the web plates through the flange angles the resultant of the vertical and horizontal forces must be found and the rivet spacing determined from this resultant. These concentrations are usually assumed as distributed over a certain distance along the flange, say 36 ins.

The change in the horizontal flange force per inch of flange length is

$$f_1 = \frac{V}{h}.$$

The vertical force per inch of flange length is

$$f_2 = \frac{\text{load}}{\text{distance distributed}}.$$

The resultant $f_r = \sqrt{f_1^2 + f_2^2}$. Rivet spacing $= \dfrac{R}{f_r}$.

137. Solve Problem 129, taking into account the load of 6000 lbs. per foot carried from the flange angles to the web by the rivets. $h = 64.75$ in. Use a moving load.

138. Given web plates $\frac{3}{8}$ in. Angles $6 \times 6 \times \frac{5}{8}$ in. Rivets $\frac{7}{8}$ in. in diameter. Shearing stress 11,000 lbs. Bearing stress 22,000 lbs. Total vertical shear at middle of panel under consideration 200,000 lbs. $h = 69$ ins. Assume load of 45,000 lbs. distributed over 36 ins. of upper flange. (This includes dead load, live load and impact.) Determine the rivet spacing. Assume that the web takes shear only.

The horizontal shear between the flange and the web is reduced when the web is assumed as assisting in carrying the bending. The change in this shear transferred through the rivets depends upon the ratio

$$\frac{A - a}{A},$$

where

A = net area of flange, total.

a = one-eighth gross web area

$$f_s = f_1 \times \frac{A - a}{A} \quad \text{and} \quad f_r = \sqrt{f_s^2 + f_2^2};$$

and, as before, rivet spacing $= \dfrac{R}{f_r}$.

139. In Problem 138 assume that one-eighth of the web acts with the flange and that at this point the net flange area is 12 sq. ins., not including one-eighth of the web. Find the required rivet spacing.

140. Determine the rivet spacing satisfying the following conditions: Vertical shear 160,000 lbs.; $h = 64.75$ ins. Net flange area, not including one-eighth of web, 20 sq. ins.; web $72 \times \frac{3}{8}$ in. Upper flange concentration 45,000 lbs. acting over 36 ins. Shearing stress 11,000 lbs. and bearing stress 22,000 lbs. per sq. in. Use rivets $\frac{7}{8}$-in. in diameter.

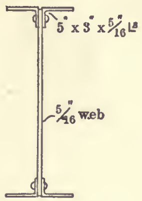


FIG. 50.

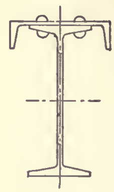


FIG. 51.

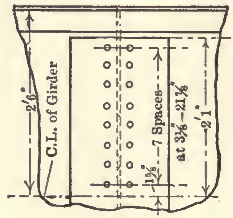


FIG. 52.

141. A girder running over two supports is cantilevered 5 ft. beyond one of them and carries a concentrated load at its end, producing an average fiber stress in the flange angles of 15,000 lbs. per sq. in. Assume the section given in Fig. 50 and determine the spacing in the flange angles of the cantilevered portion for $\frac{3}{4}$-in. rivets. Allow 10,000 lbs. per sq. in. in shear and double this in bearing.

142. Fig. 51 is made up of 1–8 in. ⊔ $11\frac{1}{4}$ lbs. per ft., area 3.35 sq. in. and 1–12-in. *I* $31\frac{1}{2}$ lbs. per ft., area 9.26 sq. ins. The average upper flange stress is 8000 lbs. at the middle of a 22-ft. span, carrying a central load. What rivet spacing should be used in securing the channel to the beam if the rivets are $\frac{3}{4}$ in., the shearing fiber stress 10,000 lbs. per sq. in. and the bearing stress double this?

143. In Problem 142 determine the rivet spacing 4 ft. from the left support if the beam carries a uniform load instead of a central one.

Web Splice

When the web is assumed as resisting bending it becomes necessary to rivet the splice plates so that they and the riveting shall replace the broken section in bending as well as in shear. See Chapter XI.

144. A girder is 5 ft. deep and has a $\frac{5}{16}$-in. web plate; one-eighth of the web is assumed as acting as flange; the average flange stress acts 29 ins. from the neutral axis and is 12,000 lbs. per sq. in. Using two splice plates, how

thick must they be made if they can be only 50 ins. deep? The fiber stress in the splice plates is to equal that in the web plate at the same distance from the neutral axis of the girder.

145. Assume the data the same as that given in Problem 144 and Fig. 52. How many vertical rows of rivets will be required to make the web splice properly carry its portion of the moment? Allow 10,000 lbs. per sq. in. in shear and 20,000 lbs. in bearing. Use $\frac{7}{8}$ in. diameter rivets.

146. A girder 6 ft. deep has a $\frac{3}{8}$-in. web plate which is to be spliced at the middle. How thick must each of two splice plates be if they can be made only $58\frac{1}{4}$ ins. deep? The fiber stress at the backs of the angles is 12,000 lbs. and the fiber stress in the splice plates is to equal the fiber stress in the corresponding position in the web plates.

147. Take the girder in Problem 146 and Fig. 53. How many rows of rivets will be required, allowing 12,000 lbs. per sq. in. shear and 24,000 lbs. in bearing? Design the splice to resist bending. Use $\frac{7}{8}$-in. diameter rivets.

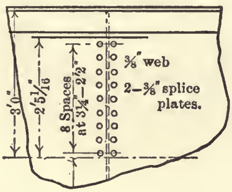


FIG. 53.

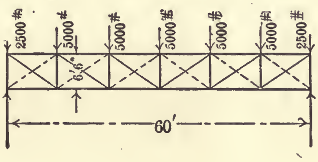


FIG. 54.

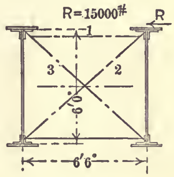


FIG. 55.

148. Assume that a girder 6 ft. deep and 60-ft. span carries a moving load of 6000 lbs. per ft. Determine the maximum shear at the middle and design the splice plates, assuming that the web takes shear only. How many $\frac{7}{8}$-in. rivets carrying 12,000 lbs. per sq. in. shear and 24,000 lbs. per sq. in. bearing will be required, the web being $\frac{3}{8}$ in. thick? Depth of splice plates is $58\frac{1}{4}$ ins.

149. The lateral bracing, Fig. 54, in a deck girder is placed in the plane of the upper flanges. Assume that the tension pieces take the load. The wind load is assumed as 300 lbs. per ft., to account for wind blowing on the train, and 30 lbs. per sq. ft. of girder. The combined load is assumed as acting upon the upper flange through the rail. (1) Determine the apex loads and (2) make a stress diagram showing how the forces acting in the wind braces may be determined. Assume vertical girders 6 ft. deep.

150. Take the data given in Problem 149 and determine the stresses in the bracing algebraically.

151. Assume that the lateral truss in line with the upper flanges transmits all the wind forces in this plane to the ends of the girder, and that from here the forces are transferred to the piers by the end bracing, Fig. 55. Find the reactions from Problem 149. Assume that $\frac{1}{2}$ R goes to 2, the other $\frac{1}{2}$ R through 1 and then to 3. Determine the forces in 1, 2 and 3, and design them, assuming struts with fixed ends and that $l = \frac{1}{2}$ the diagonal. Allow a fiber stress of 16,000 lbs. per sq. in.

Fixed and Continuous Beams

Fixed and continuous beams do not have much application in structural work; however, owing to the monolithic character of concrete work the bending moments are frequently taken at values between those for supported and fixed beams.

152. Fig. 56 is a beam fixed at one end and supported at the other; load is at the middle of the span. The point of inflection is at $X = \frac{3}{4}\,l$. Show how to find the bending-moment diagram graphically.

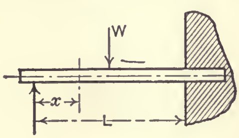


Fig. 56.

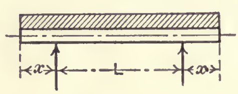


Fig. 57.

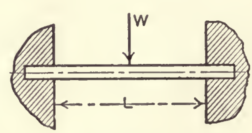


Fig. 58.

153. A beam fixed at one end and supported at the other carries a uniform load. The reaction at the fixed end is $\frac{5}{8}$ the load. Determine the bending moments graphically and locate the points of inflection.

154. The beam in Fig. 57 carries a uniform load W lbs. per ft. over its length $(L + 2\,x)$. Determine graphically the value of x in terms of L so that the central span shall fulfill the conditions of a fixed beam, *i.e.*,

$$M = \frac{WL^2}{24}$$

at the middle, and at the supports

$$M = -\frac{WL^2}{12}.$$

155. A fixed beam, Fig. 58, carries a central load W. The bending at the supports is $-M$, while for the middle the bending is $+M$. (Numerically equal but of opposite signs.) Determine graphically these moments and the points of inflection.

156. If a beam carries a uniform load and is restrained at the supports so that the bending at the middle is $\frac{1}{10} wl^2$, determine graphically the bending moment at the supports and the points of inflection.

Reinforced-concrete Designing

For the explanation of the theory and derivation of the formulæ see Chapter XIV. Also consult page 198 for the nomenclature.

157. Derive the following formulæ:

$$M_s = A \times f_s \times jd; \quad M_c = \frac{f_c}{2} \times kj \times b \times d^2$$

and

$$d = \sqrt{\frac{6\,M_c\,(z+1)^2}{b \times f_c\,(3\,z+2)}}.$$

158. Design a rectangular beam to carry a load of 800 lbs. per ft. of a 20-ft. span. Assume $M = \frac{WL}{10}$. Allow $f_s = 16{,}000$ lbs., $f_c = 500$ lbs. Take $d = 20$ ins. and assume $\frac{E_s}{E_c} = 15$. Determine the width and the area of the reinforcing steel.

159. What size rectangular beam and reinforcement will be required for a span of 16 ft. The beam ends are merely supported and the load is 1000 lbs. per ft. of span. Given $M = \frac{WL}{8}$, $\frac{E_s}{E_c} = 15$, tensile stress in steel 16,000 lbs. per sq. in., compression in concrete 500 lbs. per sq. in. and the width of the beam is to be 40 per cent of its effective depth.

160. Derive the following approximate formulæ for rectangular beams, the symbols having the significance given on page 198.

$$M_s = \tfrac{7}{8} \times A \times f_s \times d, \quad M_c = \tfrac{1}{6} f_c \times b \times d^2.$$

161. The purlins on a roof are 6 ft. center to center. How thick must a reinforced roof slab be made if the total live and dead load is 80 lbs. per sq. ft.? Assume $\frac{E_s}{E_c} = 15$; $f_c = 500$; $f_s = 14{,}000$ and $M = \frac{WL}{8}$.

162. A reinforced-concrete floor carrying a combined live and dead load of 400 lbs. per sq. ft. is placed over steel beams that are on 6-ft. centers.

Taking
$$\frac{E_s}{E_c} = 15, \quad M = \frac{WL}{8},$$

allowable compression in concrete 500 lbs. per sq. in., and tension in steel 14,000 lbs. per sq. in., find depth of slab and area of reinforcement.

163. A rectangular tank supported on its lower edges is 5 ft. wide, 6 ft. high and 13 ft. long (inside dimensions). How thick must the floor be made to hold the tank full of water? Allow $M = \frac{WL}{10}$. Compression in concrete 500 lbs. per sq. in.; tension in steel 14,000 lbs. per sq. in.

$$\frac{E_s}{E_c} = 15.$$

Find also the area of the steel reinforcement.

The common practice in concrete building construction is to use reinforced-concrete beams rather than the steel beams. In this event, owing to the monolithic character of the work, part of the floor slab acts as the compression flange of the beam. See Chapter XIV.

164. The T beams have a span of 20 ft., and are spaced 6 ft. center to center; see Fig. 59. The combined live and dead load is 250 lbs. per sq. ft. Assuming $M = \frac{WL}{10}$; $\frac{E_s}{E_c} = 15$; $f_c = 500$; $f_s = 14{,}000$; find the thickness of the slab.

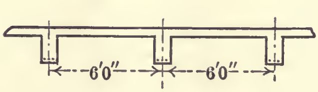


FIG. 59.

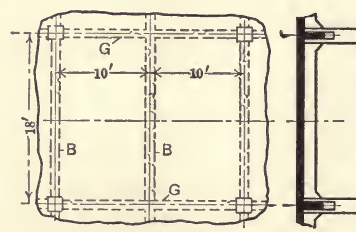


FIG. 60.

165. Take Problem 164; assume the slab 5 ins. thick and also that d = 16 ins. for the T beam; find the width of slab required for the flange of the T beam and the area of the reinforcing metal.

NOTE. — It is usual to limit the width of the slab assumed as flange of the T beam. One specification puts this limit at eight times the slab thickness but not exceeding one-third the T-beam span.

Figure 60 shows a plan of a section of a floor. The following data apply to all these problems.

$$f_c = 550 \text{ lbs.}, \qquad f_s = 14{,}000 \text{ lbs.}, \qquad \frac{E_s}{E_c} = 15.$$

Live load 150 lbs. per sq. ft. of floor, dead load 60 lbs. per sq. ft. of floor.

166. Design the slab, determining the thickness and area of metal. Take $M = \frac{WL}{10}$.

167. Assume the slab $5\frac{1}{2}$ ins. thick, and design the T beam. Assume $d =$ 19 ins. Find width of flange and area of steel.

168. Design the girder G, assuming that it carries a T beam B, on each side of it, entering it centrally, and that it takes only 85 per cent of the live load, *i.e.*, 150 lbs. per sq. ft., but entire dead load of 75 lbs. per sq. ft. Assume the depth of the girder as $d = 25$ ins. It must be at least deep enough to allow its reinforcing bars to pass under the bars in the T beams. Find the area of the reinforcement and the width B of the upper flange of the beam.

Assume the bending as four-fifths that due to a freely supported beam centrally loaded, *i.e.*, $M = \frac{WL}{5}$.

169. A beam spans 20 ft. and carries 2500 lbs. per ft. Assume that $d = 30$ ins. and find the width and reinforcing area. Take $f_s = 12{,}000$ lbs., $f_c =$ 500 lbs., $\frac{E_s}{E_c} = 15$ and $M = \frac{WL}{8}$.

170. Take the data given and found for Problem 169, and find the effect on f_s and f_c of placing the steel 2 ins. above the intended position, making $d = 28$ ins. instead of 30 ins.

171. A slab spans 10 ft. 6 ins. and carries a load of 485 lbs. per sq. ft. Assume $M = \frac{WL}{12}$ and allow $f_s = 16{,}000$ lbs., $f_c = 600$ lbs. and $\frac{E_s}{E_c} = 15$ lbs. Find the thickness of the slab and the spacing of $\frac{5}{8}$-in. diameter round rods.

Slabs should always be reinforced in both directions, the metal at right angles to the main reinforcement being used to prevent the slab from cracking. When the slabs are designed square and reinforced in both directions the maximum moment may be assumed as

$$M = \frac{WL}{20}.$$

172. A square slab reinforced in both directions has a side of 16 ft. and carries a combined live and dead load of 220 lbs. per sq. ft. Find thickness of slab and area of reinforcement. Use data of Problem 165.

173. When reinforced-concrete beams are designed assuming that the bending at the center is $\frac{WL}{10}$, what is the bending at the supports? Where in the span is the point of zero bending and what should be the minimum reinforcing over the supports expressed as a percentage of the reinforcing at the center?

In the problems thus far considered three points have not been taken into account. The horizontal shear, the vertical shear and diagonal tension. See Chapter XIV.

174. Derive the following formula for the spacing of stirrups,

$$s = \frac{T \cdot jd}{V - v_c \cdot b \cdot jd}.$$

175. Derive the following formula for the length of rod required on account of bonding,

$$l = \frac{f_s \times \delta}{2\,u}.$$

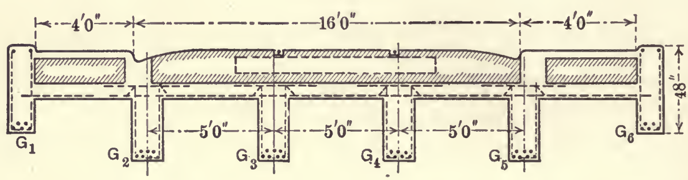
176. Derive the following formula for the length of bar required to provide flange strength,

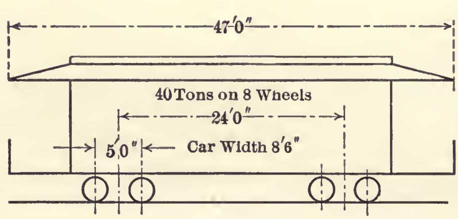
$$l = L\sqrt{\frac{a}{A}}.$$

177. Take Problem 158 and determine the vertical reinforcements, allowing a tensile fiber stress of 16,000 lbs. per sq. in. Neglect the shearing strength of the concrete. Try $\frac{3}{8}$-in. rounds bent into U stirrups. Would stirrups be needed if 50 lbs. per sq. in. was allowed in shear in concrete? What about reinforcement at the ends?

178 to 181. Design a light highway bridge of reinforced concrete. Span 32 ft. 0 ins., fill 15 ins., roadway 16 ft. with a 4-ft. wall on each side. Assume $\delta = 5\frac{1}{2}$ ins. See Figs. 61 to 63.

FIG. 61.





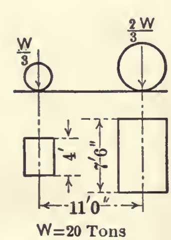


FIG. 62. FIG. 63.

178. Determine thickness of slabs under roadway. Assume live load 500 lbs. per sq. ft., concrete 60 lbs. per sq. ft., fill 120 lbs. per sq. ft. Assume

compressive stress in concrete 650 lbs. per sq. in., tensile stress in steel 16,000 lbs. per sq. in. Assume that the effect of impact is 50 per cent of live-load stresses.

$$M = \frac{WL}{10} \quad \text{and} \quad \frac{E_s}{E_c} = 15.$$

Using $\frac{1}{2}$-in. round bars, how far apart (center to center) should they be spaced? Since the bending has been taken at $M = \frac{WL}{10}$, what reinforcements should be placed over girders?

179. Design girders G_1 and G_6. Assume dead load of concrete, fill, etc., as 900 lbs. per lineal ft. of 32-ft. span. Live load 125 lbs. per sq. ft. of walk assumed as 5 ft. 0 ins. wide. Make $b = 12$ ins.

$$z = 1.64 \text{ corresponding to } f_c = 650 \text{ lbs.} \quad M = \frac{WL}{8}.$$

180. Design girders G_3 and G_4 on the following basis. Each girder to carry one-half the car load or the front wheel of the roller and one-half the weight of the rear wheels, Figs. 62 and 63. Assume dead load 1350 lbs. per ft. Take $M = \frac{WL}{8}$. Add 35 per cent to live-load stresses to allow for impact. How many $1\frac{1}{4}$-in. ⌀ bars will be required? Assume $d = 34.5$, $f_s = 16{,}000$ lbs. What width of the flange will be stressed? Make stem 14 ins. wide.

181. Draw diagram of maximum shears covering dead load and trolley load. Assume $\frac{1}{2}$-in. ⌀ U stirrups and find the spacing at the abutment and at 2, 4, 6 and 8 ft. from this end, allowing 60 lbs. per sq. in. shear on the concrete and using the formula of Problem 174.

182. In a given beam $d = 30$ ins., four $1\frac{1}{4}$-in. round reinforcing bars run parallel with the lower flange through the entire span. The maximum reaction is 44,000 lbs. What is the bond stress per square inch and is this satisfactory if the specifications allow 60 lbs. per sq. in.? How could this be provided for?

183. In a beam $d = 34$ ins., three 1-in. ⌀ bars run parallel with the lower flange through the entire span. The maximum reaction is 19,000 lbs. What is the bond stress per square inch and is this satisfactory if the specifications allow a bond stress of 60 lbs. per sq. in.? If not satisfactory, how could it be provided for?

Reinforced-concrete Columns

184. Allowing 600 lbs. per sq. in. on the concrete, what load can be placed on a column 11 ft. high, 12 × 12 ins. and reinforced by four ½-in. ⌀ bars? When a column's height does not exceed twelve times its least dimension the influence of length upon its buckling can be neglected. Assume column dimensions inside bars as 10 × 10 ins.

185. Load 420,000 lbs., reinforcement 1 per cent, height 15 ft. What size square columns allowing 600 lbs. upon concrete will be required? How many 1-in. ⌀ bars will be required? What would you make the outside dimensions of the column?

186. Take the truss in Fig. 64, place a load L at the point indicated, show the necessity for the double lacing. Explain why this is usual in the central panels and frequently not necessary at the end panels.

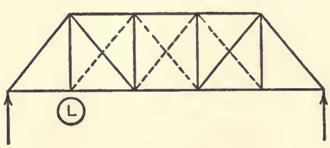


Fig. 64.

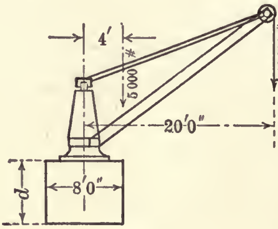


Fig. 65.

187. With Fig. 65, by means of force and equilibrium polygons find the required depth of a concrete foundation 8 ft. square, at 125 lbs. per cu. ft. so that the moment of the foundation about the toe A shall be twice that of the crane weight and live load about the same point. At the bottom of the foundation make a diagram showing the distribution of pressure on the soil.

188. Take the data given and found for Problem 187; turn the crane until the load lies in the line of a diagonal of the foundation. With the resultant pressure upon the soil as found in Problem 187, determine the maximum and minimum pressure on the foundation by the method given in Chapter XVI.

189. Take Fig. 66, and by means of a force and an equilibrium polygon determine the maximum load the locomotive crane can lift in the position given. Boom is 36 ft. long, and weighs 2000 lbs., it is attached to cab 5 ft. from center line of track and is shown at 30 degrees with the horizontal. Gauge of track is 4 ft. 8½ ins. Counterweight is 15,000 lbs. acting at center line of track. Engine, machinery, etc., weigh 18,000 lbs., and act 12 ins. to left of center line of track. Boiler and base weigh 12,000 lbs., and act 7 ft. 6 ins. to left of center line of track. Cab, etc., weigh 16,000 lbs., and act at center line of track.

190. Design a concrete footing, Fig. 67, to carry 200 tons from the column through a shoe 4 ft. square, resting on reinforced concrete. The soil can carry 3 tons per sq. ft. What spacing of $\frac{3}{4}$-in. round bars will be required? Assume

$$f_c = 550; \; f_t = 16{,}000 \text{ lbs.}; \quad \frac{E_s}{E_c} = 15.$$

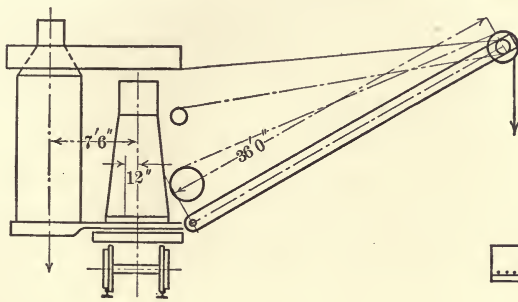


FIG. 66.

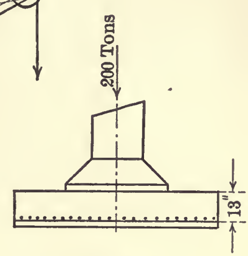


FIG. 67.

What advantages would this type of foundation have over the usual concrete pier? Consult Chapter XV.

191. In Fig. 68 assume sag 60 per cent of width, coal (bituminous) 50 lbs. per cu. ft. Area of surcharged bin $A' = 0.57\ a^2$. Assume that the load on the bin varies from the sides to the middle as the intercepts between

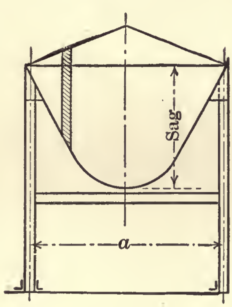


FIG. 68.

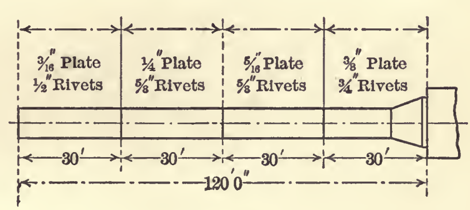


FIG. 69.

the sides of a triangle. Show by means of force and equilibrium polygons how to find the tension in the bottom of the bin plate and also in the sides at the point of suspension from the side girders.

192. A stack, Fig. 69, has outside diameter of 7 ft. and is 120 ft. high. Assume a uniform wind pressure of 30 lbs. per sq. ft. acting on it. The steel

weighs 30,000 lbs. and the concrete base is 20 ft. square. If the concrete weighs 125 lbs. per cu. ft., how deep must the foundation be if the resultant pressure falls within the kern and what is the distribution of pressure on the soil?

193. The 7-ft. stack shown in Fig. 69 is 120 ft. high, is made of $\frac{3}{16}$-in. plate for 30 ft. at the top, and each succeeding 30 ft. toward the foot of the stack is made $\frac{1}{16}$ in. thicker. What is the extreme fiber stress in the net section, 60 ft. from the top, assuming 20 per cent of the plate cut out for rivets? Calculate the pitch of rivets, allowing 8000 lbs. in shear and 16,000 lbs. in bearing and assume rivets $\frac{5}{8}$ in. in diameter.

194. In Fig. 69 investigate the fiber stress and rivet pitch for the seam 80 ft. from the top.

195. A stack is 125 ft. high. Its outside diameter is 7 ft. 6 ins. If 12 bolts are used on a circle whose radius is 65 ins., what must their diameter be? Allowable tension is 12,000 lbs. per sq. in. Weight of stack is 30,000 lbs.

ANSWERS TO PROBLEMS

5. 0.79 in.
6. 2.01 ins.
7. 3.82 ins.
8. 3.86 ins.
14. 13.5.
15. 3.90.
16. 133.4.
17. 14.6.
18. 324.0.
19. 241.1.
20. 0.0069 d^4.
21. 11,340.1.
24. $R = W, M = \dfrac{WL}{3}$ and $M_x = \dfrac{Wx^3}{3L^2}$.
27. $R = \dfrac{W}{2}, M_{\max} = \dfrac{WL}{6}$,
$M_x = \dfrac{Wx}{2} - \dfrac{2}{3}\dfrac{Wx^3}{L^2}$.
28. $R_1 = \dfrac{W}{3}, x_{\max} = 0.578\,L$,
$M_{\max} = 0.129\,WL$.
30. $R_1 = 7000$ lbs.,
$M_{\max} = 78{,}000$ ft. lbs.,
$M_{15} = 75{,}000$ ft. lbs.
32. $M_{\max} = 446{,}490$ ft. lbs.,
$M_{28} = 443{,}600$ ft. lbs.
34. 15-in. I at 45 lbs.
35. 15-in. I at 50 lbs.
36. $\dfrac{I}{e}$ required 193.2 use two 20-in. I beams at 65 lbs.
37. 15-in. I at 42 lbs.
38. 18-in. or 20-in. I at 65 lbs.
39. 20-in. I at 65 lbs.
40. $\dfrac{I}{e} = 123.8$. 20-in. I at 75 lbs.
43. 21,000 lbs.
44. Load 1460 lbs., deflection 0.161 in.
45. Load 19,200 lbs., fiber stress in fixed beam 10,670 lbs. per sq. in., deflection in supported beam 0.534 in., deflection in fixed beam 0.107 in.
46. $2\frac{1}{2}$ ins. diam.
47. 15,500 lbs.
48. 4.92 ins. back to back. $r = 3.10$.
49. $r_0 = 1.91, r_1 = 1.73$.
50. $r_0 = 1.00, r_1 = 2.53$.
51. $r = 2.0, r = 2.5$.
53. 6150, 7600 and 6550 lbs.
54. 11,800, 11,750 and 11,750 lbs.
55. 203,000 lbs.
58. 34,880 lbs.
59. 64,400 lbs.
60. 66,300 lbs.
61. $C = 16{,}740$ lbs.; $P = 19{,}100$ lbs.
63. One 12-in. channel at $20\frac{1}{2}$ lbs. horizontally and on top of one 20-in. I beam at 65 lbs. placed vertically. Lateral stress 5700 lbs. per sq. in.
66. 59.4 ins.
67. 5760 lbs.
68. 6400 lbs.
69. 16 ft. 8 ins.
70. 4740 lbs.; $\triangle = 0.486$ in.
71. 1420 lbs.
72. 10,940 lbs.
73. 8125 lbs.
74. 8450 lbs. compression.
75. 11,660 lbs. compression.
76. 7 rivets.
77. 12 rivets.
78. 10 rivets.
79. 81,000 lbs.
80. 57,600 lbs.

81. 52,500 lbs.
83. 21.8 sq. ins.
84. 24.9 sq. ins.
85. 21.7 sq. ins.
86. Top flange two $4 \times 4 \times \frac{1}{2}$-in. angles, and one plate $22 \times \frac{7}{16}$-in., lower flange two $4 \times 4 \times \frac{3}{8}$-in. angles and one plate $22 \times \frac{5}{16}$-in.
87. 10,990 in. lbs., and 916 lbs.
88. 13,400 in. lbs. and 1117 lbs.
89. The twisting moment of a round shaft may be 1.2 times that on a square shaft of the same weight per foot.
90. $\text{H.P.} = \dfrac{pd^3N}{321,300}$.
91. 21.3°.
92. $2\frac{7}{16}$ ins.
93. $2\frac{3}{8}$ ins.
94. GC = 37,600 lbs. comp., AG = 33,600 lbs. tension.
95. GH = 4300 lbs. comp., DH = 35,460 lbs. comp.
96. HI = 4800 lbs. tension and AI = 28,800 lbs. tension.
97. FL = 31,180 lbs. comp. and LM = 14,400 lbs. tension.
98. BF = 154,660 lbs. comp., FA = 109,375 lbs. tension.
99. FG = 110,470 lbs. tension, GC = 187,500 lbs. compression.
100. GH = 31,250 lbs. comp., $-HI$ = 66,280 lbs. comp.
114. AJ = 166,670 lbs. comp., KJ = 25,000 lbs. tens., IF = 104,100 lbs. tens.
129. 18 rivets.
130. 3, the spacing is commonly not allowed to exceed 4 to 6 ins.
135. 4.6 ins.
136. 4.8 ins.
137. 3.5 ins.
138. 2.28 ins.
139. 2.8 ins.
140. 2.93 ins.
141. 4.38 ins.
142. 14.67 ins.
143. 9.6 ins.
144. 0.27 in.
145. 2 rows.
146. Two plates each $\frac{3}{8}$ in. thick.
147. 2 rows.
148. 6 rivets.
149. Check by method of coefficients.
158. b = 13.5 ins. and A = 1.34 sq. ins.
159. d = 24 ins., b = 9.5 ins., A = 1.14 sq. ins.
161. d = 2.2 ins. and A = 0.166 sq. in.
162. d = 4.9 ins. and A = 0.372 sq. in.
169. b = 20 ins. and A = 4.8 sq. ins.
170. f_c = 555 lbs. per sq. in. and f_s = 12,800 lbs.
171. d = 6.85 ins. Spacing = 6.77 ins.
178. d = 4.65 ins., $\frac{1}{2}$-in. ∅ bars 5.5-in. centers.
179. d = 38.0 ins., five — $\frac{15}{16}$-in. ∅ bars.
180. Eight $1\frac{1}{4}$-in. ∅ bars or nine $1\frac{1}{8}$-in. ∅ bars.
182. 106 lbs. per sq. in.
183. 68 lbs. per sq. in.
184. 66,590 lbs.
185. 25 ins. square and eight 1-in. ∅ bars.

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